Classroom Videos

from

The Children's Math Worlds Research Project:

Building Classrooms That Create

Mathematical Understanding, Competence, and Confidence

Background of the Classroom Videos

The National Science Foundation funded A Video Research Report of the Children's Math Worlds Research Project to show the levels of learning possible by children from backgrounds of poverty who experience good teaching. The classroom videos were taken in four public schools, three high-poverty urban schools with some to most students speaking a native language other than English and a suburban school with immigrants from many countries. The classrooms you will see were built by the continuing efforts of many dedicated teachers, students, parents, school administrators, and members of the Children's Math Worlds Research Project team. In many classrooms children are wearing what look like uniforms because those schools have rules about what clothing can be worn to school in order to combat gang influence. Our examples show teachers and students who are in various places along the path to a collaborative classroom with high levels of student explanations of mathematical thinking.

In 1993 the Children's Math Worlds Research Project began to work in English-speaking and in Spanish-speaking urban classrooms. We expanded to suburban classrooms, and in a ten-year period worked in kindergarten through grade 5 in 5 states with 200 teachers and 4000 students. We developed teaching and learning materials that built on and extended student understandings. We worked collaboratively with teachers to develop an ambitious curriculum that can support high levels of mathematical understanding by all students. We continually revised the curriculum year after year based on what we learned in classrooms and from teachers and students. Language and ideas of students, teachers, and parents were woven into the curriculum as it was revised each year.

The project had a strong emphasis on children explaining their thinking and on using math drawings developed in the project to support student thinking and explaining. None of our teachers had learned math in this way, so they were all brave pioneers in learning how to create such classrooms. All classrooms are complicated places, and no teaching segment can be perfect. We can all have ideas about how to change things the next time. I am enormously proud of and grateful to these teachers for being willing to share their classrooms so that everyone can see what students can do if we support them in their thinking.

The **Teaching Progressions** on this website show in more detail the learning progressions students experienced to reach the points shown on these Classroom Videos. The Kindergarten through Grade 5 Children's Math Worlds curriculum and materials are available from Houghton Mifflin Harcourt under the name *Math Expressions*.

The creative, tireless, and dedicated members of the Children's Math Worlds Research Project teams through the years included many Northwestern students who worked to make student

pages in Word and to translate them into Spanish. Many research team members helped to write lessons and student pages, translated pages into Spanish, and/or worked in the classrooms gathering data about student and teacher learning or worked with teachers about our teaching approaches. These include Laura Grandau, Steven T. Smith, Janet Fleetwood, Ana Maria Lo Cicero, Pilar Ron, Yolanda De La Cruz, Kim Hufferd-Ackles, Kay Hughes, Kristen Hudson, Andrew Izsák, Peggy Tinzmann, Cathy Feingold, and Rebecca Steeby. The efforts of all team members are manifest in the learning of the teachers and students in this video research presentation. Funding for the Children's Math Worlds Project was provided by the National Science Foundation, the Office of Educational Research and Improvement, the Spencer Foundation, the Annenberg Initiative, Houghton Mifflin, the Dwight D. Eisenhower Professional Development Program, and the McDonnell Foundation. Co-principal investigators on some of these grants were Yolanda De La Cruz, Miriam Sherin, Bruce Sherin, Jim Spillane, Tony Bryk, and Raul Zaritsky. The filming of the Classroom Videos was by Kartemquin, Chicago, Illinois, directed by James A. Morrissette, DP, the producer of Hoop Dreams, with Raul Zaritsky, Second Camera. Production was by Karen C. Fuson & Raul Zaritsky. Designing, editing, programming, and DVD production was by Raul Zaritsky with Brian Uphoff.

Central Aspects of the Research Results

The Children's Math Worlds curriculum continually links to real-world situations through the word problems in the program and those generated by students. We developed some inexpensive manipulatives to facilitate sense-making by students and later in the project developed large dry-erase MathBoards with conceptual supports to enable students to build sense-making rapidly for math drawings that they can draw and share with the rest of the class when they solve and explain their thinking. These math drawings (or "proof pictures") are central in our project. They enable students to link meanings to formal solution methods and notations, providing a bridge from concrete to more abstract thinking. After some time, students do not make drawings but they can still use mental visual images from the drawings to explain and make sense of their formal numerical methods. We always begin a new math topic with students making their own math drawings. Then we introduce the kinds of drawings that we have found to be powerful in our classroom research. For multidigit numbers these are drawings of hundreds, tens, and ones. For word problems these are math tools that show the structure of the real-world situation given in the word problem. For multidigit multiplication the drawings are area models of rectangles, and for fractions they are length models using unit fractions. These central meaning-making aspects of the project are shown and discussed in the various parts of the **Classroom Videos** and can also be seen in the **Teaching Progressions** for specific math domains in the Common Core State Standards.

You can see in the Classroom Videos how students and teachers lead attention across and connect the modalities involved in learning mathematical concepts: words that are said, symbols that are written, and visual images that are drawn or built from things. Much of the classroom research involved finding powerful visual images that direct attention to the major attributes of the mathematical concepts and do not mislead. Similar efforts were directed toward accessible and mathematically-desirable ways to record written methods with symbols. And students contributed their own meaningful language, which we use at certain points in learning (such as calling division *unmultiplying*). These research results can be found in the many project papers in the website **Publications** section. Here you can see them in action with children and teachers.

Overview of the Organization of the Classroom Videos

Here are brief descriptions of the ten major groups of videos.
More details about the parts within each group are given after this overview.

Overviews of Our Major Results

A Classroom Components describes and shows many examples of aspects of lessons that we found to be crucial for understanding and moving to fluency: Building Concepts, Math Talk, Student Leaders, Quick Practice, and Helping Community. We present the five components in separate sections, but they interact synergistically in the classroom. We have found that implementing these components enables children from all backgrounds to learn mathematics. [40 minutes; 7 parts]

B Math Explanations shows examples from many different classrooms interspersed with explanations by me of main aspects of the research-based Children's Math Worlds approaches to each topic. It summarizes some major results in finding accessible computational methods for all students. These accessible methods are more easily understood by students than are many of the more complex methods commonly taught. These "accessible algorithms" can be used with any curriculum, as can the math drawings used by our students and shown in the classroom components. [45 minutes; 6 parts]

C Longer Classroom Teaching Examples show students making drawings for and then explaining each of the Children's Math Worlds approaches described in **B Math Explanations.** [51 minutes; 5 parts]

Grade-Level Videos

D Kindergarten shows children using special meaning-making visual supports from the Children's Math Worlds Project: objects grouped in ten, five, and singles; secret-code cards in which the ones number goes on the 0 in 10; fingers using groups of ten, five, and singles. [20 minutes; 8 parts]

E G1 Single-Digit Addition and Subtraction shows first graders finding and discussing partners of 9 and using the Children's Math Worlds progression of methods and math drawings to represent and solve word problems and numeral problems with the total a teen number. [31 minutes; 5 parts]

F G3 Single-Digit Multiplication and Division shows third graders finding, explaining, and using patterns of 4 and of 9 to multiply and divide. They also write, share, and discuss word problems for three types of division situations. We also see fourth graders use comparison bars to represent and solve additive and multiplicative comparison problems. [57 minutes; 6 parts]

G Place Value and Multidigit Addition and Subtraction shows first and second graders using dime, nickel, and penny strips to practice counting to 2-digit numbers and second graders comparing numbers and finding all eight related equations for 25 + 75 = 100. Second graders explain the Children's Math Worlds methods for adding 2-digit and 3-digit numbers and for subtracting 3-digit numbers and relate the written methods to math drawings. Fourth graders explain three different written subtraction methods for 7-digit numbers. [32 minutes; 8 parts]

H G3 & G5 Fractions and Ratios shows third and fifth graders finding and explaining equivalent fractions in different ways: using rows of the multiplication table to show many equivalent fractions and drawing fraction bars and/or fraction number lines for a pair of equivalent fractions. Fifth graders practice a unit fraction times a whole number and solve puzzles made from two intersecting rows and columns of a multiplication table. [19:40 minutes; 4 parts]

Interviews with Classroom Teachers

I Teacher Interviews Grade 1 shows Grade 1 teachers discussing innovative and important aspects of the teaching they are doing for the Children's Math Worlds Project and how they build their classrooms. Parts 1, 4, and 6 are classes in which all children are English language learners (Part 6 is a combined Grade 1 and 2 class with many different languages). Parts 2, 3, and 5 are classes with some English language learners using several different languages. You can see the classrooms of these teachers in action in some parts of the **Classroom Videos E G1 Single-Digit Addition and Subtraction, G Place Value and Multidigit Addition and Subtraction,** and **A Classroom Components**. [18:32 minutes; 6 parts]

J Teacher Interviews Grades 2 and 3 shows Grade 2 and Grade 3 teachers discussing innovative and important aspects of the teaching they are doing for the Children's Math Worlds Project and how they build their classrooms. Part 3 is a class in which all children are English language learners. Parts 2, 5, and 6 are classes with many English language learners with several different languages. Parts 1 and 4 are classes with some English language learners. You can see the classrooms of these teachers in action in some parts of the **Classroom Videos F G3 Single-Digit Multiplication and Division, G Place Value and Multidigit Addition and Subtraction,** and **A Classroom Components**. [25:22 minutes; 6 parts]

Contents of Each of the Ten Major Groups of Classroom Videos

Overviews of Our Major Results

A Classroom Components

These videos describe and show five aspects of lessons that we found to be crucial for understanding and moving to fluency: Building Concepts, Math Talk, Student Leaders, Quick Practice, and Helping Community. We present the five components in separate sections, but they interact synergistically in the classroom. We have found that implementing these components enables children from all backgrounds to learn mathematics. [40 minutes; 7 parts]

Part 1 The Children's Math Worlds Research Project (CMW) describes the research project and shows visual images to portray crucial components of the project further described in Parts 2 through 7. [1.5 minutes]

Part 2 Building Concepts shows students building meanings by relating math drawings and word problems to written symbols and using other visual supports like secret-code cards and 1 to 20 boards. [7.5 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 0:30]
b. Kindergarteners make teen numbers with blocks of ten attached cubes and single cubes and by finger flashing (ten fingers to the left and some fingers to the right). They say each result as *ten plus one equals eleven* or *ten plus two equals twelve*. [0:30 to 2:13]
c. A Grade 1 class makes up a word problems for 58 + 36. Each child makes a math drawing for the situation and solves numerically in his/her own way. Four students solve at the board. Students on the rug use large dry-erase Math Boards around which goes a Number Path from 1 to 100 in groups of tens with two groups of five in each ten. A girl explains her math drawing and her Show Subtotals method that goes from the left: 80 written above 14 with 94 below these. This is one of the two accessible algorithms that the Children's Math Worlds Research Project shows students. The girl groups to make a ten-stick by connecting 5 vertical ones from the 8 ones to five vertical ones in the 6 ones. [2:13 to 4:30]

d. A second grader and her teacher discuss adding using small penny/dime strips and secret-code cards. [4:30 to 4:56]
e. Second graders make 358 with secret-code cards (as 300 + 50 + 8 with 8 top of the 0 in 50 and the 58 on top of the 00 in the 300). A girl uses 300 and 5 and 8 and then has to find the 50 instead of the 5. [4:56 to 5:38]
f. A second grader not yet fluent in English gives a story problem for 80 – 47. She stumbles on the English word "used" and changes to another word that will tell subtraction. We can see the power of having children give word problems—this develops sense-making and English vocabulary in use. The teacher then retells the story with enthusiasm and with correct English and relates by pointing the word problem to the numerals in the subtraction problem. [5:38 to 6:06]
g. The teacher sends two third graders to the board to solve 346 – 189 while the rest of the class solves at their seats. We see the students at the board begin to make their math drawings. Later, another student comes up to say that one student ungrouped left-to-right and the other student ungrouped right-to-left. More of this segment is in **C Longer Classroom Teaching Examples Part 2 G3 Multidigit Subtraction**. [6:06 to 7:02]
h. Third graders use paper fraction bars to show equivalent fractions for 4/6 and explain these to each other. [7:02 to 7:30]

Part 3 Math Talk shows students explaining their math thinking by relating written methods to math drawings they made and then responding to questions from classmates about their explanations. [11 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 1:06]
b. Grade 2 students solve at their seats while Student Helpers help some students. A boy explains his vertical bar graph on the board. Several students ask questions about comparing different numbers on the bar graph. Comparing language is difficult even for nature English-speakers, and we hear typical errors that will eventually become correct language. This school has at least 15 different languages spoken by different students. [1:06 to 2:46]
c. A first-grade boy explains 58 + 36 using his own invented method: 58 is written in numerals and 34 in 10-sticks and circles. He says, "58 needs 2 more to be 60, so I borrowed 2 from the 36 and I gave it to the 58 (58 is crossed out and 60 written there and 2 of the ones in 36 are crossed out), and now 58 is a 60 and I have to count 30 more so 60 plus 30 equals 90 and plus 4 more is 94." [2:46 to 3:24]
d. A first-grade girl explains 58 + 36 using the Children's Math Worlds accessible method New Groups Below: the new 1 ten is written below on the line under the 36 and is added after the 5 tens and 3 tens are added. Her ones are in 5-groups so you can see how many more she needs to make a ten. She says she needs to get 2 more to make a ten, and she gets it by circling two of the ones in 36 to make a group with her 8 ones. [3:24 to 4:47]
e. A second-grade girl represents with a math drawing and solves 80 – 47. She ungroups a ten by drawing ten small balls on the ten-stick to make ten ones; she calls this "making a trade". She records this above her problem by crossing out the 8 and writing 7 and crossing out the 0 and writing 10 above it. In response to questions from the class, she gives a clear explanation of why she needs to ungroup: to get enough ones to subtract 7 of them. She circles what was taken away (7 ones and 4 tens) to leave 33. [4:47 to 6:33]
f. A second-grade boy explains 48 + 36. He forgets to write the new 1 ten above the tens column. A question from a girl asks him about that. He calls the 1 a one, and the teacher

does not have him call it "1 ten" [she did later in Learning Math Talk part e]. Another good question comes from a girl who looks at the teacher afterwards as if she's a co-teacher and is proud that the boy answered correctly. [6:33 to 7:38]

g. A third-grade girl explains a 3-step word problem involving two divisions and a subtraction and answers questions from classmates. [7:38 to 8:39]
h. A fourth-grade boy uses the Area Method: He explains 43 x 67 using his area drawing that shows lengths of ten and of one. He adds the partial products to get the answer outside the rectangle. Classmates ask questions. [8:39 to 10:49]

Part 4 Learning Math Talk shows examples of the Math Talk Community where students need more support from the teacher or from other students. [6.5 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 1:04]
b. A second-grade girl answers a question about 48 + 36: Why didn't you just put the ten next to the 4 (to make it be 814 instead of 84)? She answers but needs the teacher's help a bit to explain that then it would be 814. Then the teacher explains to the class that a boy who has solved 80 – 47 can't remember how he did it and asks the class, "Who can help?" [1:04 to 1:54]
c. Two girls in a multilingual Grade 1 and 2 class practice, with help from the teacher, asking and helping each other answer questions about a word problem with extra information in it. We see the problem written on the board. [1:54 to 2:44]
d. A teacher works with a third grader at her seat eliciting a word problem for 700 – 374. The girl then explains her proof drawing using unpacking language. The class is working well independently as are students making drawings at board. [2:44 to 3:34]
e. In this follow-up to Math Talk part f, the second grader explaining 48 + 36 who did not use place-value language gets help from the teacher to use place-value language. [3:34 to 4:33]
f. A fifth grader while explaining her number-line drawing and numerical method for 4/5 x 2/3 finds an error and corrects and explains it. The teacher comments that this is why we explain our thinking---so that we can find and fix our mistakes. Two classmates ask questions. [4:33 to 6:19]

Part 5 Helping Community shows various ways in which students work together to solve problems or a teacher encourages such helping or Student Leaders help other students while everyone solves. In a Helping Community everyone is a teacher and a learner. A Helping Community is a crucial part of a classroom that supports all students to learn. [5 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 0:35]
b. Second-grade Student Leaders help while the class draws bar graphs at their seats. [0:35 to 1:23]
c. Three first graders in a Spanish bilingual class solve a 2-step word problem about rocks; they work well together and are enthusiastic. [1:23 to 2:03]
d. The teacher of a grade 1 and 2 multilingual class encourages her students to work together as they solve word problems: "You can ask your friend next to you." And she reminds them to label their problem. [2:03 to 2:26]
e. The teacher encourages two second graders to explain to each other how they solved 48 + 36 with math drawings and a written method. [2:26 to 2:48]
f. Third graders work together in pairs to make up word problems about subtracting 3-digit numbers. [2:48 to 3:22]

g. Two second graders each help the child sitting beside them to solve a problem about perimeter. [3:22 to 3:54]
h. A third-grade teacher asks for volunteers to be student helpers walking around the class helping students finish their page of different kinds of division word problems. The helpers help students who ask for help. [3:54 to 4:48]

Part 6 Quick Practice shows teacher-led or student-led short practice sessions focused on crucial grade-level concepts and skills. Each Quick Practice repeats for several days, and then new concepts are addressed. When choral responses are given, the leader gives some signal so that everyone can respond together and not give away the answer. [5 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 0:36]
b. A fourth-grade teacher leads "Say, write, compare" for 6-digit #s. She dictates 6 digits to make one number and then 6 digits to make the other number. Students write these numbers and put a blank between them in which to write the < or >. Each student says the comparing sentence in both directions, thus practicing saying 6-digit numbers and the greater than/less than words. [0:36 to 1:47]
c. A third-grade teacher leads oral single-digit division practice. [1:47 to 2:00]
d. Third graders work in lines at the board. Each line writes cumulative repeated additions of 3 (…, 15 + 3 = 18, 18 + 3 = 21, etc.) or repeated subtractions of 3 (…, 15 – 3 = 12, 12 – 3 = 9, …). [2:00 to 2:21]
e. The teacher of a grade 1 and 2 multilingual class leads oral practice of finding an unknown addend for a teen number (9 + ? = 15). [2:21 to 2:39]
f. Fourth-grade Student Leaders lead oral division practice in which students give the unknown factor on their fingers. [2:39 to 3:06]
g. A second-grade Student Leader leads practice with scrambled expanded notation (e.g., 5 + 900 + 40 =). [3:06 to 3:51]
h. Third graders practice multiplications and divisions in helping pairs. One student asks the other student oral questions from a sheet and records the performance at the end. [3:51 to 4:59]
i. A second-grade teacher leads counting by tens in the hundreds (we see 930 to 1000). [4:59 to 5:14]

Part 7 Student Leaders shows four first-grade Student Leaders leading Quick Practices with different several activities. All students in classrooms in every grade eventually become Student Leaders and lead effective community practice. Sometimes Student Leaders make up the practice problems, and at other times the problems are given by the program or the teacher. This segment only shows Grade 1, but Student Leaders are shown in other videos. [2:46 minutes]

a. Introduction with footage of different teachers and classes. [0:00 to 0:33]
b. Four first graders lead different Quick Practices [0:33 to 2:46]

B Math Explanations

These videos show examples from many different classrooms interspersed with explanations by me of main aspects of the research-based Children's Math Worlds approaches to each topic. Classroom videos showing more detail about each of these topics are in **C Longer Classroom Teaching Examples** except for Part 4 Single-Digit Multiplication and Division, whose videos are in **F G3 Single-digit Multiplication and Division Parts 1 through 5**.

[45 minutes; 6 parts]

Part 1 Multidigit Addition explains the research-based mathematically-desirable and accessible written methods developed in the Children's Math Worlds Project (New Groups Below and Show All Subtotals) and the math drawings that support meaning-making for students by showing the quantities and the process of making and recording new groups with these quantities. Advantages of the accessible methods over the current common method are explained. Showing and discussing different algorithms allows students to understand the fundamental concepts of multidigit addition rather than just memorizing one algorithm. Each accessible algorithm shows these fundamental concepts in different ways. These concepts are first, that you must add like quantities to each other (ones to ones, tens to tens, hundreds to hundreds, etc.). Second, because our base-10 number system only allows us to write 9 or less of a given quantity in any position, if you get ten or more of one kind of quantity, you must make 1 group of ten and write that 1 new group in the next left column. [9 minutes]

Part 2 Multidigit Subtraction shows the research-based mathematically-desirable and accessible written method developed in the Children's Math Worlds Project (First Ungroup Everywhere Needed) and the drawings that support meaning-making for students by showing the quantities and the process of making and recording new groups with these quantities. Advantages of the accessible method over the current common method are explained.

Many students prefer to do mathematical processes from left to right because that is how they read and write. The Children's Math Worlds accessible algorithm for subtraction enables students to go in either direction, helps them understand the two basic concepts involved in multidigit subtraction, and helps them avoid the most common error in subtraction, subtracting the smaller number from the larger even if the smaller number is on the top. In our algorithm, all of the ungrouping is done first, from the left or from the right, and then all of the subtracting is done, from the left or from the right. This helps students understand that these are the two major concepts in multidigit subtraction, and they are less likely to make the top from bottom error because they check whether every column is ready to subtract <u>before</u> they subtract. Our accessible algorithm is a minor modification of the common alternating method of ungrouping in the ones place, subtracting the ones, ungrouping in the tens place, subtracting the tens, etc. Such alternations are more complex than doing all necessary ungrouping first.

In our research we have found that many students using other programs view multidigit subtraction problems as separate vertical columns of digits and have an inadequate understanding of the top number as a whole number. Therefore we initially have students draw a large oval around the top number to help them see that ungrouping in subtraction is just rewriting the top number in a way that is ready for the subtraction. We call this oval with a handle a "magnifying glass" that helps them look at the top number and prepare it for subtraction. [4 minutes]

Part 3 Word Problems shows the sense-making about word problem situations as students use their own math drawings, special Children's Math Worlds math drawings, and equations to represent and then solve the situation. These classroom examples show in action the Children's Math Worlds major strategy for solving word problems: make sense of the situation and make some kind of drawing if it will help you. When teachers emphasize representing <u>the situation</u>, students have to listen to or read the word problem and try to make sense of its situation. In our classroom research we have found that children from all backgrounds can learn to solve word problems if they are given many opportunities to solve and explain them and are given math tools to help represent the problem situations. For addition and subtraction situations, these tools are equations, Math Mountains that show the total on the top and the addends as two legs of the mountain, and comparison bars. Equations represent the situations (e.g., $\square + 6 = 14$) and can

then be solved or rewritten if needed to represent the solution method (e.g., as 6 + ☐ = 14 or 14 – 6 = ☐). For multiplication and division students use equations, equal groups drawings, area/array drawings, and comparison bars. For all situations, students need to write as well as solve word problems. [7 minutes]

Part 4 Single-Digit Multiplication and Division describes the three important aspects of learning multiplications and divisions: finding patterns for each number, real-world situations, and sustained well-organized practice. The patterns for multiples of each number are determined by how that number fits with our base-ten 10 number system. Fives fit well and are easy to learn, sevens do not and are difficult. Students can explore patterns for each number. The nines are particularly rich because 9 is one less than 10, leading to a pattern in which the ones decrease as the tens increase. A crucial aspect is building fluency for multiplications and divisions by using a well-organized sustained program of practice. Both whole-class practice and individual practice are important. We start practice with divisions right away because they are often easier than their related multiplication. For example, a student knows that 56 ÷ 8 is going to be a number like 6 or 7 or 8, but finding the exact number for the multiplication of 8 x 7 is more complex.

The term "memorizing multiplication and division facts" can be misleading. Calling these "facts" sounds as if these are all random unrelated pieces of knowledge, and using the word "memorizing" seems as if the only choice for learning them is rote memorization. I show a multiplication table made using the alphabet with A as 1, B as 2, etc. This shows how many patterns there are in multiplications. Students who have an opportunity to study and discuss these patterns build up complex individual mental networks of knowledge about individual numbers. Of course practice to remember individual multiplications or divisions is important. But such practice is more effective if it is intertwined with work on word problems and on patterns. [11 minutes]

Part 5 Multidigit Multiplication shows the four research-based mathematically-desirable and accessible written methods developed in the Children's Math Worlds Research Project and the math drawings that support meaning-making for students. We use an area model for multidigit multiplication because it shows how each digit in one number multiplies each digit in the other number. Students initially develop their own methods, but we also show them three written methods that capture important mathematical ideas and that we have found to be understood by students. We have found that the area method is preferred by many less-advanced students because they sometimes get confused about which numbers to multiply by which in the other numerical methods. In the area method, students find and write the four products within the rectangle and then just add them up outside the rectangle. In the algebraic method, students multiply each by each horizontally as when multiplying using algebraic notation. Most students choose the expanded notation method because they feel more comfortable seeing the tens and ones in each number and writing out the products they are finding. This method can be done from the right or from the left. Students often find it easier to start from the left because they get the largest product first and then they can align all of the other products under this number. The common 2-row method used in many programs omits two of the sense-making steps in the expanded notation method, begins by multiplying from the right, and collapses the multiplications into two rows. This abstract and intricate method is difficult for many fourth graders to understand, but later they can move toward it by collapsing steps of the expanded notation method. Students see and discuss how this works in the **Longer Classroom Teaching Example Part 4 G4 Multidigit Multiplication** video. [8.5 minutes]

Part 6 Fractions describes how unit fractions and length models work to support sense-making about equivalent fractions and addition, subtraction, and multiplication of fractions. This segment explains two core conceptual features of the Children's Math Worlds approach to fractions: unit fractions and length models. Students make many errors with fractions because they generalize their whole number knowledge to fractions. Seeing fractions as composed of unit fractions using length models, initially with fraction bars and later with number lines, can support student understanding of all four operations on fractions, as shown by examples here and in **Longer Classroom Teaching Example Part 5 Multiplication of Fractions**. Coordinating the changes involved in making equivalent fractions is difficult for students. Here the fraction bar drawings and written unit fractions support an explanation that the equivalent fraction 3/6 has more but smaller unit fractions than the 1/2. More about equivalent fractions is in **H G3 & G5 Fractions and Ratios.** Understanding equivalent fractions forms the basis for adding, subtracting, and comparing fractions with different denominators. A general method of multiplying fractions is to multiply the top numbers and multiply the bottom numbers. Understanding why this is true of all fractions is supported by drawing a length model and taking the first fraction of each of the unit fractions that make the second fraction (see **C Longer Classroom Teaching Examples Part 5 G5 Fraction Multiplication).** [19:40 minutes]

C Longer Classroom Teaching Examples

These videos show students making drawings for and then explaining each of the Children's Math Worlds (CMW) approaches described in **B Math Explanations.** [51 minutes; 5 parts]

Part 1 G1 Multidigit Addition shows first graders making math drawings and explaining the CMW research-based mathematically-desirable and accessible written methods New Groups Below and Show All Totals. This video shows how well even first graders can explain their thinking using place value language and concepts. [6.5 minutes]

Part 2 G3 Multidigit Subtraction shows third graders making math drawings and explaining the CMW research-based mathematically-desirable and accessible written method First Ungroup Everywhere Needed and how this method allows students to move from left to right or from right to left as they ungroup and then as they subtract. [11 minutes]

Part 3 G2 Comparison Word Problems shows second graders representing, solving, and explaining different methods to solve a comparison word problem. They use their own methods and drawings, equations, and the CMW research-based matching and Math Mountain drawings to show the situation. [9 minutes]

Part 4 G4 Multidigit Multiplication shows fourth graders making math drawings that support meaning-making and explaining the research-based mathematically-desirable and accessible written methods developed in the Children's Math Worlds Project. The class then compares and relate methods. [9 minutes]

Part 5 G5 Fraction Multiplication shows students making math drawings to support multiplication of fractions meaningfully and then two students explain their drawings and their methods. The teacher then leads a discussion using the Commutative Property for whole number multiplication to show that the Commutative Property also is true for fraction multiplication. [15 minutes]

Grade-Level Videos

D Kindergarten

These videos show children using special meaning-making visual supports from the Children's Math Worlds Project: objects grouped in ten, five, and singles; secret-code cards in which the ones number goes on the 0 in 10; fingers using groups of ten, five, and singles. [20 minutes; 8 parts]

Part 1 K Counting Mat shows children using colored tiles to show the number 8 in different ways. They use the 5-group tile with single tiles and/or use dots/no dots or colors or spatial lay-out. The teacher develops language by emphasizing and extending children's descriptions of their eight tiles, e.g., above/below, left/right, top/bottom. [4 minutes]

Part 2 K Word Problems shows children acting out a word problem a child gives using a family meal scenario. Then a child tells and acts out a problem using plastic strawberries. The class gives the addition and subtraction equation and shows each equation with fingers. [2:40 minutes]

Part 3 K Dots to Fingers shows a Student Leader holding cards with dots in 5-patterns and the class shows with fingers the same number as shown by the dots. [1 minute]

Part 4 K Patterns in Teen Numbers shows children describing patterns they see in a Teen Pattern Poster that shows teen numbers as groups of ten and some ones. Numbers greater than 5 are shown as groups of 5, making many patterns children can describe. [3 minutes]

Part 5 K Teen Number Cards uses big Secret-Code Cards in which the ones card fits on top of the 0 on the tens card to show the teen number. Children show ten fingers for 10 and then fingers for the ones. Then the teacher moves the ones card on top of the 0 in the tens card and the class says that teen number. [1:40 minutes]

Part 6 K Finger Tens in Teen Numbers shows children flashing ten fingers to the left and then ones fingers to the right moving from 11 to 20, saying each teen number in the form "ten and three more is thirteen." [1:35 minutes]

Part 7 K 1 to 20 Mats shows children arranging small cubes using 5-groups to show 6, 7, 8, 9, and 10. Then the teacher asks them to describe what they have done and to relate their patterns to fingers. Children count from 1 to 20 and relate objects, written numerals, and spoken number words. [4:30 minutes]

Part 8 K Teen Mats shows children making teen numbers with a ten rod and small cubes and describing teen numbers as ten ones plus some ones (e.g., 12 equals ten and two). [1:40 minutes]

E G1 Single-Digit Addition and Subtraction

These videos show first graders finding and discussing partners of 9 and using the Children's Math Worlds progression of methods and math drawings to represent and solve word problems and numeral problems with the total a teen number. [31 minutes; 5 parts]

Part 1 G1 Break Apart 9 with Stairsteps shows first graders using Stairsteps (foam inch lengths that go from 1 to 10 inches) to make partners (addends) of 9 and then discussing patterns in equations showing these partners (9 = 1 + 8, 9 = 2 + 7, etc.). [6:30 minutes]

Part 2 G1 More Than Word Problem shows four first graders solving and explaining a comparison problem using a length, a Math Mountain, or a matching drawing and three different equations: 14 – 8 = 6, 8 + ☐ = 14, and 6 = 14 – 8. [8:20 minutes]

Part 3 G1 Fewer Than Word Problem shows three first graders solving and explaining a comparison problem using a Math Mountain or a matching drawing and three vertical computations: an unknown addend computation 8 + ☐ = 14, 14 – 8 = 6, and 6 + 8 = 14. [6 minutes]

Part 4 G1 Math Mountain Make a Ten shows first graders using and explaining the make-a-ten strategy to solve 8 + ? = 14 and 7 + 9 = ? within a Number Ghost game in which the Number Ghost (a child) erases a total or a partner in a Math Mountain while children hide their eyes. [4:15 minutes]

Part 5 G1 Teen Subtraction Methods shows a Student Leader asking first graders to solve and explain subtractions with a teen total in their own way. They show counting on to the total, making a ten, and doubles or doubles plus 1. [6 minutes]

F G3 Single-Digit Multiplication and Division

These videos show third graders finding, explaining, and using patterns of 4 and of 9 to multiply and divide; they also write, share, and discuss word problems for three types of division situations. We also see fourth graders use comparison bars to represent and solve additive and multiplicative comparison problems. [57 minutes; 6 parts]

Part 1 G3 Count By 4s shows a teacher leading third graders in counting by 4s to multiply and divide and asking them to solve and explain examples. They multiply by stopping when their fingers show the known multiplier of 4; the unknown product is the number they say. They divide by stopping counting by 4s at the known product; they look at their fingers to see the unknown multiplier. [5 minutes]

Part 2 G3 Class Find 9s Patterns shows third graders finding and explaining many patterns they see on the 120 Poster as they make groups of 9 and relate these groups to products of 9 = 10 – 1: 2 x 9 = 20 – 2 = 18, 3 x 9 = 30 – 3 = 27, etc.. [18 minutes].

Part 3 G3 Teacher Summarizes 9s Patterns shows the teacher summarizing the patterns for 9s children had found and described in Part 2. She then discusses why she values this activity for her students. [9:40 minutes]

Part 4 G3 Teaching Finger 9s shows the teacher building on the patterns students found relating 9s products to 10 – 1 products by developing and explaining how to show the tens and ones with fingers. She then shows and explains the Short-Cut Finger 9s that use the multiplier finger to separate the tens fingers from the ones fingers to show the product. [11:30 minutes]

Part 5 G3 Division Word Problems shows third graders sharing division word problems they made up for the different kinds of division situations: grouping, array, and multiplicative comparison. Each student explains how they solved the problem they wrote, and classmates ask questions. [8:30 minutes]

Part 6 G4 Comparison Problems shows fourth graders drawing comparison bars to compare amounts using addition/subtraction (more than/less than) and using multiplication/division (times as many as). [4:15 minutes]

G Place Value and Multidigit Addition and Subtraction

These videos show first and second graders using dime, nickel, and penny strips to practice counting to 2-digit numbers and second graders comparing numbers and finding all eight related equations for 25 + 75 = 100. Second graders explain the Children's Math Worlds methods for adding 2-digit and 3-digit numbers and for subtracting 3-digit numbers and relate the written methods to math drawings. Fourth graders explain three different written subtraction methods for 7-digit numbers. [32 minutes; 8 parts]

Part 1 G1 Dimes and Pennies shows Quick Practice with first graders showing values of a dime, nickel, and penny with fingers, and then counting by tens and fives and ones while flashing fingers to show the value of the increasing counts. The class then shows values of coin

problems and of dime strips and pennies by counting by tens while flashing ten fingers, freezing, then counting by ones and flashing one finger each time. [4:20 minutes]

Part 2 Place-Value Routine shows two second-grade Student Leaders each choosing a 2-digit number, making it with dime and nickel strips and single pennies, and the class counts by tens, fives, and ones to find the number. The same numbers are made and counted on a meter stick showing decimeters and centimeters (tens and ones). Students then say which number is more and which is less. [3:15 minutes]

Part 3 G2 8 Related Equations shows two second-grade Student Leaders leading a Quick Practice eliciting from the class and then writing the 8 related addition or subtraction equations from a Math Mountain: 25 + 75 = 100, 100 = 25 + 75, 75 + 25 = 100, 100 = 75 + 25, 100 – 25 = 75, 75 = 100 – 25, 100 – 75 = 25, 25 = 100 – 75 = 25. [5 minutes]

Part 4 G2 4 Methods for 67 + 86 shows second graders in a Spanish-speaking class explaining in English four Children's Math Worlds addition methods: New Groups Below, New Groups Above, Show Subtotals from the left, and Show Subtotals from the right. The teacher supports students to use hundreds, tens, and ones language when they are explaining. [10:30 minutes]

Part 5 G2 Explaining 580 + 276 shows a second grader explaining his math drawing that shows the hundreds, tens, and ones in 580 + 276 and then explaining his written addition method beside the drawing. [1:35 minutes]

Part 6 G2 Explaining 600 – 247 shows a second grader explaining her math drawing of hundreds, tens, and ones and how she used it to solve 600 – 247 by ungrouping 1 hundred and then 1 ten to get enough to subtract the tens and the ones. She related her steps in the drawing to her steps in her written method. [1 minute]

Part 7 G3 Explaining 700 –374 shows a third grader make up a word problem for 700 – 374 and then a girl explains her math drawing and her written method and answers two questions from classmates. [2:30 minutes

Part 8 G4 Explaining 7-Digit Subtraction shows three fourth graders using and then explaining different methods to ungroup 5,472,639 – 2,375,841 (ungrouping left to right, ungrouping right to left, ungrouping left to right without writing the first ungrouping step) and to subtract (left to right or right to left). Students discuss why they got the same answer. [13 minutes]

H G3 & G5 Fractions and Ratios

These videos show third and fifth graders finding and explaining equivalent fractions in different ways: using rows of the multiplication table to show many equivalent fractions and drawing fraction bars and/or fraction number lines. Fifth graders practice a unit fraction times a whole number and solve puzzles made from two intersecting rows and columns of a multiplication table. [19:40 minutes; 4 parts]

Part 1 G3 Equivalent Fractions shows third graders using rows cut from a multiplication table to see and explain many equivalent fractions as different multiples of the same simplest fraction. Students then make math drawings using fraction bars to show and explain why 2/7 = 6/21 and end by stating the general pattern: The smaller the denominator, the bigger the parts (the unit fractions). [10:30 minutes]

Part 2 G5 Equivalent Fractions shows a fifth grader explaining fraction bar and number line drawings to show that ½ = 3/6. The teacher then summarizes the opposite relation using the drawings: It takes three of the smaller 1/6ths to make the one larger half. [2:50 minutes]

Part 3 G5 Unit Fraction x Whole Number shows a Quick Practice in which two Student Leaders elicit answers to unit fraction x whole number problems such as 1/6 x 42. [0:42 minutes]
Part 4 G5 Multiplication Table Puzzles shows fifth graders dividing or multiplying to find the unknown number in puzzles made from intersections of two rows and two columns of a multiplication table. These puzzles show proportions and are part of the Children's Math Worlds approach to ratio and proportion based on the multiplication table (see **Teaching Progressions Ratio and Proportion Parts 1 and 2** and *Math Expressions Grade 6* for the full approach). [5:35 minutes]

Interviews with Classroom Teachers

I Teacher Interviews Grade 1

These videos show Grade 1 teachers discussing innovative and important aspects of the teaching they are doing for the Children's Math Worlds Project and how they build their classrooms. Parts 1, 4, and 6 are classes in which all children are English language learners (Part 6 is a combined Grade 1 and 2 class with many different languages). Parts 2, 3, and 5 are classes with some English language learners using several different languages. You can see the classrooms of these teachers in action in some parts of the **Classroom Videos E G1 Single-Digit Addition and Subtraction, G Place Value and Multidigit Addition and Subtraction,** and **A Classroom Components**. [18:32 minutes; 6 parts]
Part 1 G1 Building a Community of Math Learners [0.00 to 1:36]
Doing More Than I Expected ("I didn't think that it would work.") [1:36 to 2:41]
My Students Surprised Others [2:41 to 3:20]; Workshops with the Parents [3:20 to 4:14]
Part 2 G1 Building Math Helpers [0:00 to 0:35]; It's OK to Make Mistakes [0:35 to 1:22]
Visual Models Helping Explaining [1:22 to 1:49]
Part 3 G1 Get the Children Involved [0:00 to 1:09]
Part 4 G1 Learning English in Math Class [0:00 to 1:42]; Building Helpers [1:42 to 3:01]
Part 5 G1 Word Problems [0:00 to 0:59]; Student Leaders [0:59 to 2:32]
Part 6 G1&2 Many Languages in a Classroom [0:00 to 2:33]
Helping Community [2:33 to 4:33]; High Expectations [4:33 to 5:47]

J Teacher Interviews Grades 2 and 3

These videos show Grade 2 and Grade 3 teachers discussing innovative and important aspects of the teaching they are doing for the Children's Math Worlds Project and how they build their classrooms. Part 3 is a class in which all children are English language learners. Parts 2, 5, and 6 are classes with many English language learners with several different languages. Parts 1 and 4 are classes with some English language learners. You can see the classrooms of these teachers in action in some parts of the **Classroom Videos F G3 Single-Digit Multiplication and Division, G Place Value and Multidigit Addition and Subtraction,** and **A Classroom Components**. [25:22 minutes; 6 parts]
Part 1 G2 Shifting From Being a Traditional Teacher [0:00 to 4:20]
Sharing after Practice [4:20 to 5:51]; Introducing Vocabulary [5:51 to 7:13]
Part 2 G2 A Second Grade Math Community [0:00 to 1:27]
They can go up to the board and explain what they've done. [1:27 to 2:27]
Part 3 G2 Building Vocabulary [0:00 to 0:18]; Explaining Like the Teacher [0:18 to 0:55]

Part 4 G2 Using Different Methods [0:00 to 1:41]
The Importance of the Drawings [1:41 to 2:39]
Building and Explaining by Students [2:39 to 4:03]; Teaching the Parents Too [4:03 to 5:00]
Part 5 G3 Building a CMW ESL Classroom [0:00 to 2:39]
Student Helpers & Teacher Modeling [2:39 to 4:49]
Learning Deeply and Making Connections [4:49 to 6:32]
Part 6 G3 Building a Math Talk Community [0:00 to 1:00]; Building the Listener [1:00 to 3:15]