Teaching Progressions:
Explanations of the Learning Progressions in the U.S. Common Core State Standards (CCSS) and of the Research-Based Visual Supports in Math Expressions

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These Teaching Progressions focus on the U.S. Common Core State Standards (CCSS) progressions in learning within and across grades and how my research-based PK to Grade 6 math program *Math Expressions* supports these learning progressions. The Teaching Progressions can be used by anyone (district leaders, teachers, consultants) who is helping people understand the learning progressions in the CCSS and the visual supports that can help students learn. These Teaching Progressions are also appropriate for the several states and other countries who are using high-quality standards that differ from the CCSS because the research underlying these Teaching Progressions is appropriate for all high-quality Prekindergarten to Grade 6 standards. The Teaching Progressions can be used in parts as described below. Uses can range from grade-level focused meetings to overviews across grades for all teachers. They can be used in a succession of meetings over time or half-day or full-day or multiple-day meetings. Teachers can watch them before or after such meetings and go at their own pace to permit reflection as needed.

The Common Core State Standards (CCSS) do not have standards for Prekindergarten (PK). But a National Research Council Report *Mathematics learning in early childhood: Paths toward excellence and equity* (National Research Council, 2009) summarized research about mathematics teaching and learning in the prekindergarten years, from age 2 to age 8. This report identified teaching-learning paths of foundational and achievable goals for the two crucial math domains it identified for this age group: a) Number, Relations, and Operations and b) Geometry, Spatial Thinking, and Measurement, with more mathematics learning time devoted to number than to other topics. These recommendations were a major basis for the Common Core State Standards in K, 1, and 2. The PK Teaching Progression on this website outlines research-based learning activities that allow PK children to move easily into K and be prepared to master K math goals. I briefly discuss how K, 1, 2 and later grades build on these PK goals.

These Teaching Progressions show some of the CCSS 2013 *Math Expressions* pages, but the content applies to all previous books and to the 2018 *Math Expressions* books. Other programs may use different visual supports, but the attributes of the visual supports and the learning paths will be similar across all programs that implement the CCSS or any other set of standards for PK to Grade 6 because these are all related. A useful professional development activity is to discuss similarities and differences in visual supports shown in the Teaching Progressions and used in a different math program. The visual supports shown in the Teaching Progressions were used in classrooms in research supported by the National Science Foundation and so anyone can use these supports. The Teaching Progressions were created by Dr. Karen Fuson with Pam Richards and have narration for each slide by Dr. Karen Fuson. There are 22 hours of these Teaching Progressions explaining learning and teaching paths from Kindergarten through Grade 6 in all of the math domains in the Common Core State Standards and another hour for the Prekindergarten Teaching Progression.

Teaching Progressions are described below the list.

On the website click on Teaching Progressions at the top of the Home page.

The Math Talk Community: Parts 1 and 2 [22 + 14 minutes]

Overview of the Common Core State Standards (CCSS) and *Math Expressions*: Parts 1 and 2 [14 + 30 minutes]

Math Expressions and Operations and Algebraic Thinking (OA): Parts 1, 2, 3, and 4 [60, 60, 20, 20 minutes]

Math Expressions and Number and Operations in Base Ten (NBT): Parts 1, 2, 3, 4, 5, 6, 7 [53, 75, 62, 27, 23, 49, 27 minutes]

Math Expressions and Number and Operations—Fractions (NF): Parts 1, 2, 3, and 4 [24, 32, 44, 33 minutes]

Math Expressions and Ratios and Proportional Relationships (RP): Parts 1 and 2 [64 + 48 minutes]

Math Expressions and Measurement and Data (MD): Parts 1, 2, 3, 4, 5, 6, 7, 8 [28, 32, 48, 22, 31, 50, 27, 44 minutes]

Math Expressions and Geometry (G): Parts 1, 2, 3, 4, 5 [67, 58, 44, 24, 44 minutes]

Math Expressions and Prekindergarten*:* Preparing All Children for Kindergarten Success [73 minutes]

The Math Talk Community
Parts 1 and 2
Both parts contain video segments showing Math Talk.

MTC Part 1 The Math Talk Community overviews what a Math Talk Community is and why is it central to *Math Expressions*, to the Common Core State Standards, and to any high-quality standards. The Solve and Discuss classroom structure is explained, and the four components of the Math Talk Community are described. How the teacher and students assist learning by all is summarized, and how the CCSS 8 Mathematical Practices are used in the Math Talk Community is overviewed. Five short videos of Math Talk are shown near the beginning. This part takes about 22 minutes with the videos.

MTC Part 2 Getting Started with Math Talk draws on tips from experienced *Math Expressions* teachers. This part goes more deeply into details of how to build and lead a Math Talk Community. Three short videos of Learning Math Talk are shown near the beginning. This part takes about 14 minutes with the videos.

Overview of the Common Core State Standards (CCSS) and *Math Expressions*
Parts 1 and 2

Overview Part 1 is a general overview of what the Common Core State Standards are and why they were needed. Some of their central features are identified, and how these Common Core State Standards relate to *Math Expressions* is summarized. This part could be used with parents or teachers or administrators. Viewers should watch Part 1 first. This part takes 14 minutes.

Overview Part 2 provides a quick overview of each of the CCSS math domains and how *Math Expressions* supports each domain. Teachers and administrators could watch all of these in succession or focus on particular domains. More in-depth looks at the domains are provided in other Teaching Progressions described below.
Slide 1 is a cover page and slide 2 lists the CCSS math domains. [37 seconds; starts at 0:00]
Slides 3-10: OA: Operations and Algebraic Thinking [9 minutes: starts at 0:37]
Slides 11-18: NBT: Number and Operations in Base Ten [10 minutes; starts at 9:31]
Slides 19-24: NF: Number and Operations– Fractions [6 minutes; starts at 19:10]
Slides 25-31: MD: Measurement and Data and G: Geometry [4 minutes; starts at 25:14]
Slide 32 gives the website where future information will appear. [16 seconds; starts at 29:40]
Part 2 takes 30 minutes.

Math Expressions and Operations and Algebraic Thinking (OA) in the Common Core State Standards: Parts 1, 2, 3, and 4

OA Part 1 Problem Situations and Problem Solving
OA Part 2 The Grades K to 2 Learning Paths for Addition and Subtraction
OA Part 3 The Grade 3 Learning Path for Multiplication and Division
OA Part 4 Properties and Advanced Problem Solving

OA Part 1 Problem Situations and Problem Solving describes the range of problem situations for different operations and how problem solving varies across these problem situations. The OA Common Core State Standards identify situational meanings of operations for addition/subtraction and for multiplication/division. Three situations are specified for each pair of these operations. Within a given situation, any of the three quantities can be the unknown. Students make math drawings to show a problem situation. They also learn the research-based diagrams used in *Math Expressions* to support problem solving. These diagrams allow students to generalize situational understandings to larger numbers, fractions, decimals and establish relationships across all of these situations and quantities. Part 1 describes the OA learning path for problem solving from Kindergarten through Grade 5 for single-step problems using any kinds of quantities. All teachers should see this part to understand problem solving in OA and in *Math Expressions*. This part takes about an hour because of the complexities involved in the problem types.

OA Part 2 The Grades K to 2 Learning Paths for Addition and Subtraction focuses on the Kindergarten, Grade 1, and Grade 2 learning paths for single-digit addition and subtraction calculations. Each operation has levels of increasing abbreviation, abstraction, and internalization reflected in three levels of solution methods specified in the OA standards for Kindergarten, Grade 1, and Grade 2. These levels come from research done all over the world and are used by children everywhere.
Slides 1-6: Overview the levels in the learning path [11 minutes; starts at 0:00]

Slides 7-14: Kindergarten activities and learning supports for the key Kindergarten CC and OA standards [20 minutes; starts at 11:12]
Slides 15-28: Grade 1 activities and learning supports for the key Grade 1 OA standards [25 minutes; starts at 31:17]
Slides 28-29: Grade 2 activities for fluently adding and subtracting within 20 [3 minutes; starts at 55:16]
Slides 30-31: Wrapping up Part 2 [2 minutes; starts at 57:56]
Part 2 takes about an hour. Kindergarten and Grade 1 teachers should watch all of it.
Grade 2 teachers should watch at least from slide 15 on to understand the Level 2 and Level 3 solution methods that will be brought to fluency in Grade 2.

OA Part 3 The Grade 3 Learning Path for Multiplication and Division focuses on the learning path in Grade 3 for single-digit multiplication and division computation. This learning path begins in Grade 3, and students are to come to fluency by the end of Grade 3. The *Math Expressions* supports for this learning path and for reaching fluency are summarized.
Part 3 takes about 20 minutes.
Grade 3 teachers should watch this Part 3, and it would be helpful also for Grade 4 teachers.

OA Part 4 Properties and Advanced Problem Solving focuses on properties and advanced problem solving in OA. How the commutative, associative, and distributive properties help students with single-digit computations and with problem solving are discussed. Different meanings of the equal sign for different problem situations are described, and the support within *Math Expressions* for different equations forms is overviewed. Representing and solving two-step and multistep problems from Grades 2 to 4 is overviewed, as is the small CCSS focus on patterns and expressions in Grades 3 to 5. Some aspects of Part 4 are helpful to teachers in each grade. Part 4 takes about 20 minutes.

Math Expressions and Number and Operations in Base Ten (NBT) in the Common Core State Standards: Parts 1, 2, 3, 4, 5, 6, 7

NBT Part 1: Overview of the Numbers and Operations in the Base Ten Domain (NBT)
NBT Part 2: Place Value and Multidigit Addition and Subtraction in Grades K to 4
NBT Part 3: Place Value and Multidigit Multiplication and Division in Grades 3 to 6
NBT Part 4: Place Value and Decimal Operations in Grade 4
NBT Part 5: Place Value and Decimal Meanings, Addition/Subtraction, and Rounding in Grade 5
NBT Part 6: Place Value and Decimal Multiplication/Division in Grade 5
NBT Part 7: Place Value and Decimal Operations in Grade 6

NBT Part 1: Overview of the Numbers and Operations in the Base Ten Domain (NBT) overviews the *Math Expressions* supports for place value and the approaches within the Common Core State Standards and *Math Expressions* to multidigit addition, subtraction, multiplication, and division. Math drawings that support students to understand and explain written methods are summarized, and acceptable variations in writing the standard algorithm for fluency are overviewed. Teachers at all grade levels should watch this overview. [53 minutes]
Slides 1-6: Introduction [3 minutes; starts at 0:00]
Slides 7-17: Overview of Place Value Concepts and Supports [10 minutes; starts at 2:38]
Slides 18-21: Math Talk Community and Math Practices [3 minutes; starts at 12:33]

Slides 22-28: Meaningful Development of Standard Algorithms in the CCSS [9 minutes; starts at 15:53]
Slides 29-41: Overview of *Math Expressions* Multidigit Addition and Subtraction Methods [12 minutes; starts at 24:20]
Slides 42-46: Overview of *Math Expressions* Multidigit Multiplication and Division Methods [10 minutes: starts at 36:25]
Slides 47-50: Summary [6 minutes; starts at 46:51]

NBT Part 2: Place Value and Multidigit Addition and Subtraction in Grades K to 4 focuses on place value and methods of multidigit addition and subtraction in Grades K to Grade 4. The whole learning path of visual supports for understanding place value is discussed. Math drawings and accessible methods of adding and subtracting are related and explained. Teachers from K to Grade 4 should watch and discuss certain parts and would benefit from all parts. [75 minutes]
Slides 1-7: Overview of NBT K, G1, G2, G3, G4 teachers watch these [7 minutes; starts at 0:00]
Slides 8 to 32: Kindergarten K and G1 teachers watch these [15 minutes; starts at 6:23]
[8-18: Count to 100 place value, daily routines, quick practice; 19-32: Tens in teen numbers]
Slides 33-59: Grade 1 K, G1, and G2 teachers watch these [21 minutes; starts at 22:51]
[33-40: G1 Count to 120 place value, daily routines, quick practice;
41-59: G1 adding and comparing within 100; starts at 28:33]
Slides 60-80: Grade 2 G1, G2, G3, and G4 teachers watch these [22 minutes; starts at 44:13]
[60-71: G2 place value and adding within 1000; 72-80: G2 subtracting within 1000]
Slides 81-90: Grades 3 and 4 G2, G3, and G4 teachers watch these [8 minutes; starts at 1:05:53] [81-83: G3; 84-90: G4]
Slides 91 and 92: Point of Use Teaching Background All teachers read this [starts at 1:14:06]

NBT Part 3: Place Value and Multidigit Multiplication and Division in Grades 3 to 6 focuses on place value and methods of multidigit multiplication and division in Grades 3 to 6. Math drawings and accessible methods of multiplying and dividing are related and explained. Teachers from K to Grade 4 should watch and discuss certain parts and would benefit from all parts. [62 minutes]
Slides 1-3: Overview of Part 3 K, G1, G2, G3, G4 teachers watch these [1 minute; starts at 0:00]
Slide 4: Grade 3 G3, G4, and G5 teachers watch this [2 minutes; starts at 1:16]
Slides 5-38: Grade 4 G3, G4, and G5 teachers watch these [42 minutes; starts at 3:03]
Slides 39-54: Grade 5 G4, G5, and G6 teachers watch these [15 minutes; starts at 45:18]
Slide 55: Grade 6 G4, G5, and G6 teachers watch this [1 minute; starts at 59:51]
Slides 56 and 57: Point of Use Teaching Background All teachers read this [1 minute; starts at 1:00:30]

NBT Part 4: Place Value and Decimal Operations in Grade 4 focuses on how Grade 4 students build meanings for decimals as a place value notation for fractions with denominators of 10 and 100. They use concepts of money to understand, read, and write tenths and hundredths as dimes and pennies with the support of secret-code cards that layer to show decimal values. Students relate fraction and decimal notation on number-line diagrams. They use all of these visual supports to reason about the size of decimal values when comparing them, and practice

and discuss the decimal values in order to overcome the common error of looking at decimal numbers as whole numbers and not considering their place values (e.g., overcoming the error 0.07 > 0.3 because 7 > 3). The *Math Expressions* place value poster helps them consider and discuss the symmetry around the ones place instead of around the decimal point and draw conclusions about where they can write a 0 in a number without changing the value (before a whole number and after a decimal number). [26:47 minutes]

NBT Part 5: Place Value and Decimal Meanings, Addition/Subtraction, and Rounding in Grade 5 focuses on how Grade 5 students extend and deepen connections between fraction and decimal concepts and notations including thousandths using money on secret-code cards and metric lengths. They extend their methods for adding and subtracting whole numbers to adding and subtracting decimal numbers by relating money values to a written method and explaining the method using place value and properties of operations. They develop, discuss, and use efficient, accurate, and generalizable methods by generalizing the place value relationships in both directions. They extend rounding to decimal numbers and use a rounding frame of writing the numbers to the nearest tenth or hundredth or thousandth just below and just above the number to see easily which is closer. [23:14 minutes]

NBT Part 6: Place Value and Decimal Multiplication/Division in Grade 5 focuses on how Grade 5 students extend their methods for multiplying and dividing whole numbers to multiplying and dividing decimal numbers by relating money values to a written method and explaining the method using place value and properties of operations. They examine and generalize patterns in shift rules that tell in what direction and how many places a decimal or whole number factor or dividend moves when multiplied or divided by 10, 100, and 1000 and by 0.1, 0.01, and 0.001. They develop, discuss, and use efficient, accurate, and generalizable methods by using the shift rules and creating and justifying patterns about the number of decimal places in the answer. Students express patterns using expanded and exponential notation. They discuss how multiplication as scaling extends from fractions to decimal numbers and explain why multiplying by a decimal less than 1 results in a product smaller than the given number. And they practice identifying word problem situations as multiplying or dividing situations even though the results of multiplying by a decimal number less than 1 is opposite to the result for whole numbers (multiplying results in a smaller number and dividing results in a larger number). [48:33 minutes]

NBT Part 7: Place Value and Decimal Operations in Grade 6 focuses on how Grade 6 students review Grade 5 shift rules and patterns in decimal operations by using the *Math Expressions* poster showing money for each place value. Students overview all operations with fractions to relate them to the new general division of fractions and become fluent with operations on fractions before extending these operations in Grade 7 (see Fractions NF Part 6 for more details). Students also relate operations with decimals to operations with fractions to understand how the meanings are alike (e.g., you add like units) but the notations create different methods (you find a new equivalent unit in different ways). This enriches and deepens student understanding of number systems and helps students fluently add, subtract, multiply, and divide with decimals. [26:37 minutes]

Math Expressions and Number and Operations—Fractions (NF) in the Common Core State Standards: Parts 1, 2, 3, and 4

NF Part 1: The Grade 3 Learning Path for Fractions
NF Part 2: The Grade 4 Learning Path for Fractions
NF Part 3: The Grade 5 Learning Path for Fractions Everything Except Division
NF Part 4: The Grade 5 and Grade 6 Learning Paths for Fraction Division

NF overviews the *Math Expressions* supports for unit fraction understanding and the approaches within the Common Core State Standards and *Math Expressions* to understanding and explaining composing/decomposing, comparing, adding, subtracting, multiplying, and dividing fractions. Math drawings that support students to understand and explain are shown. Research has identified many errors commonly made by students with fractions, many of which come from wrong generalizations from operations with whole numbers. These errors are reviewed, and approaches within *Math Expressions* to prevent or overcome these errors are discussed. Teachers from Grade 3 to Grade 6 should watch and discuss their grade and any grades earlier than their grade. They would also benefit from parts after their grade to see where their students will go. [133 minutes]

NF Part 1: The Grade 3 Learning Path for Fractions overviews Grade 3 concepts of fractions as numbers composed of unit fractions 1/d, finding simple equivalent fractions, and comparing for fractions with like numerators or like denominators. [23:45 minutes]

NF Part 2: The Grade 4 Learning Path for Fractions overviews general fraction equivalence and general comparing fractions, adding and subtracting like denominators, and whole number times a fraction. [32:03 minutes]

NF Part 3: The Grade 5 Learning Path for Fractions Everything Except Division overviews general addition and subtraction (find equivalent fractions if needed), general multiplication of fractions, and multiplication as scaling (resizing). [44:19 minutes]

NF Part 4: The Grade 5 and Grade 6 Learning Path for Fraction Division overviews Grade 5 division of whole numbers results in a fraction, Grade 5 division with unit fractions, and Grade 6 general division of fractions. [33:16 minutes]

Math Expressions and Ratios and Proportional Relationships (RP) in the Common Core State Standards

RP Part 1 Unit 1 Grade 6 overviews the approaches within the Common Core State Standards and *Math Expressions* to understanding and explaining ratio and proportion problem solving. The CCSS specify a relatively new approach extending from multiplication and division and the multiplication table summarized in the Grade 6 Critical Area 1: "Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates." This approach draws on research of various kinds including research in the Children's Math Worlds Project that was the basis for *Math Expressions*. [64 minutes]

RP Part 2 Unit 7 Grade 6 helps students relate and contrast ratios and fractions concepts and notations. Students expand their visual supports and proportion solution methods and use these flexibly to represent and solve more difficult ratio situations including proportion word problems, percentage problems, and converting measurement units. [48 minutes]

Both parts are primarily for Grade 6 teachers, but Grade 5 teachers would benefit from understanding this approach so that they can see where their students are headed.

Math Expressions and Measurement and Data (MD) in the Common Core State Standards: Parts 1, 2, 3, 4, 5, 6, 7

MD Part 1: Geometric Measurement Kindergarten to Grade 2: Length
MD Part 2: Geometric Measurement Grade 3 and Grade 4: Length and Area
MD Part 3: Geometric Measurement Grade 5 and Grade 6: Area
MD Part 4: Geometric Measurement Grade 6: Surface Area
MD Part 5: Geometric Measurement Grade 5 and Grade 6: Volume
MD Part 6: Other Measurement Grade 1 to Grade 5: Money and Time
MD Part 7: Other Measurement Grade 3 to Grade 5: Liquid Capacity and Mass/Weight
MD Part 8: Represent and Interpret Data Kindergarten to Grade 5

The MD standards have three major aspects: geometric measurement (Parts 1 to 5), other measures (Parts 6 and 7), and represent and interpret data (Part 8).
Geometric measurement is a lovely progression from K to G2 length to G3 and G4 length and area to G5 and G6 area to G6 surface area to G5 and G6 volume. Length units build up square units and cubic units. The angle measure concepts in Grade 4 are discussed in the Geometry Teaching Progression.
Other measures include the familiar measures money and time (Part 6) and metric and customary liquid volume and mass/weight (Part 7).
Represent and interpret data K to G5 moves from K and G1 children classifying things into categories and counting and comparing things to G2 and G3 focusing on picture and bar graphs using length units (G2) and then scale multiples (G3) to G2 to G5 discussing line plots that use length units, first whole units and then fractions.
Research has identified many errors commonly made by students. These errors are reviewed, and approaches within *Math Expressions* to prevent or overcome these errors are discussed.
Teachers from Kindergarten to Grade 6 should watch and discuss their grade and any grades earlier than their grade. They would also benefit from parts after their grade to see where their students will go. [4 hours 42 minutes]

MD Part 1: Geometric Measurement Kindergarten to Grade 2: Length overviews how *Math Expressions* supports the Common Core State Standards learning path on length from Kindergarten to Grade 2. Seeing and counting length units and comparing lengths are the major emphases. All of these are foundational for all kinds of comparison problems. Understanding length units is necessary for the work on area and volume in the following grades. [27:59 minutes]

MD Part 2: Geometric Measurement Grade 3 and Grade 4: Length and Area overviews how *Math Expressions* supports the Common Core State Standards on length and area in Grade 3

as area is developed as filling with square units and related to multiplication and differentiated from the length units used in perimeter. In Grade 4 these ideas are consolidated into formulas for area and perimeter developed and used by students. [31:56 minutes]

MD Part 3: Geometric Measurement Grade 5 and Grade 6: Area overviews how *Math Expressions* supports the Common Core State Standards on area in Grades 5 and 6. Grade 5 students multiply fractional side lengths to find areas of rectangles. Grade 6 students engage in a fascinating learning path to find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. They apply these techniques in the context of solving real-world and mathematical problems. [48:24 minutes]

MD Part 4: Geometric Measurement Grade 6: Surface Area overviews how *Math Expressions* supports the Common Core State Standards in Grade 6 to represent 3D figures using 2D nets made up of rectangles and triangles and use the nets to find the 2D surface area of these 3D figures. Students apply these techniques in the context of solving real-world and mathematical problems. Students draw on the knowledge they developed about the area of various shapes to find surface area. [22:15 minutes]

MD Part 5: Geometric Measurement Grade 5 and Grade 6: Volume overviews how *Math Expressions* supports the Common Core State Standards in Grade 5 to find the volume of a right rectangular prism with whole number edge lengths by packing it with unit cubes of the appropriate edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Students understand and use the formulas V = l w h and V = b h to find volumes of right rectangular prisms in the context of solving real-world and mathematical problems. Grade 6 students extend these ideas to right rectangular prisms with fractional edge lengths. [31:06 minutes]

MD Part 6: Other Measurement Grade 1 to Grade 5: Money and Time overviews how *Math Expressions* supports the Common Core State Standards on money and time. Money is used to support base-ten concepts in Kindergarten and Grade 1, developed intensively in Grade 2, and used in Grade 4 to support decimal and fraction ideas. Time is developed in Grades 1, 2, and 3, moving from hours and half-hours in Grade 1 to time to 5 minutes in Grade 2 to time to minutes and intervals of time in Grade 3. The many difficulties with concepts of time and the tools used to tell time are discussed and ways to overcome these difficulties are described. [50.09 minutes]

MD Part 7: Other Measurement Grade 3 to Grade 5: Liquid Capacity and Mass/Weight overviews how *Math Expressions* supports the Common Core State Standards on the metric and the customary systems of measure for liquid capacity and mass/weight. In Grade 3 the major units for liquid capacity and mass/weight are introduced. In Grade 4 students focus on all of the metric base-ten relationships, on the meanings of a few important units in the metric measures, and on the main customary measures of liquid capacity and weight. In Grade 5 students can consider the lovely relationships across the systems for which we need the idea of a cube and volume, which are Grade 5 topics. [27:02 minutes]

MD Part 8: Represent and Interpret Data Kindergarten to Grade 5 overviews how *Math Expressions* supports the Common Core State Standards on data. Kindergarten and Grade 1 children classify things into categories and count and compare things. Grade 2 and Grade 3

students focus on picture and bar graphs using length units (G2) and then scale multiples (G3). Students in Grades 2 through Grade 5 discuss line plots, which use length units, first whole number unit lengths and then fractional unit lengths. [43:59 minutes]

Math Expressions and Geometry (G) in the Common Core State Standards

Geometry Part 1: K and G1 Identify, Describe, and Compose Shapes
Geometry Part 2: G2 and G3 Attributes, Compose, Partition into Equal Shares
Geometry Part 3: G4 and 4.MD 5, 6, 7 Shapes and Angles
Geometry Part 4: G5 Classify 2D Shapes in a Hierarchy and Graph Points in the First Quadrant of the Coordinate Plane
Geometry Part 5: G6 Graph Points in All Four Quadrants of the Coordinate Plane and Number Systems 5, 6, 7, 8: Negative Numbers and Graphing

There is a great deal of geometry in the Measurement Data MD standards. The first five parts of the MD Teaching Progression described above discuss Geometric Measurement from Kindergarten through Grade 6. Geometric measurement is a lovely progression from K to G2 length to G3 and G4 length and area to G5 and G6 area to G6 surface area to G5 and G6 volume. Length units build up square units and cubic units. The angle measure concepts in Grade 4 are discussed here in Part 3 of the Geometry Teaching Progression. The Geometry standards focus on four major aspects. In Kindergarten through Grade 2 children focus on analyzing, naming, describing attributes of, and composing/decomposing shapes. Students in Grades 3, 4, and 5 build on this knowledge as they classify subcategories in Grade 3, classify using properties in Grade 4, and classify in a hierarchy in Grade 5. Students in Grades 1, 2, and 3 partition a shape into equal shares/parts. Finally students graph on the coordinate plane, in the first positive quadrant in Grade 5 and in all four quadrants in Grade 6. [3 hours 57 minutes]

Geometry Part 1: K and G1: Identify, Describe, and Compose Shapes Children work with a few examples of shapes to identify, describe, and compose them. They then see many examples of a shape and see shapes in different orientations to generalize that shape. They also see non-examples of a shape to understand that shape. Students need considerable experience with square units and the related rectangles, right triangles, and isosceles triangles made from these units. These are units and visual subunits for MD Geometric Measurement and are crucial in later mathematics. Children enjoy composing with these shapes. [67 minutes]

Geometry Part 2: G2 and G3: Attributes, Compose, Partition into Equal Shares
Students continue to have considerable experience with square units and the related rectangles, right triangles, and isosceles triangles made from these units. They also discuss, recognize, and draw 2D shapes with 3, 4, 5, and 6 sides/angles, and draw squares inside rectangles to partition rectangles into rows and columns, leading into area. Work partitioning into equal shares supports understanding of unit fractions. Students begin to classify shapes in subcategories. [53 minutes]

Geometry Part 3: G4 and 4.MD 5, 6, 7: Shapes and Angles The Grade 4 MD standards 5, 6, 7 are about measuring angles. This relates to the focus in the Grade 4 geometry standards on angles in shapes, so those three MD standards about angles are discussed here. Students measure angles with a protractor, and add and subtract angles that share a common ray. They draw points,

lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines and identify these in two-dimensional figures. They identify and draw lines of symmetry. [44 minutes]

Geometry Part 4: G5 Classify 2D Shapes in a Hierarchy and Graph Points in the First Quadrant of the Coordinate Plane Students classify shapes into a hierarchy based on their properties. They see and discuss different ways to show hierarchies. They extend the kinds of shapes they discuss so that they can discuss all major classes of 2D shapes using many properties. Students use a horizontal and a vertical number line with positive numbers to define a coordinate plane on which they can place ordered pairs of numbers. They relate graphs to horizontal and vertical tables of pairs of numbers they can graph on the coordinate plane. [24 minutes]

Geometry Part 5: G6 Graph Points in All Four Quadrants of the Coordinate Plane and Number Systems 5, 6, 7, 8: Negative Numbers and Graphing Students develop understandings of count and measure situations for and meanings of negative numbers. They order positive and negative numbers and absolute values of positive and negative numbers. They use a horizontal and a vertical number line with positive and negative rational numbers (a/b where $b \neq 0$) to define a coordinate plane on which they can place ordered pairs of numbers in all four quadrants. They graph points and polygons and find distances between points with the same first coordinate or the same second coordinate. [44 minutes]

Math Expressions and Prekindergarten (PK): Preparing All Children for Kindergarten Success

The Common Core State Standards (CCSS) do not have standards for Prekindergarten (PK). But a National Research Council Report *Mathematics learning in early childhood: Paths toward excellence and equity* (National Research Council, 2009) summarized research about mathematics teaching and learning in the prekindergarten years, from age 2 to age 8. This report identified teaching-learning paths of foundational and achievable goals for the two crucial math domains it identified for this age group: a) Number, Relations, and Operations and b) Geometry, Spatial Thinking, and Measurement, with more mathematics learning time devoted to number than to other topics. These recommendations were a major basis for the Common Core State Standards in K, 1, and 2. In this PK Teaching Progression I outline a learning path of research-based learning activities that allow PK children to learn the National Research Council's recommended foundational and achievable goals and move easily into Kindergarten and be prepared to master Kindergarten math goals. I briefly discuss how K, 1, 2 and later grades build on these PK goals. [73 minutes]