Kindergarten and Grade 1 Children Living in Poverty Can Learn the CCSS NBT Concepts

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For more details about NBT concepts and visual supports for the learning progression, or for details about other CCSS domains, please see the 13 hours of audio-visual Teaching Progressions I have made. You can find links to these and to papers and other presentations at karenfusonmath.net

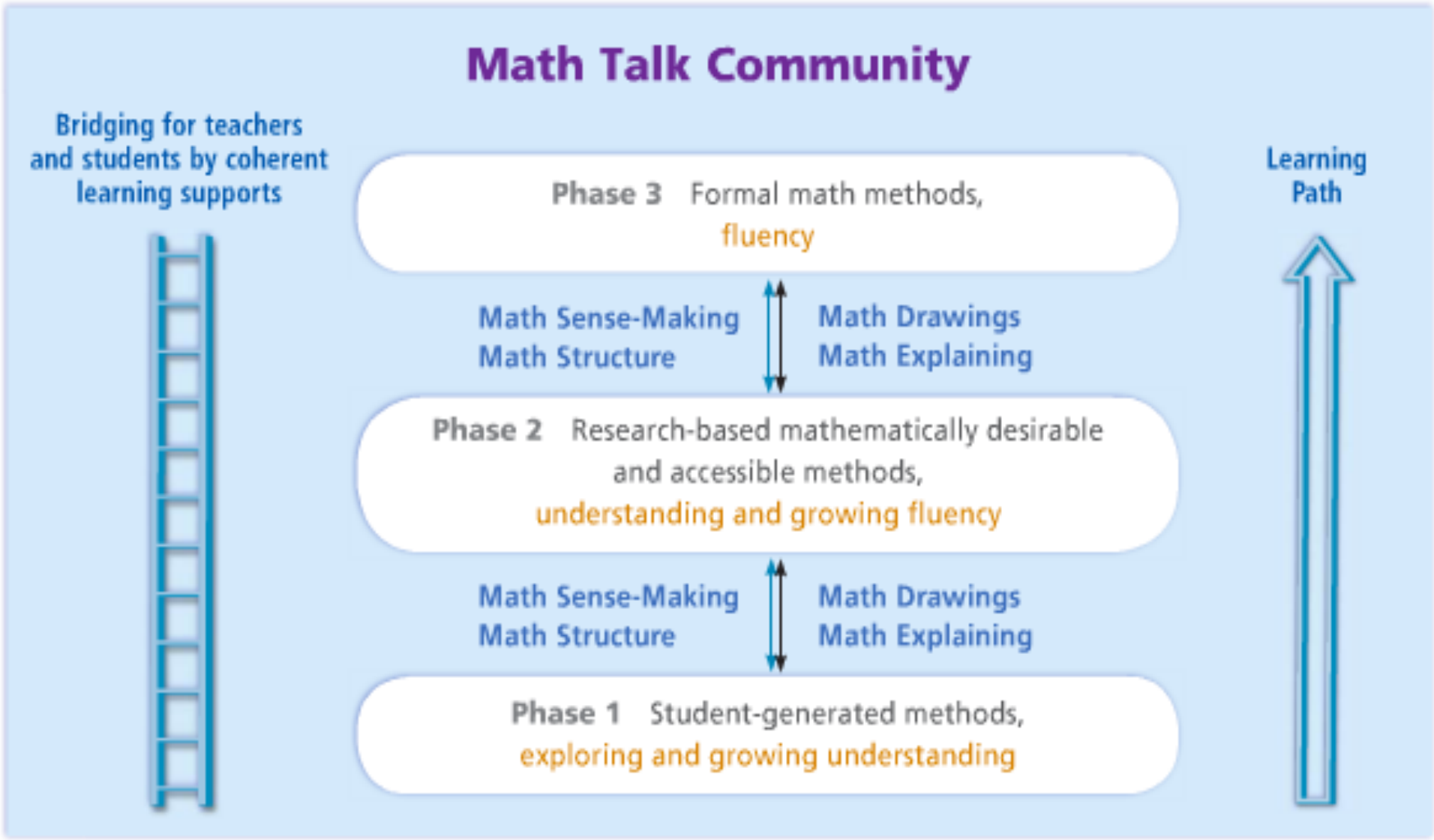
This presentation is also posted there.

Also see these NCTM books for more information about NBT concepts:

Focus in kindergarten:
Teaching with curriculum focal points

Focus in grade 1:
Teaching with curriculum focal points

Focus in grade 2:
Teaching with curriculum focal points


Math Talk Community
Bridging for teachers and students by coherent learning supports
Phase 3 Formal math methods, fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 2 Research-based mathematically desirable and accessible methods, understanding and growing fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 1 Student-generated methods, exploring and growing understanding
Learning Path

Common Core Mathematical Practices Used in a Math Talk Community

Math Sense-Making: Make sense and use appropriate precision 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings: Model and use tools** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure: See structure and generalize 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining: Reason, explain, and question** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.

Figure 2

The Math Practices in action

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

Can I do some part of this better tomorrow?

K NBT.1 and 1.NBT.1 and 2

K.CC.1: Count to 100 by ones and by tens.

K.NBT.1: Work with numbers 11–19 to gain foundations for place value: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 +8); understand that these numbers are composed of ten ones and one, two, …, or nine ones.

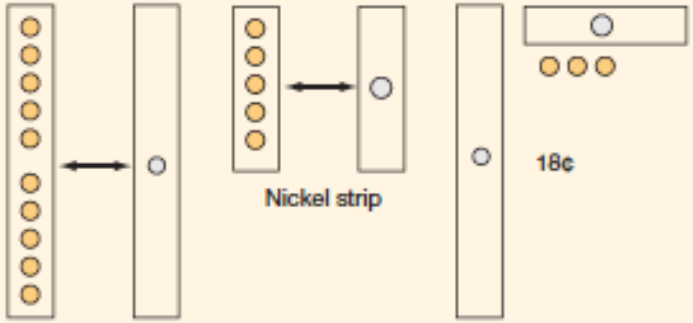
1.NBT. 1: Extend the counting sequence: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

1.NBT.2: Understand place value. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

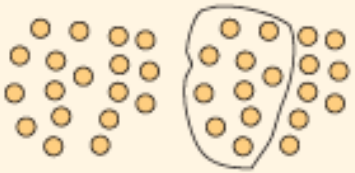
a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, …, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, …, or nine tens (and 0 ones).

B. See ten and some ones

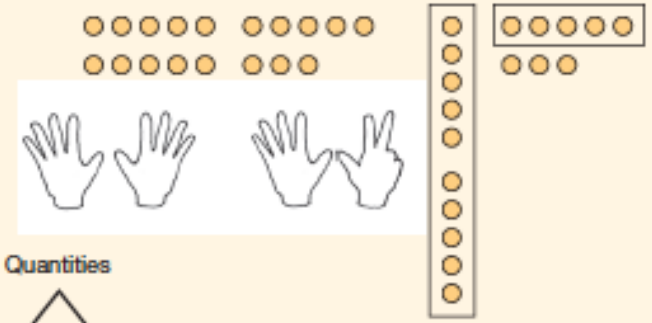
D. Coin values as a 5-group of pennies = a nickel and two 5-groups = a dime



A. Unitary



C. See 5-groups within ten and the ones



Quantities

10 to 19

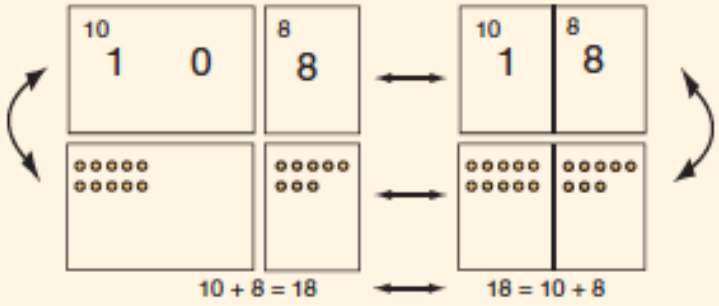
Count Words

Written Numerals

ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen
[say teen, not ten]

The ones are first in the words and second in the written numerals.

A. Eighteen
B, C, D. Ten and eight
1 ten 8 ones



Concepts of Numbers over Ten

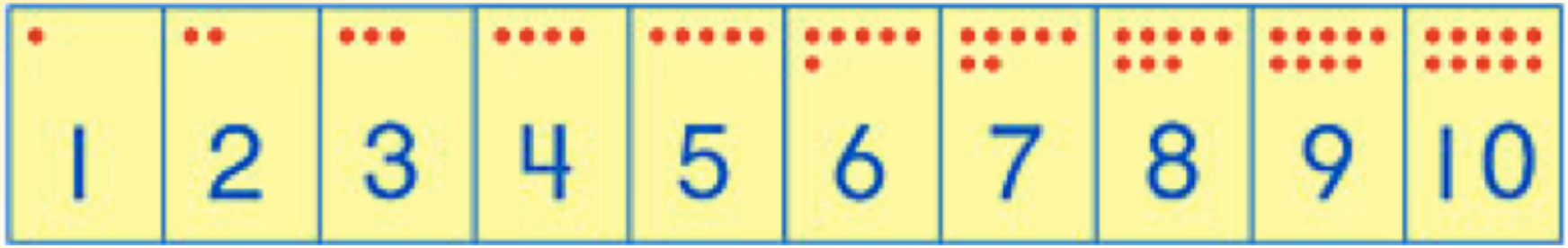
Unitary: a collection of single units
Kindergarten and Grade 1: Count these units by ones using English count words

Tens-Total and Ones: Separate the ones from the tens-total:
Count by ones using English count words but notice the total of the separated tens-total
Kindergarten: for *11 to 19: ten and eight is eighteen* 18 = 10 + 8
Grade 1: for *20 to 99: seventy and nine is seventy nine* 64 = 60 + 4

Sequence-Count-by-Tens and by Ones: count each group of ten ones by English tens words
Kindergarten: *ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety*
Grade 1: Count by tens and then count on by ones from the final tens word:
ten, twenty, thirty, forty, fifty, sixty SHIFT THE COUNTING UNIT
sixty-one, sixty-two, sixty-three, sixty-four

Separate-Count-the-Tens and Ones: count or see each tens quantity as 1 ten and use tens and ones words to give the multiunit value: *one ten, two tens, three tens, four tens, five tens six tens* and *one, two, three, four ones* to make *six tens four ones*

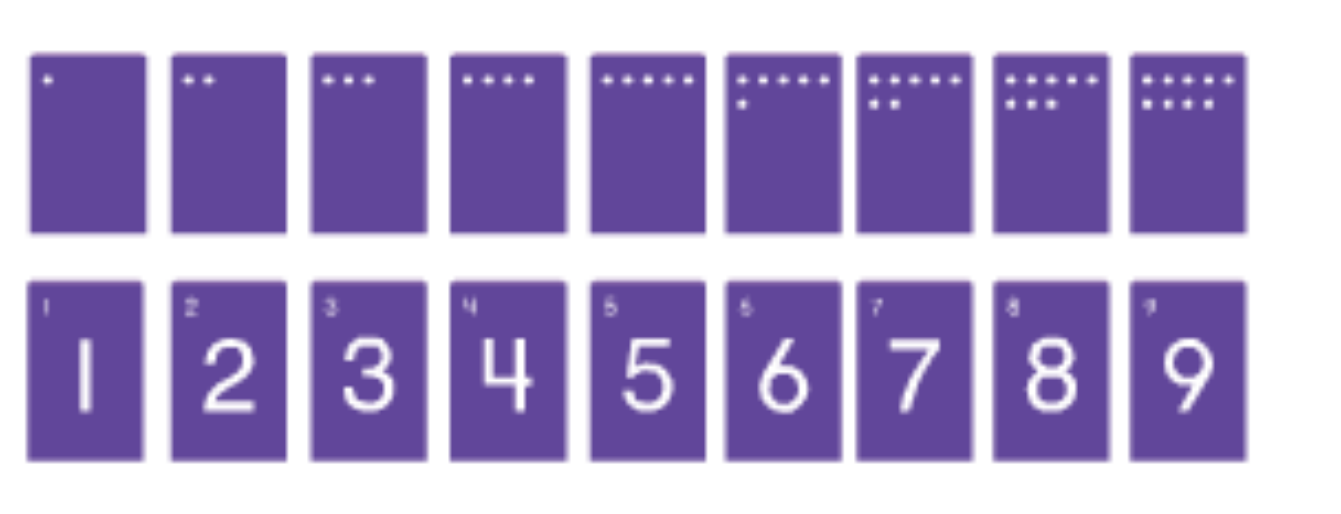
K and 1 Seeing 1 to 10

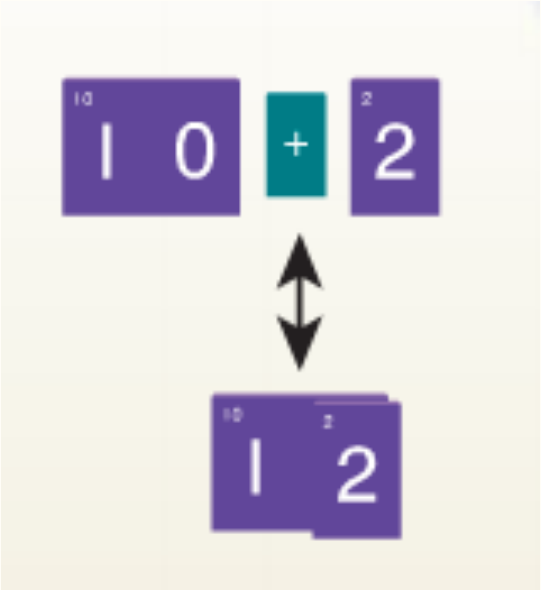


This large Number Parade was on the wall and used in many activities.

See the 5-groups that make 6 through 10.

K Number Tiles

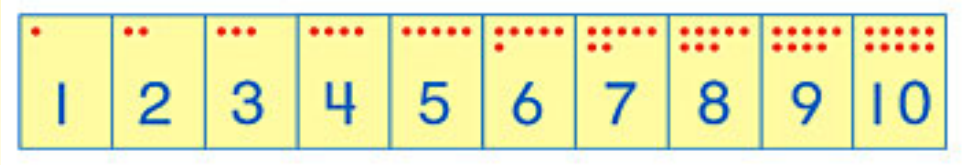




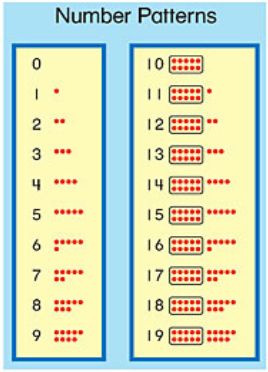
K Number Patterns in Order

Visual Supports for Patterns in Numbers and Quantities in Order

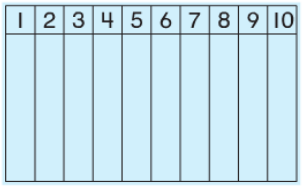
Number Parade



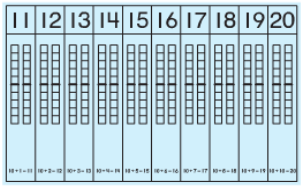
Number Patterns to 19



1-20 board

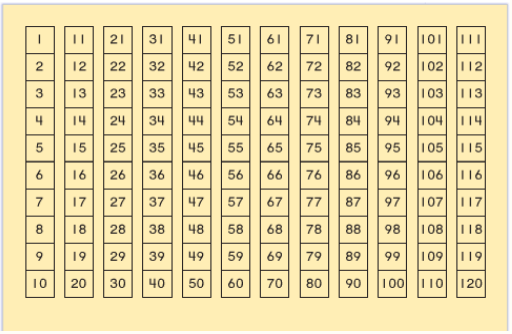


1–20 Board (front)



1–20 Board (back)

120 poster



K Daily Routines, Quick Practice, and Student Leaders

Learning the patterns and specific decade words in **the count to 100** or to 120 is very time-consuming.

Understanding the left-right patterns in **the written numerals** is also complex.

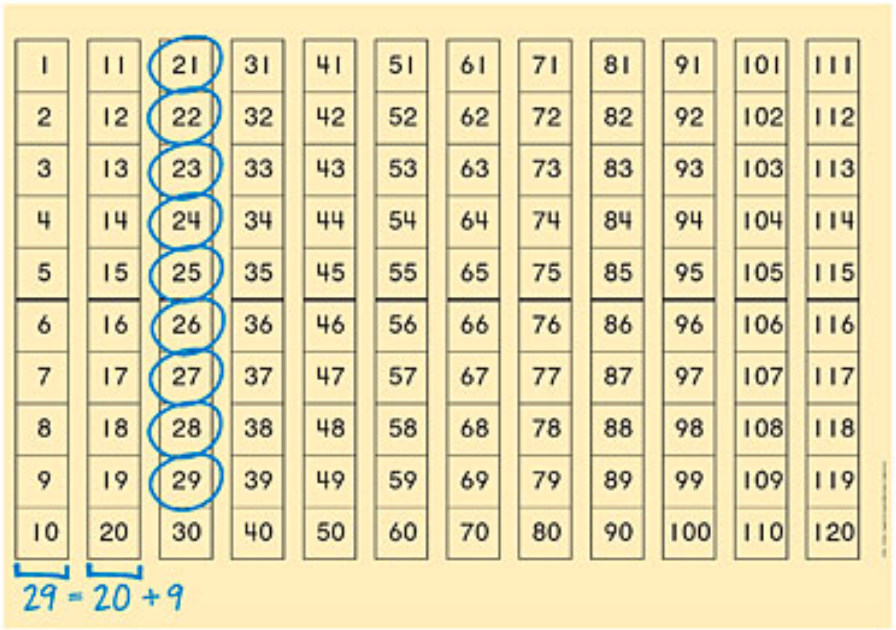
Seeing **tens and ones** and relating words, numerals, and quantities takes time and structured experiences.

So *Math Expressions* has a special **Daily Routine** for this and also has **Quick Practices** that begin class to provide all of this learning time.

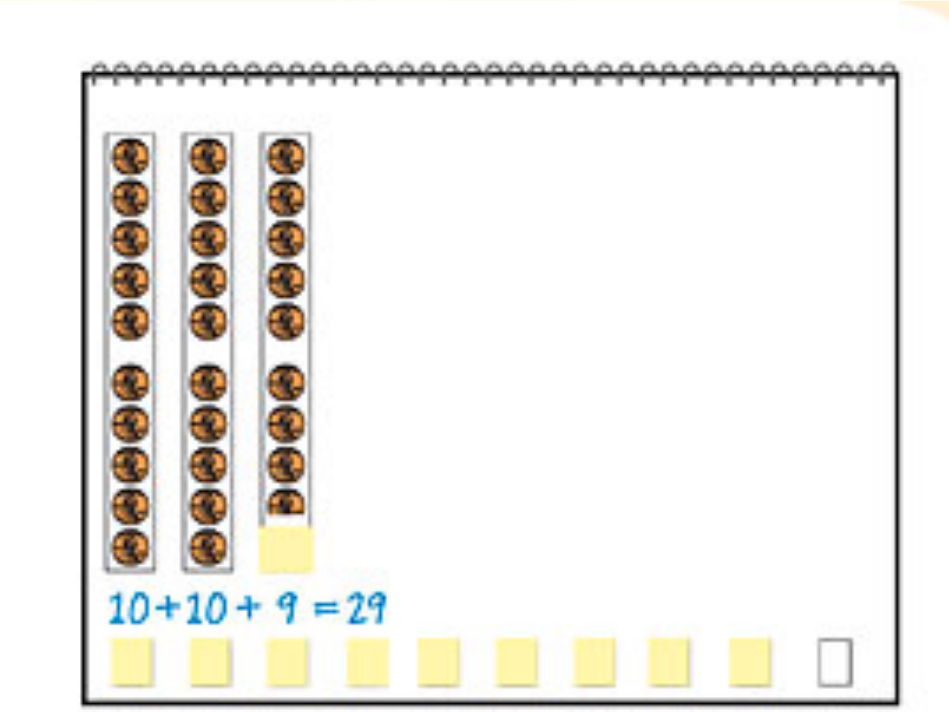
Student Leaders lead many of these activities, providing leadership experience and a feeling of community to these crucial class activities.

K Daily Routines A

120 Poster

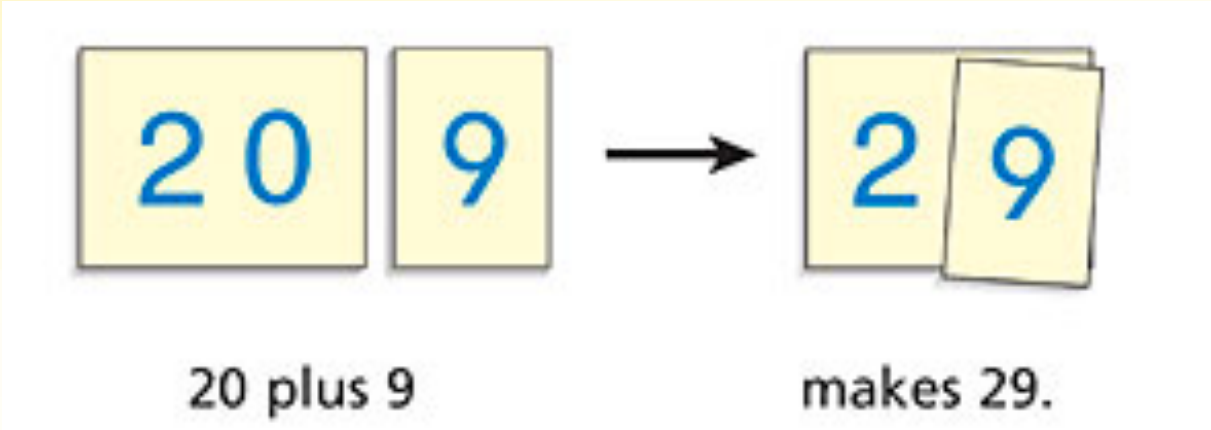


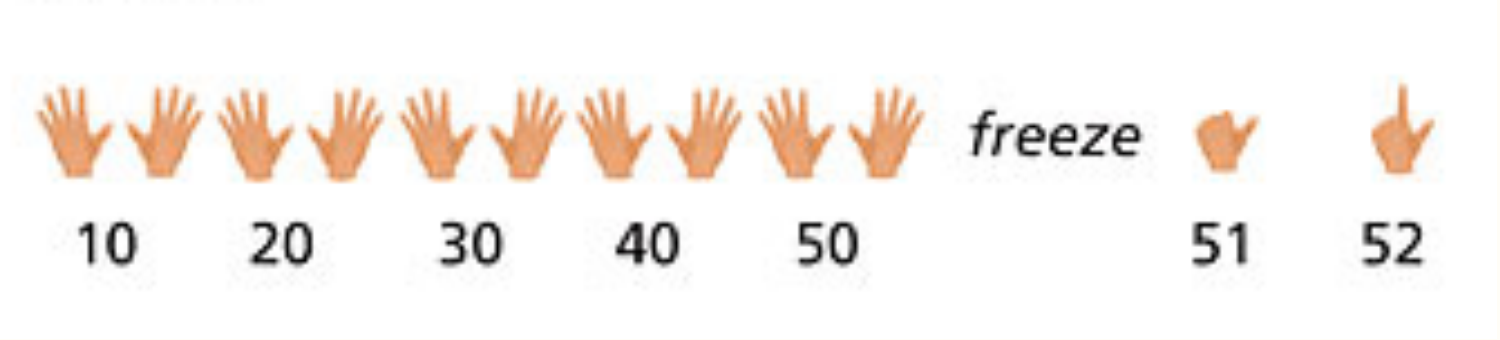
Money Flip Chart



K Daily Routines B

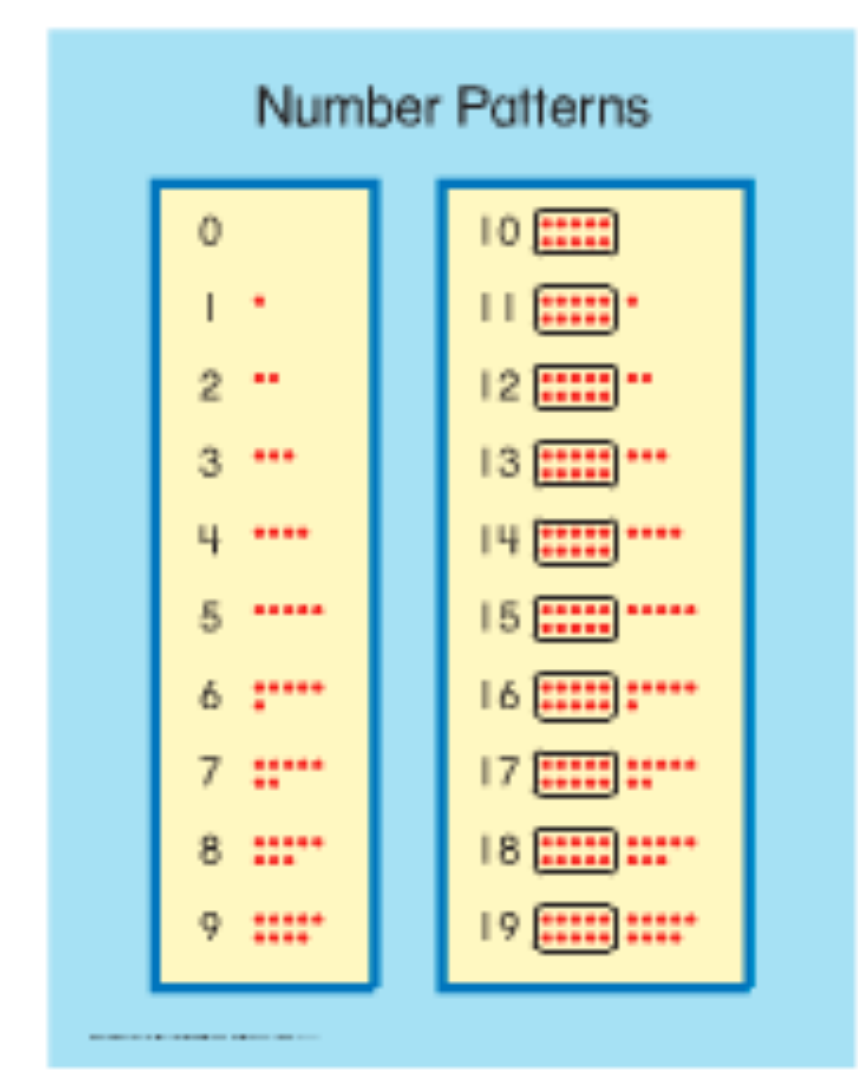
Giant Number Cards





K Count from 11 to 20 Quick Practice Unit 2

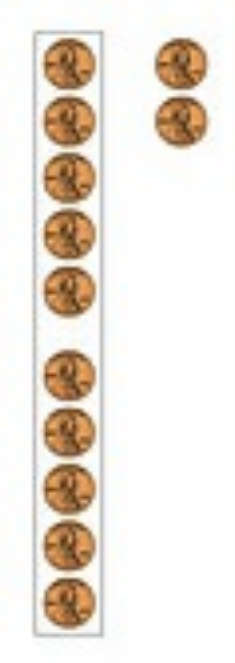
Number Pattern Poster

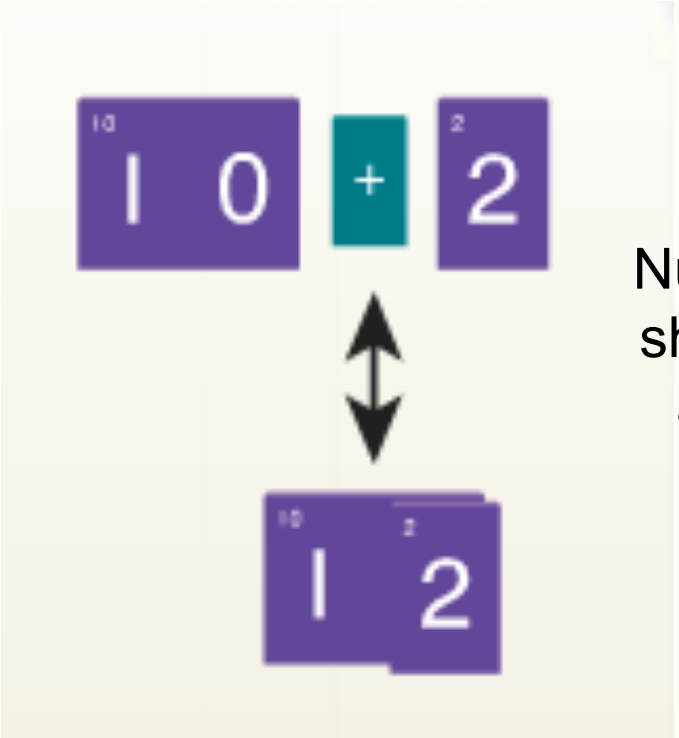


K Unit 3: See Teen Numbers as Ten Ones and More Ones

The Ten Bug helps children see a group of ten things or two groups of five (that make ten).

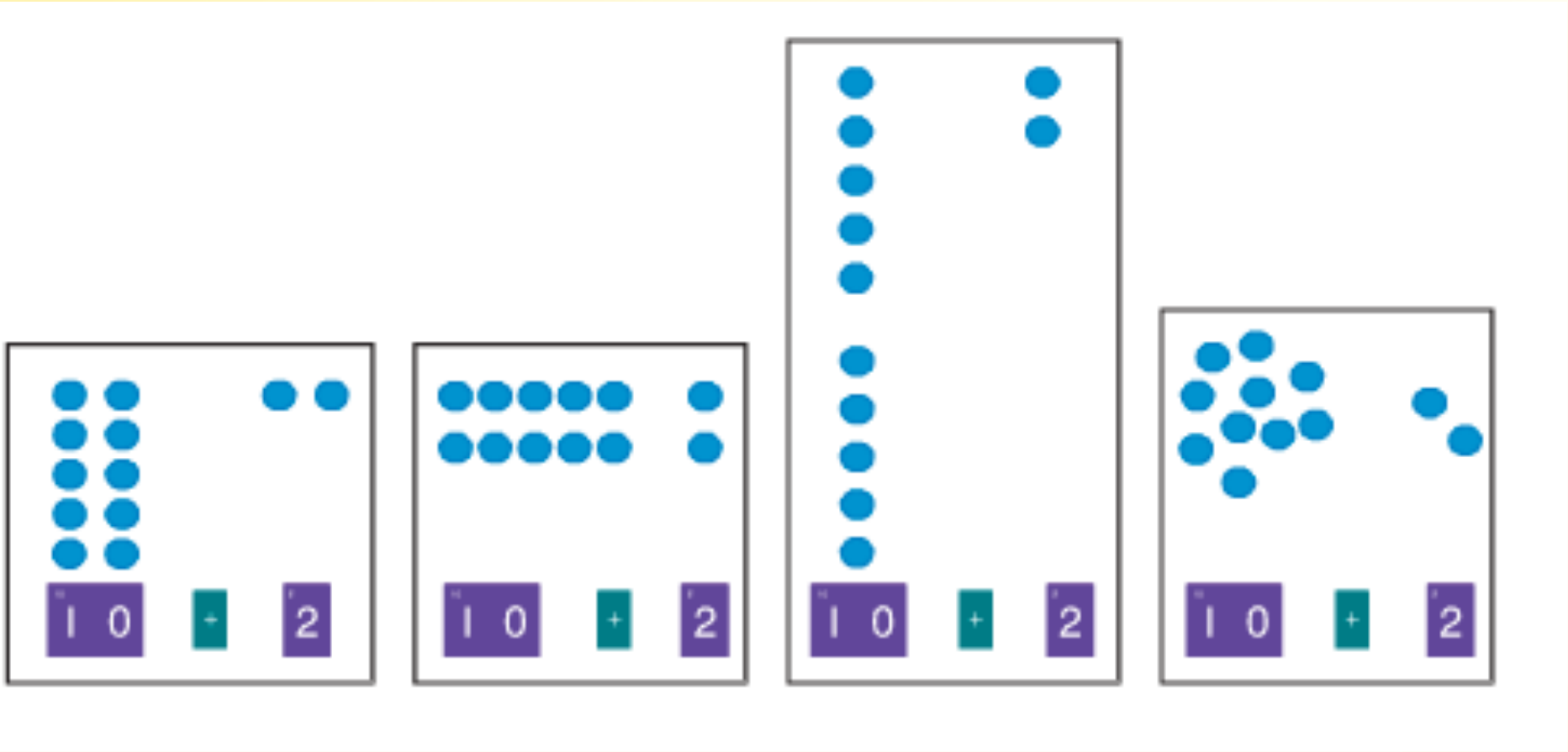
The
10-Penny Strip
and
2 loose pennies



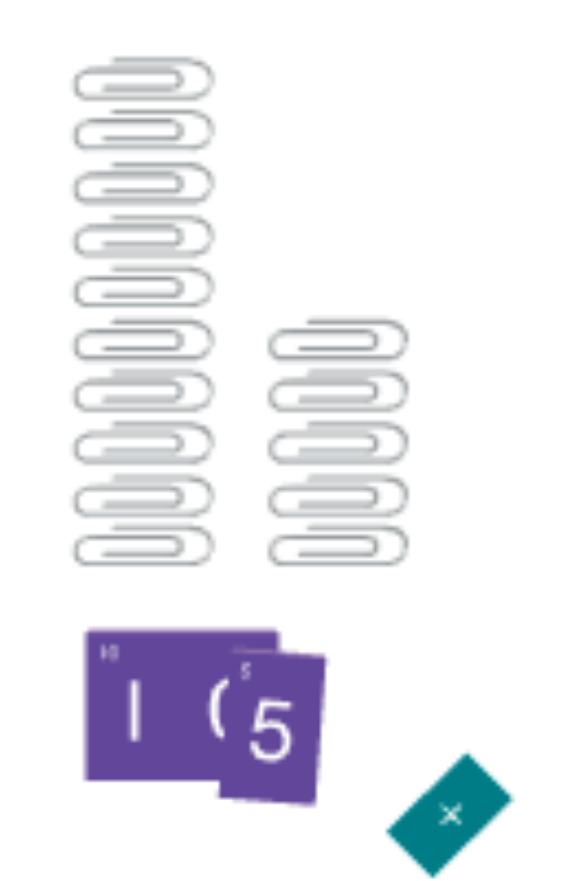


Number Tiles show the ten and ones.

K see the 10 ones in different ways including as 2 groups of 5



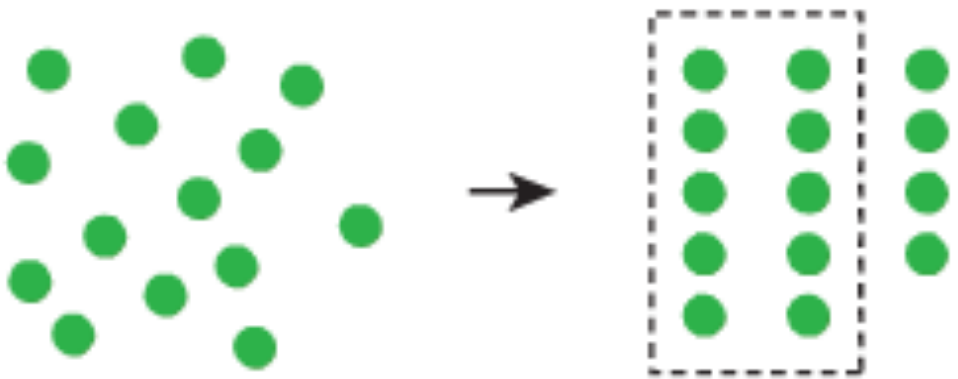
K Make Teen Numbers with Real Objects



Math Talk Questions:

- Why did we make a group of ten in each teen number?
- Does it matter what objects we use to show a teen number?
- Does it matter how we arrange the objects?

K Make Teen Numbers at Home



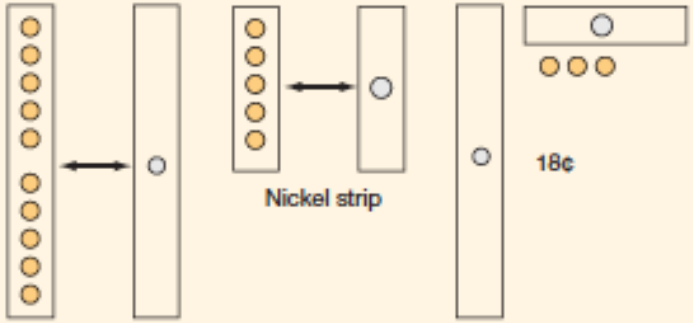
Have your child count out objects for a teen number and separate the 10-group.

Have your child show the teen number with the number cards

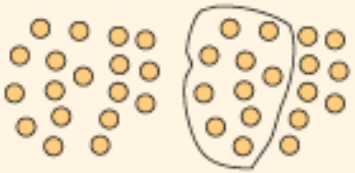
K The 1 to 20 Board

B. See ten and some ones

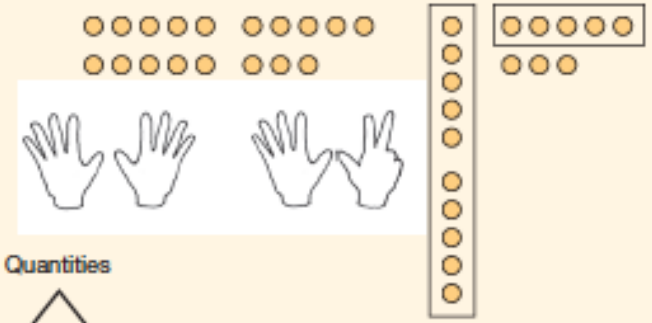
D. Coin values as a 5-group of pennies = a nickel and two 5-groups = a dime



A. Unitary



C. See 5-groups within ten and the ones



Quantities

10 to 19

Count Words

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ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen
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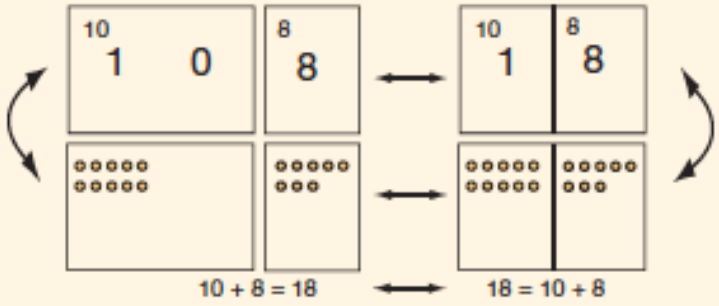


Percentage Showing Ten Chips as the Meaning for the 1 in 16

Samples	Immediate showing of ten	After 1, 2, or 3 prompts	Total showing 1 as ten
Kamii (1989) suburban students			
n = 32 Grade 2	16	ng	
n = 40 Grade 3	30	ng	
Math Expressions Kindergarten			
n = 85 Mean percentage for all children	34	25	59
By school			
n = 24 Full-day B	75	21	96
n = 22 Full-day C	23	23	46
n = 39 Half-day A	5	31	36
Math Expressions Grade 1			
n = 25	64	32	96

Note. The Kamii (1989) samples were from an upper-middle class school with high test scores. The *Math Expressions* kindergarten children were from three schools (A, B, C) with high levels of students with free lunch and many English language learners.

Percentage of Kindergarten Understanders of Embedded-10 Cardinality

Task	Ho & Fuson (1998) Sample			*Math Expressions*
	Chinese Hong Kong $n = 36$	English $n = 18$	U. S. $n = 12$	Half-day and Full-day $n = 82$
Understanders of Embedded-10 Cardinality: Gave rapid and accurate answers to all 10 + n problems and did not give rapid and accurate answers to all 4 + n problems	39%	0%	0%	35%

Note. All of the Ho and Fuson (1998) samples were middle-class children. The *Math Expressions* sample was randomly drawn from four half-day and four full-day classrooms in three schools (A, B, and C) with many English-language learners and almost all children from homes qualifying for free lunch. This task falls centrally within the following CCSS: K.NBT.1 knowledge.

Counting Words

In Chinese

One, two, three, …, nine, ten
ten one, ten two, ten three, ten four,
ten five, ten six, ten seven, ten eight,
ten nine

In Spanish

Uno, dos, tres, ..., nueve, diez
once, doce, trece, catorce, quince,
dieciseis, diecisiete, dieciocho,
diecinueve

Cardinal to counting meanings

Cardinal words counting word

Ten and six is ten six
10 + 6 = 16

In the interview about 16:
Dieciseis es diez y seis
16 is 10 + 6

Percentage Use of Correct Representations on the Miura 2-Digit Task by Cognitive Representation Category

Trial 1 Representation	Miura (1988) upper-middle-class Grade 1				Miura	*Math Expressions* low-SES		
					Kinder garten	Kindergarten		Grade 1
	U. S. *n* = 24	PRC *n* = 25	Japan *n* = 24	Korea *n* = 40	Korea *n* = 20	Half-day *n* = 39	Full-day *n* = 46	*n* = 23
Canonical base-10: 1 ten 3 ones 2 tens 8 ones	8	81	72	83	34	40	79	94
Noncanonical base-10: 10 + 3 ones 1 ten 18 ones	1	9	10	11	7	25	6	2
Unitary collection: 13 ones 28 ones	91	10	18	6	59	35	15	4

Note. This task falls centrally within the following CCSS: K.NBT.1 and 1.NBT.2 knowledge.

K NBT.1 and 1.NBT.1 and 2

K.CC.1: Count to 100 by ones and by tens.

K.NBT.1: Work with numbers 11–19 to gain foundations for place value: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 +8); understand that these numbers are composed of ten ones and one, two, …, or nine ones.

1.NBT. 1: Extend the counting sequence: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

1.NBT.2: Understand place value. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, …, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, …, or nine tens (and 0 ones).

1.NBT.4 and Critical Area

Critical Area (2): Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. … They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). …

1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

G1 Use place value understanding and properties of operations to add and subtract. 1 NBT.4, 5, 6

5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

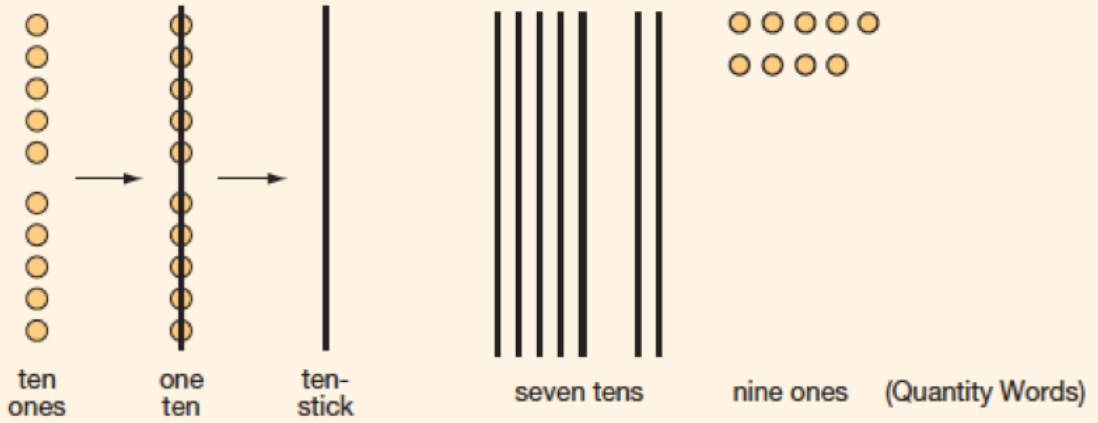
No subtracting without regrouping

because these lead to the top from bottom errors in Grade 2.

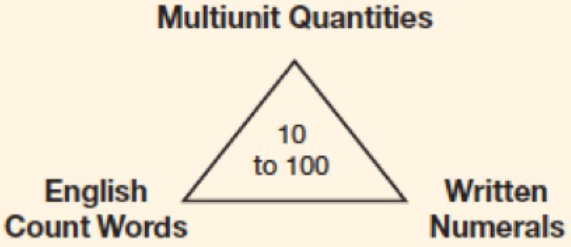
G2 Multidigit Subtraction Common Error

$$\begin{array}{r} 83 \\ -\ 57 \\ \hline 34 \end{array}$$

$$\begin{array}{r} 346 \\ -\ 157 \\ \hline 211 \end{array}$$



See and Say the Multiunit Values



Named Value (Expanded Notation)

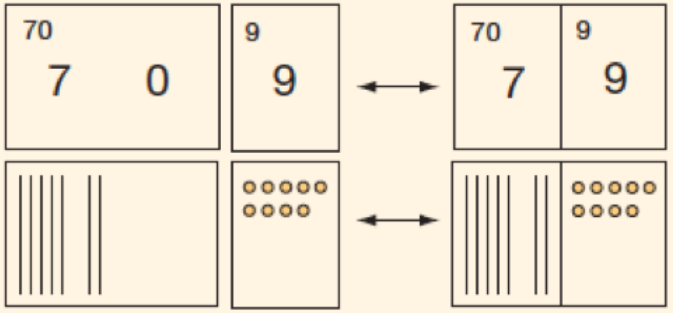
seventy nine

Concatenated Single Digits

79 (looks like seven nine)

$70 + 9 = 79$

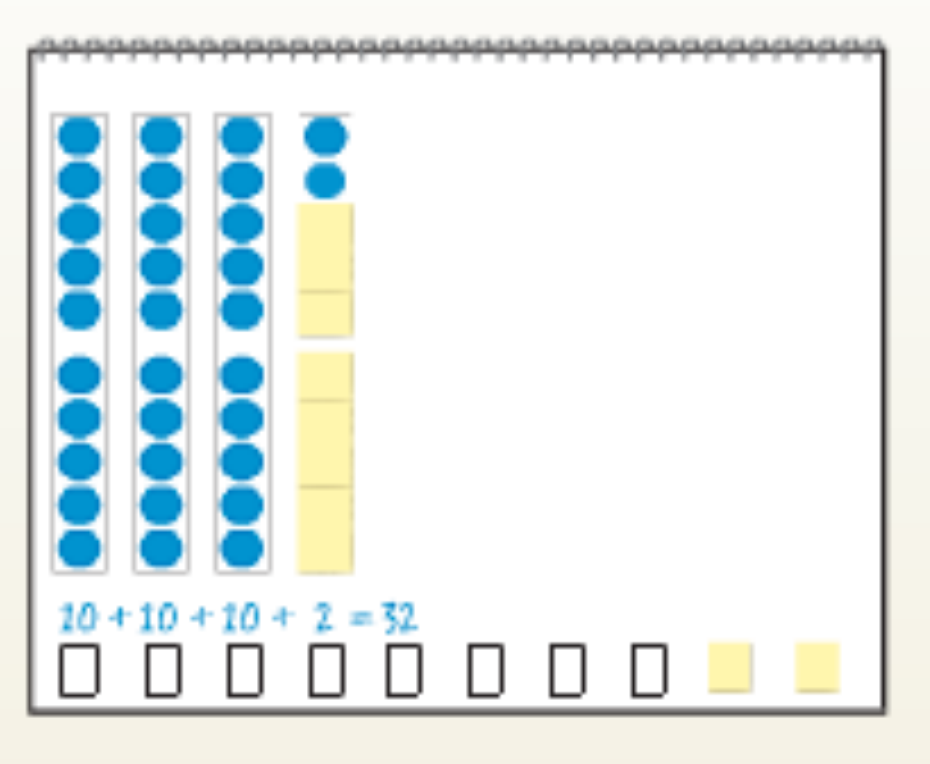
$79 = 70 + 9$

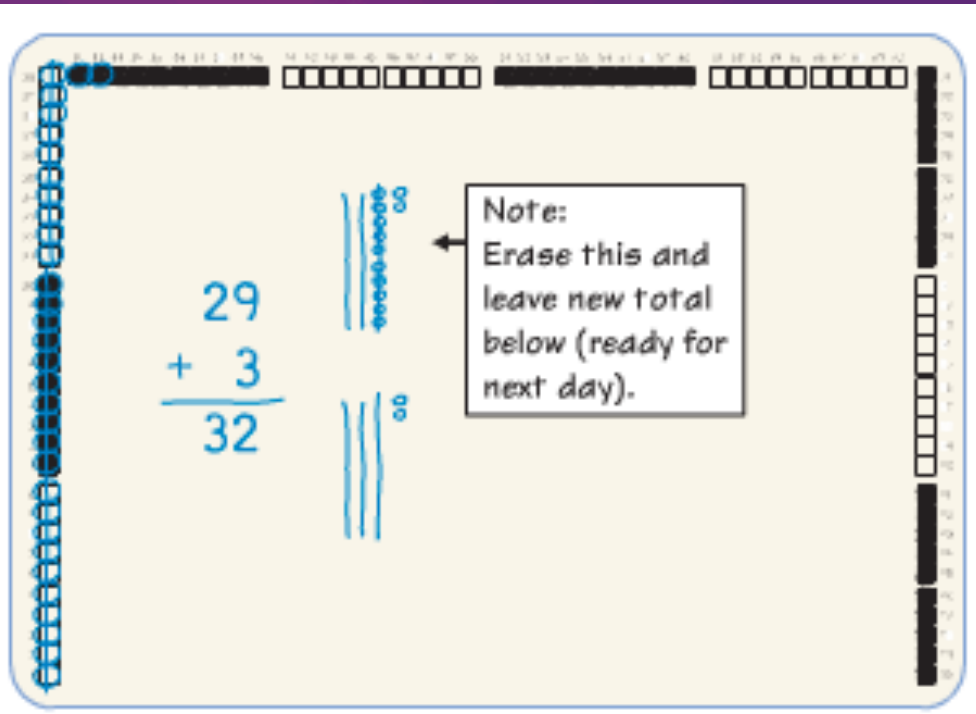


Secret-Code Cards

G1 Daily Routine 1.NBT.1 and 1.NBT.2

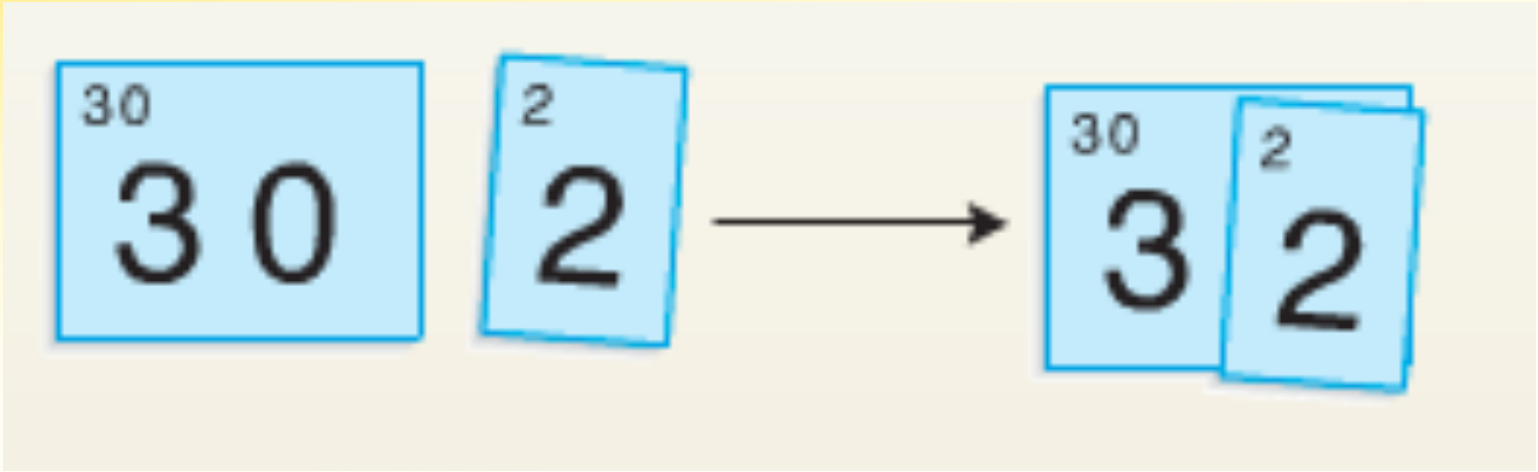
Counting Tens and Ones Flip Chart



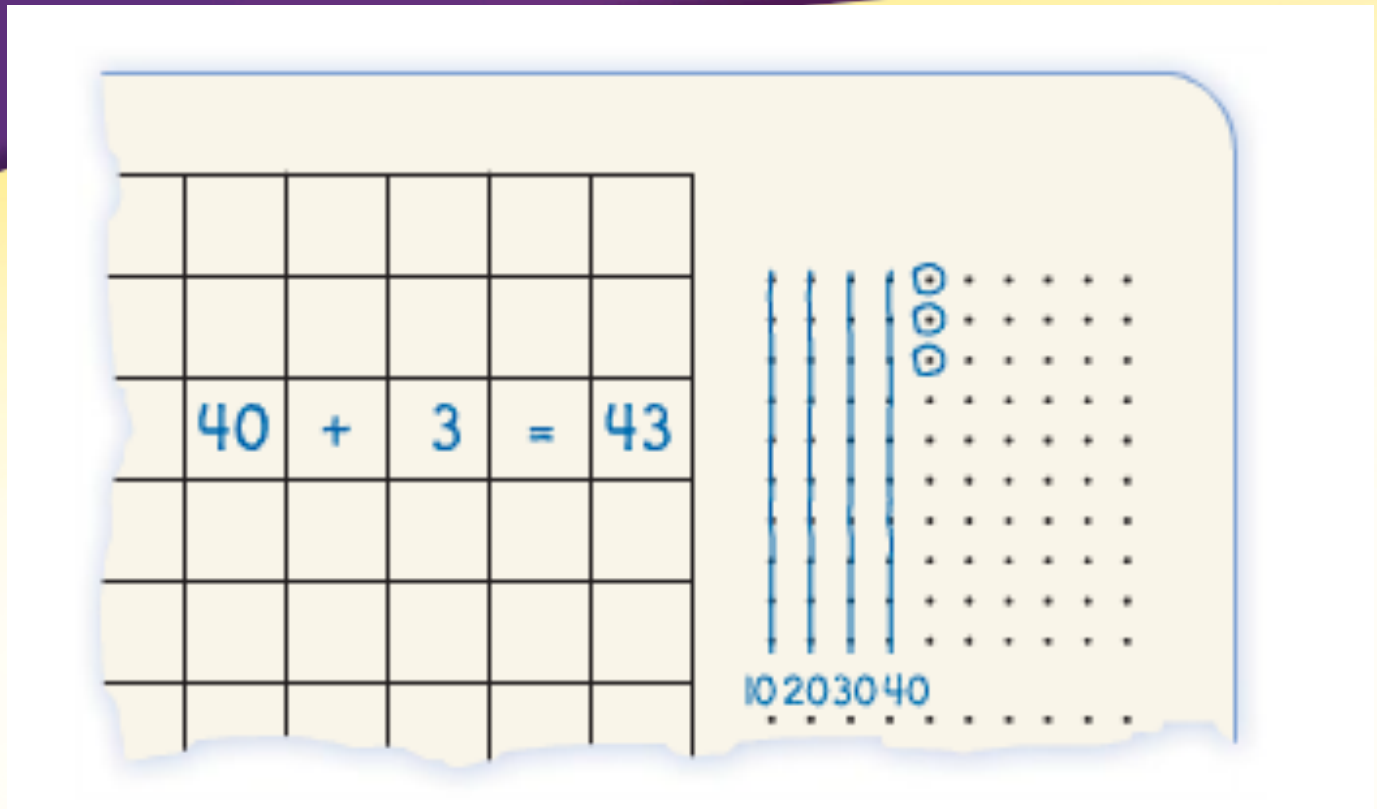


Number Path
on Math Board

G1 Secret Code Cards

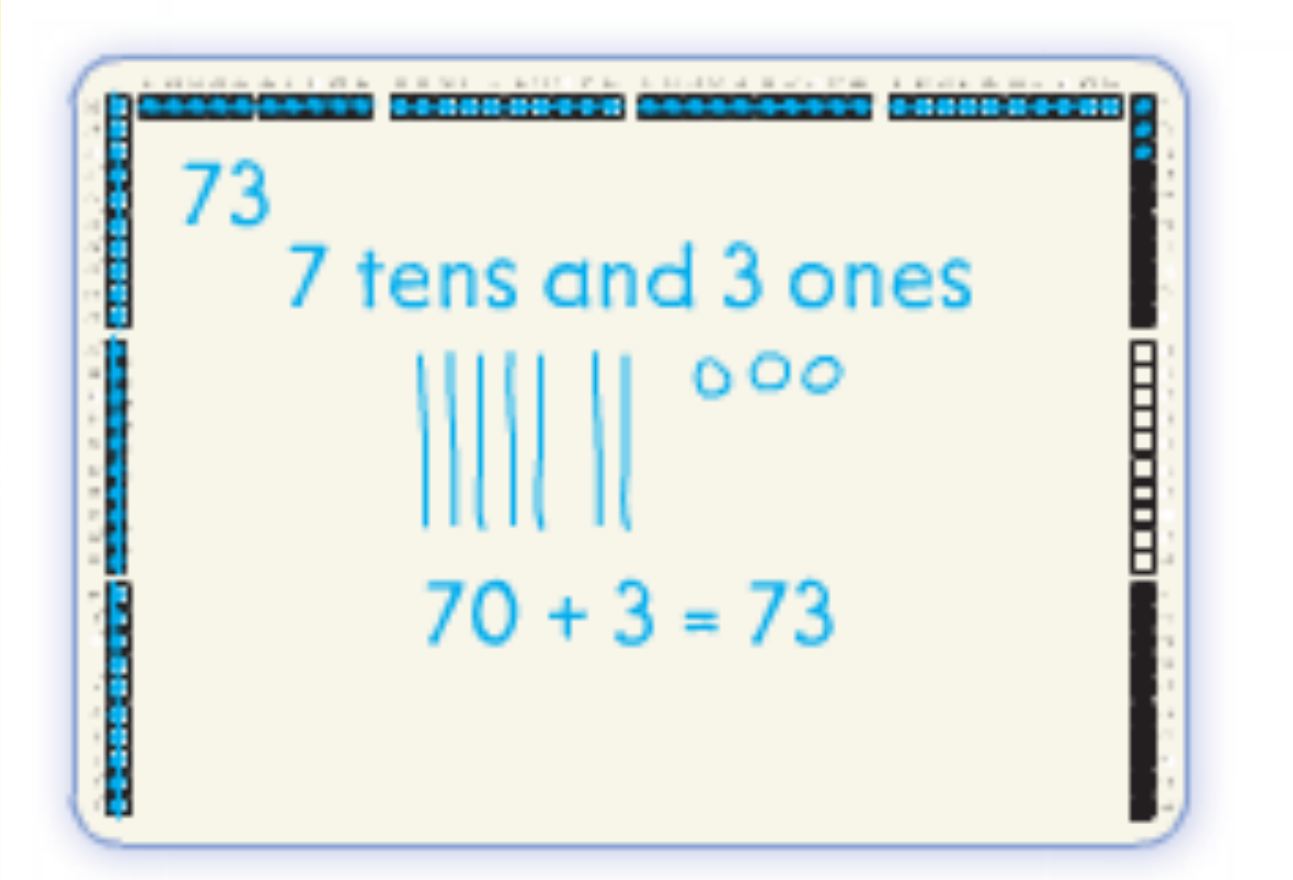


G1 Drawing Tens and Ones 1.NBT.2

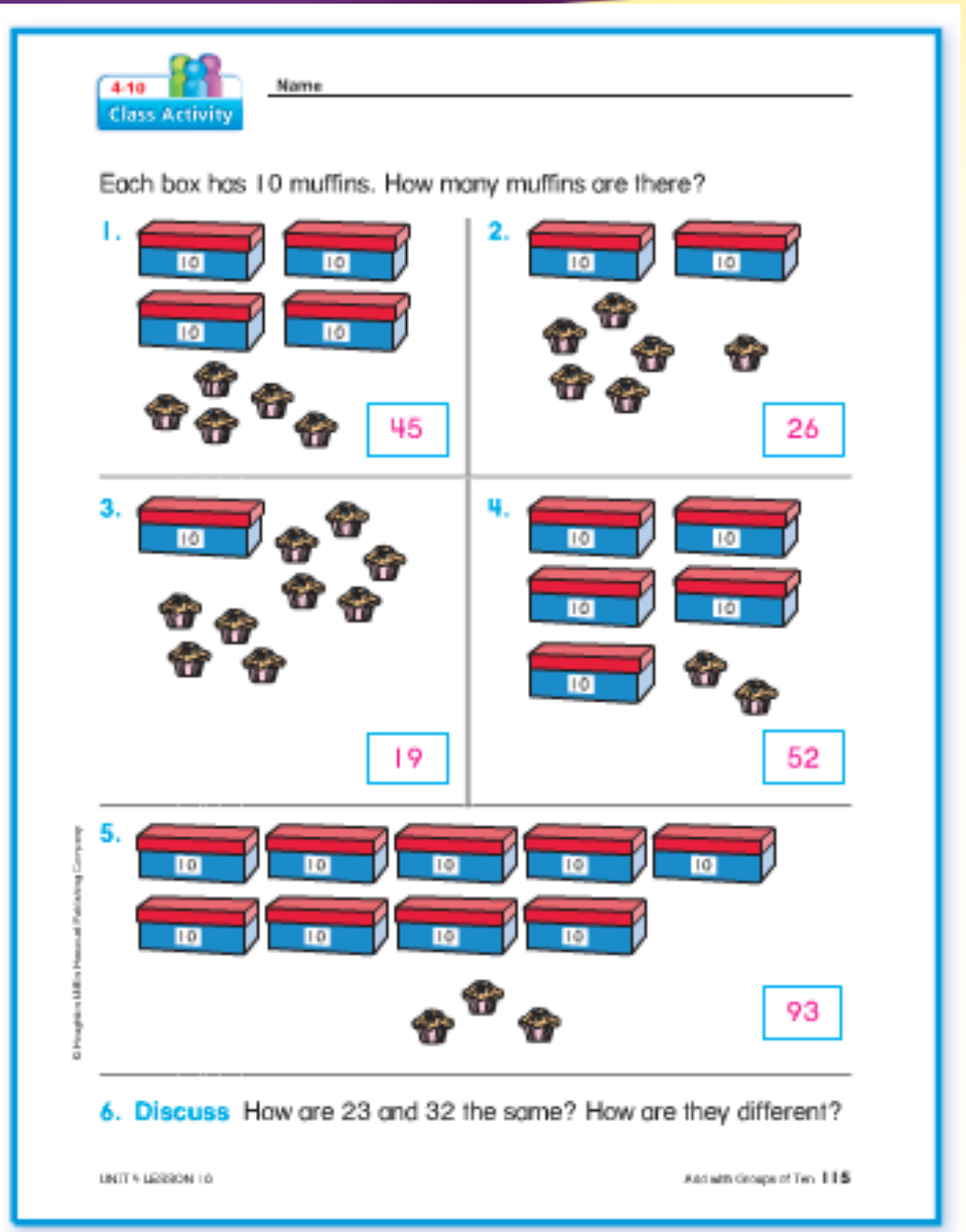


G1 Tens and Ones on the Number Path 1.NBT.2

Number Path



G1 Unseen Tens and Some Ones
1.NBT.2



4-10
Class Activity

Name _____

Each box has 10 muffins. How many muffins are there?

1.

45

2.

26

3.

19

4.

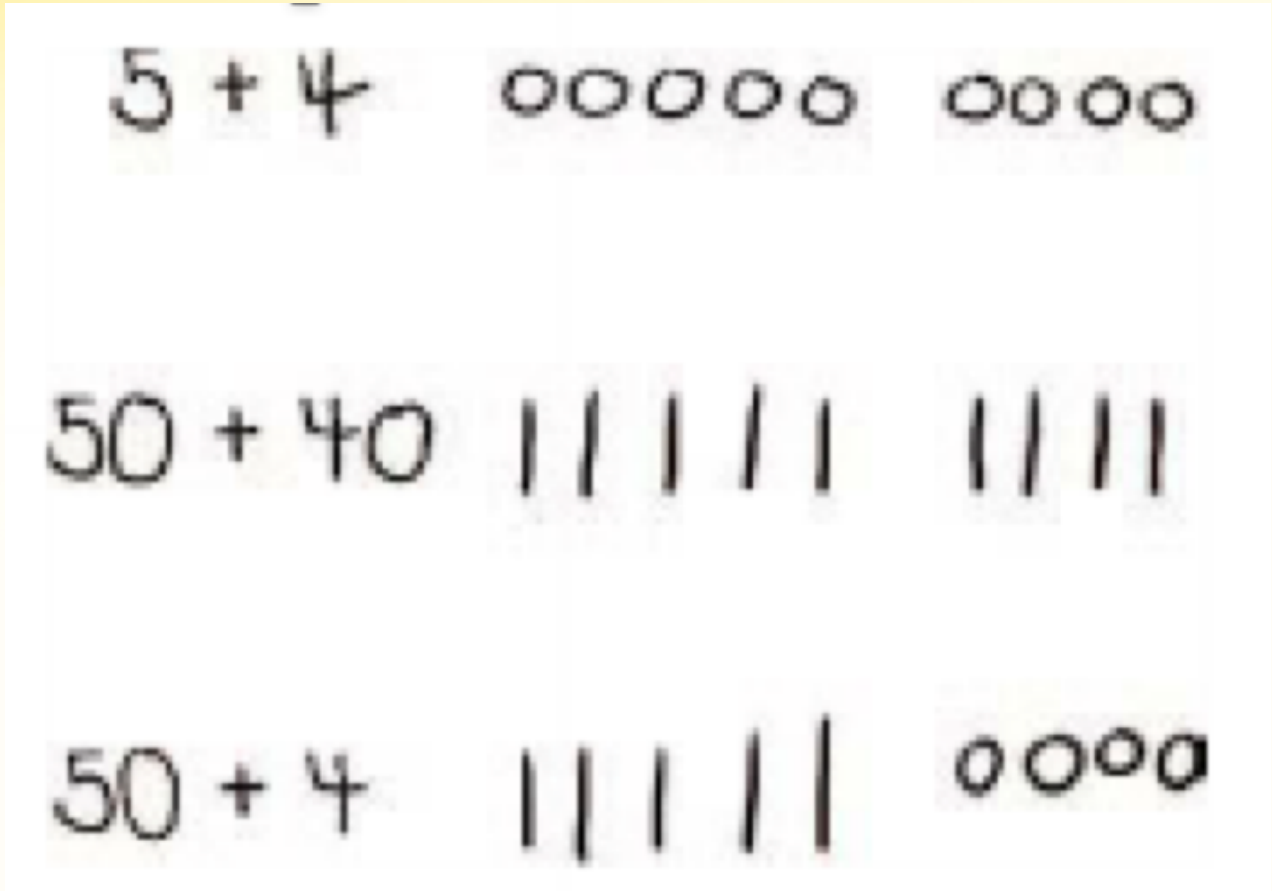
52

5.

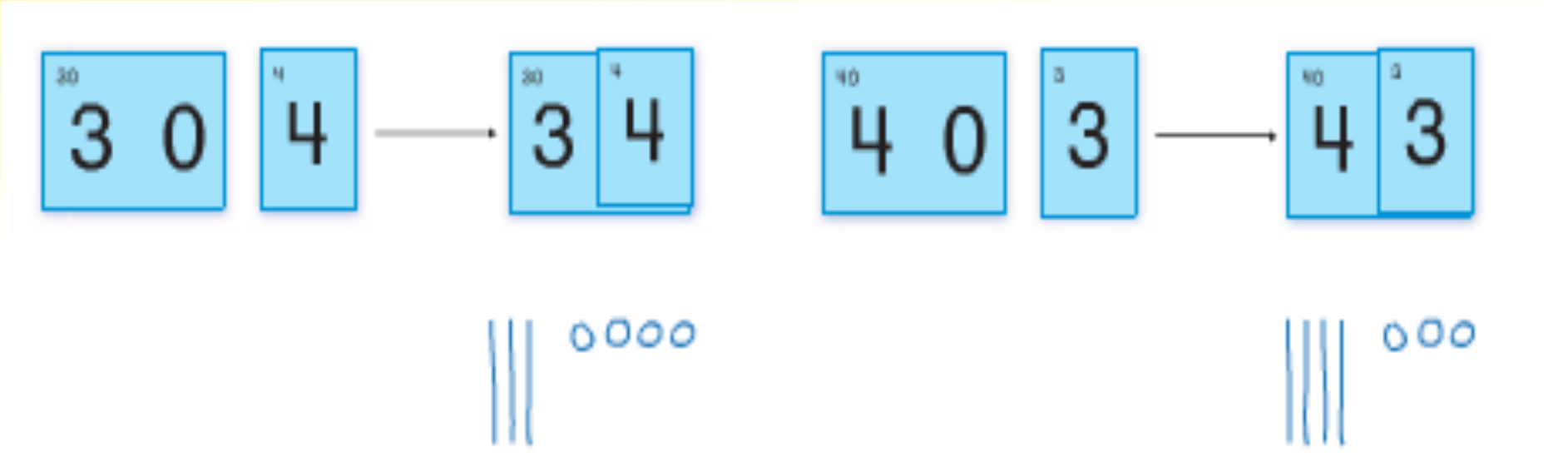
93

6. **Discuss** How are 23 and 32 the same? How are they different?

G1 Adding Tens and Ones 1.NBT.2 and 1.NBT.4



G1 Comparing 2-Digit Numbers
1.NBT.3



Percentage Showing Ten Chips as the Meaning for the 1 in 16

Samples	Immediate showing of ten	After 1, 2, or 3 prompts	Total showing 1 as ten
Kamii (1989) suburban students			
n = 32 Grade 2	16	ng	
n = 40 Grade 3	30	ng	
Math Expressions Kindergarten			
n = 85 Mean percentage for all children	34	25	59
By school			
n = 24 Full-day B	75	21	96
n = 22 Full-day C	23	23	46
n = 39 Half-day A	5	31	36
Math Expressions Grade 1			
n = 25	64	32	96

Note. The Kamii (1989) samples were from an upper-middle class school with high test scores. The *Math Expressions* kindergarten children were from three schools (A, B, C) with high levels of students with free lunch and many English language learners.

Percentage Use of Correct Representations on the Miura 2-Digit Task by Cognitive Representation Category

Trial 1 Representation	Miura (1988) upper-middle-class Grade 1				Miura	*Math Expressions* low-SES		
					Kinder garten	Kindergarten		Grade 1
	U. S. *n* = 24	PRC *n* = 25	Japan *n* = 24	Korea *n* = 40	Korea *n* = 20	Half-day *n* = 39	Full-day *n* = 46	*n* = 23
Canonical base-10: 1 ten 3 ones 2 tens 8 ones	8	81	72	83	34	40	79	94
Noncanonical base-10: 10 + 3 ones 1 ten 18 ones	1	9	10	11	7	25	6	2
Unitary collection: 13 ones 28 ones	91	10	18	6	59	35	15	4

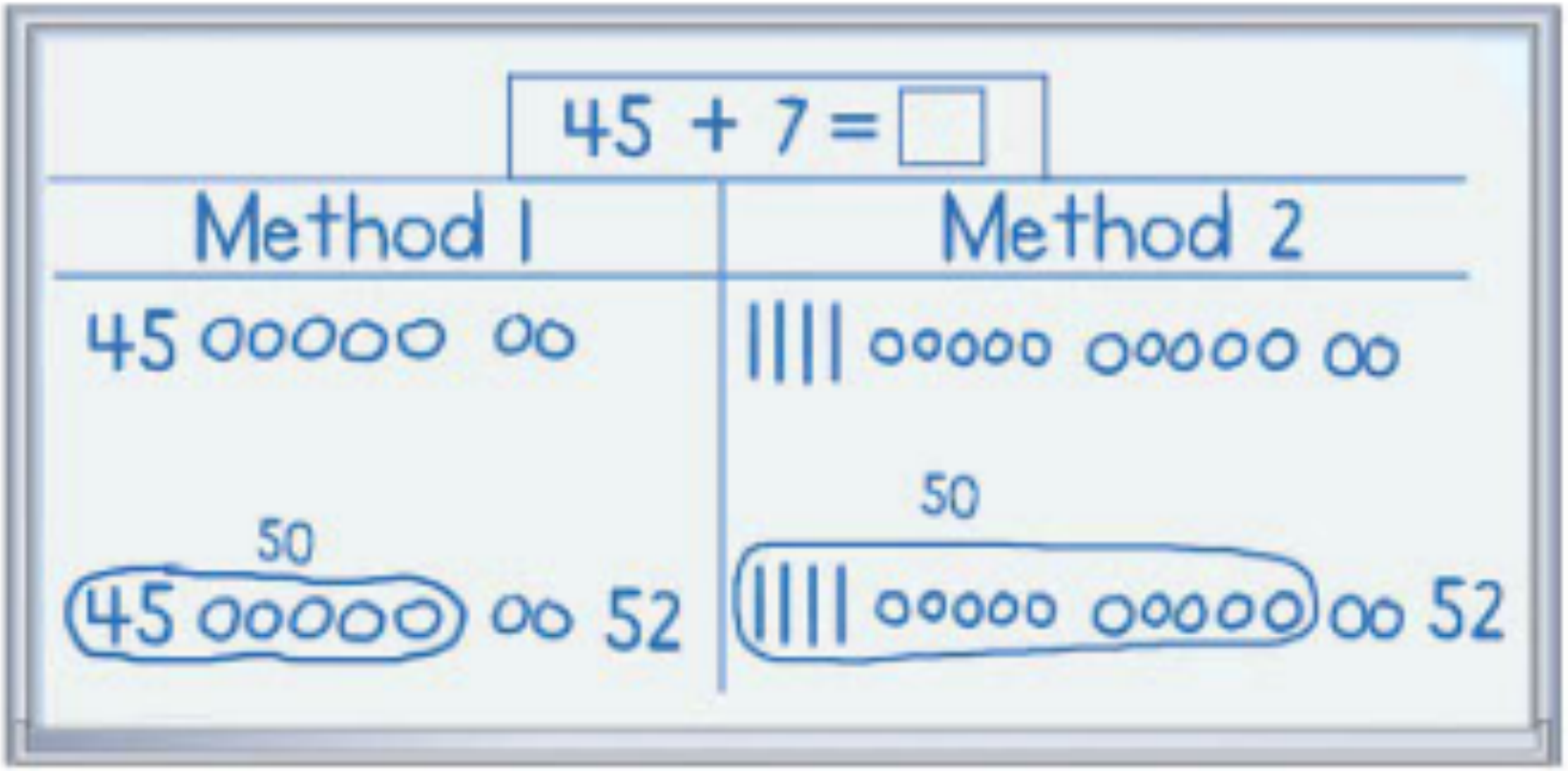
Note. This task falls centrally within the following CCSS: K.NBT.1 and 1.NBT.2 knowledge.

Percentage of Children in the Chandler and Kamii Sample and in Math Expressions Grade 1 Giving 4 Pennies as Change From a Dime for a 6ç Purchase

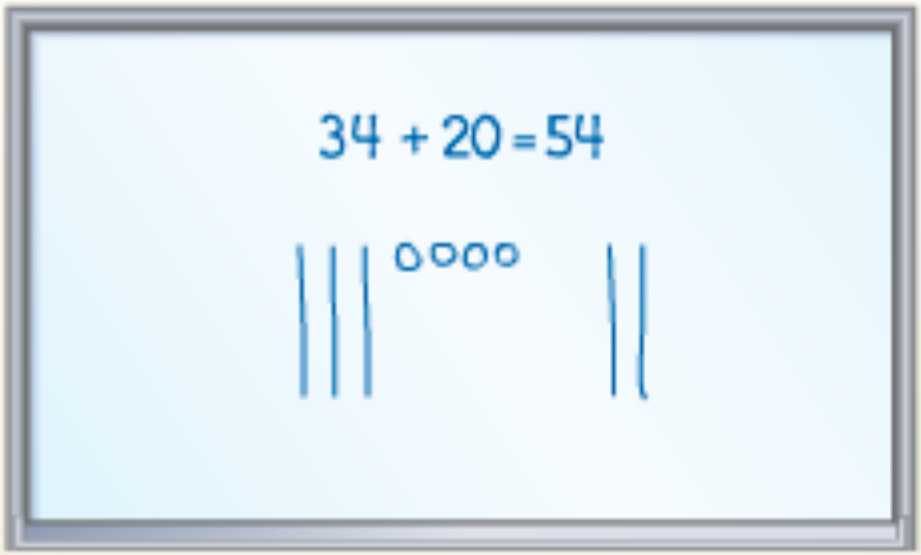
Task	Chandler and Kamii Sample					*Math Expressions* Sample
	Kindergarten n = 19	Grade 1 n = 21	Grade 2 n = 20	Grade 3 n = 19	Grade 4 n = 19	Grade 1 n = 20
Gives 4 pennies as change from a dime for a 6ç purchase	5***	14***	50***	74*	84	100

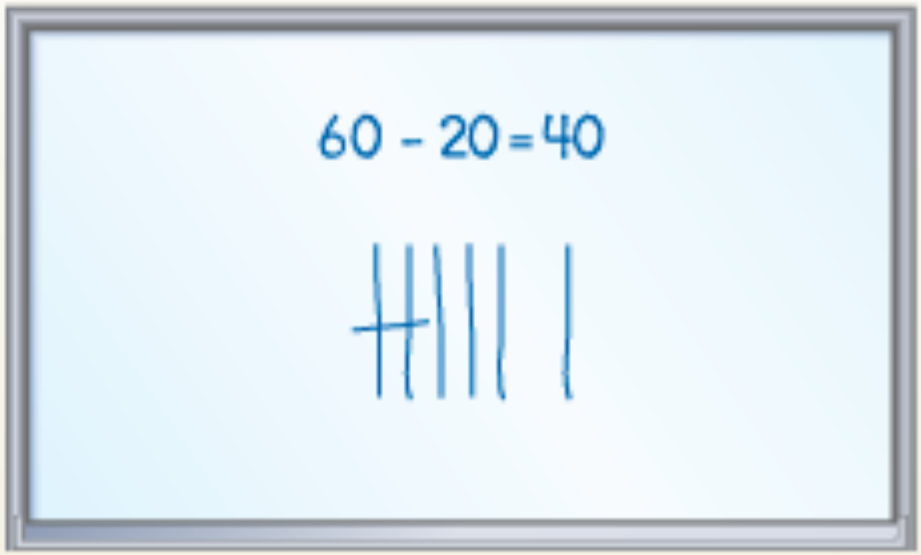
Note. Chi-square analyses on the number of children giving 4 pennies found that significantly more *Math Expressions* first graders did so than did the Chandler and Kamii kindergarten, grade 1, grade 2, and grade 3 children $p<0.001$ and 0.05 levels for *** and *, respectively. These tasks fall centrally within the following CCSS: 1.OA.1, 6 and 2.MD.8.

G1 2-Digit + 1-Digit Numbers 1.NBT.4

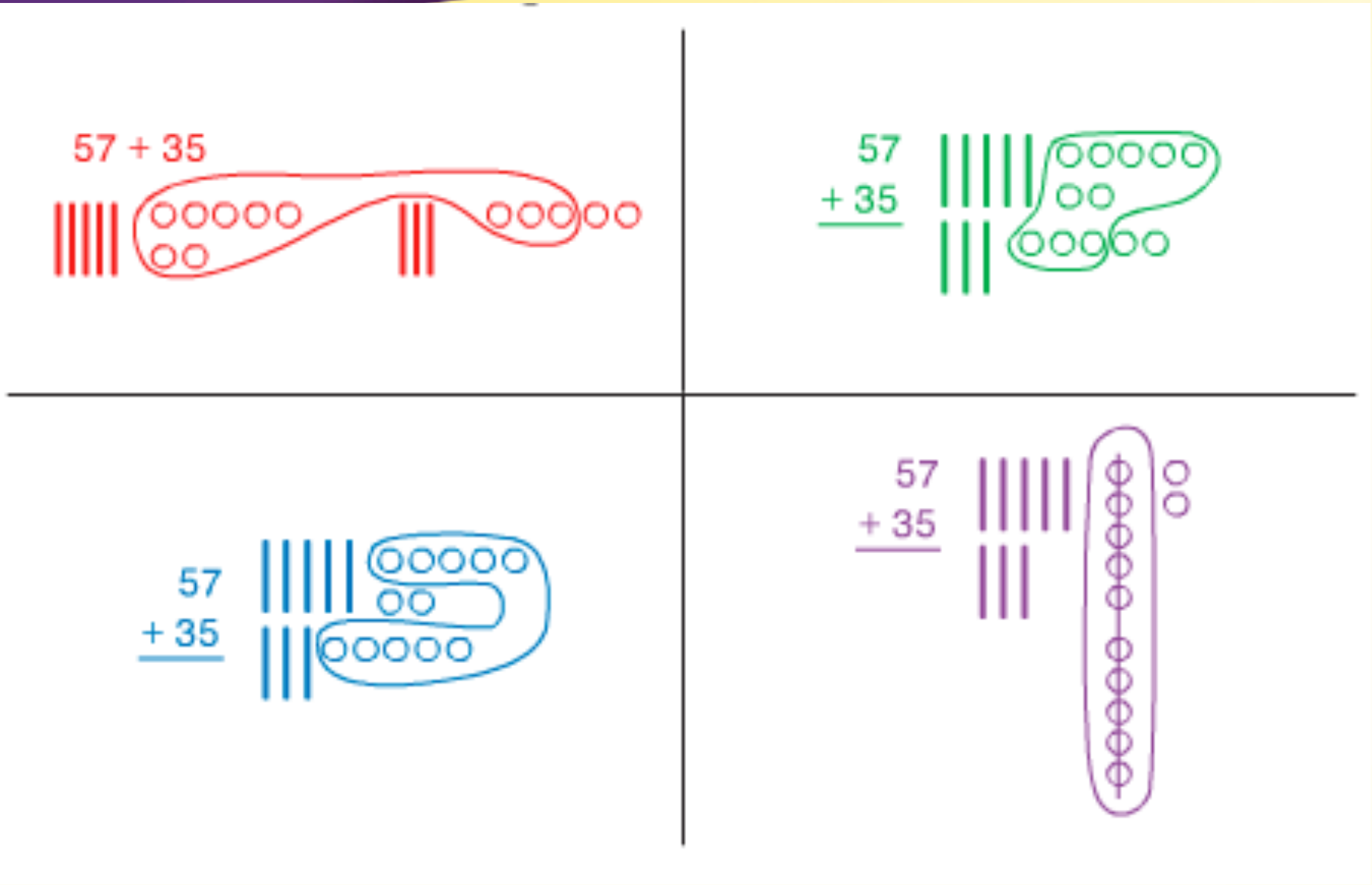


G1 Add and Subtract Tens 1.NBT.4 and 1.NBT.6



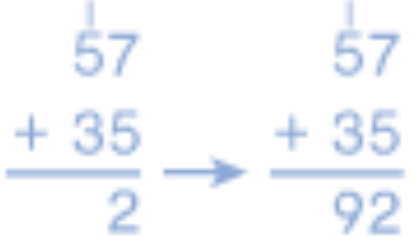


G1 Possible Student Drawings 1.NBT.4

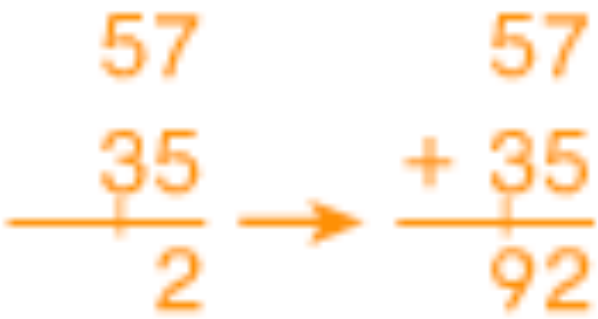


G1 Possible Written Methods 1.NBT.4

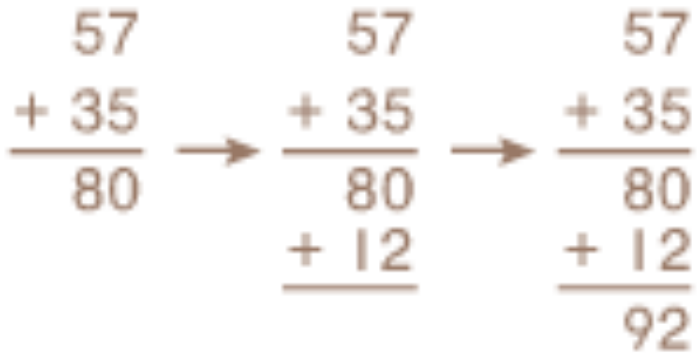
New Group Above



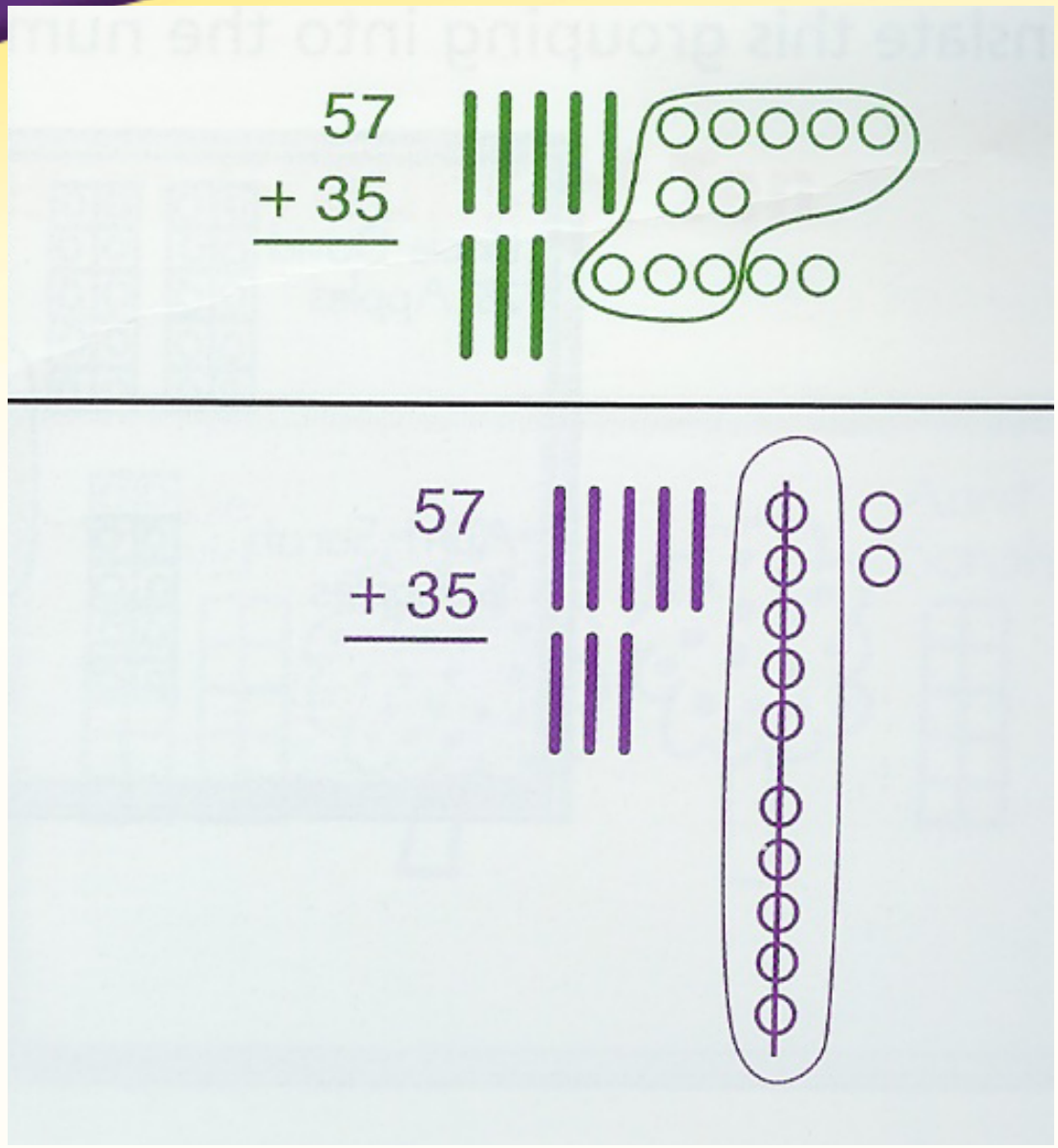
New Group Below



Show All Totals



G1 Show All Totals 1.NBT.4



$$\begin{array}{r} 57 \\ +\ 35 \\ \hline 80 \\ 12 \\ \hline 92 \end{array}$$

G1 New Groups Below 1.NBT.4

$$\begin{array}{r} 58 \\ +\ 36 \\ \hline 1 \\ \hline 4 \end{array}$$

G1 New Groups Below 1.NBT.4

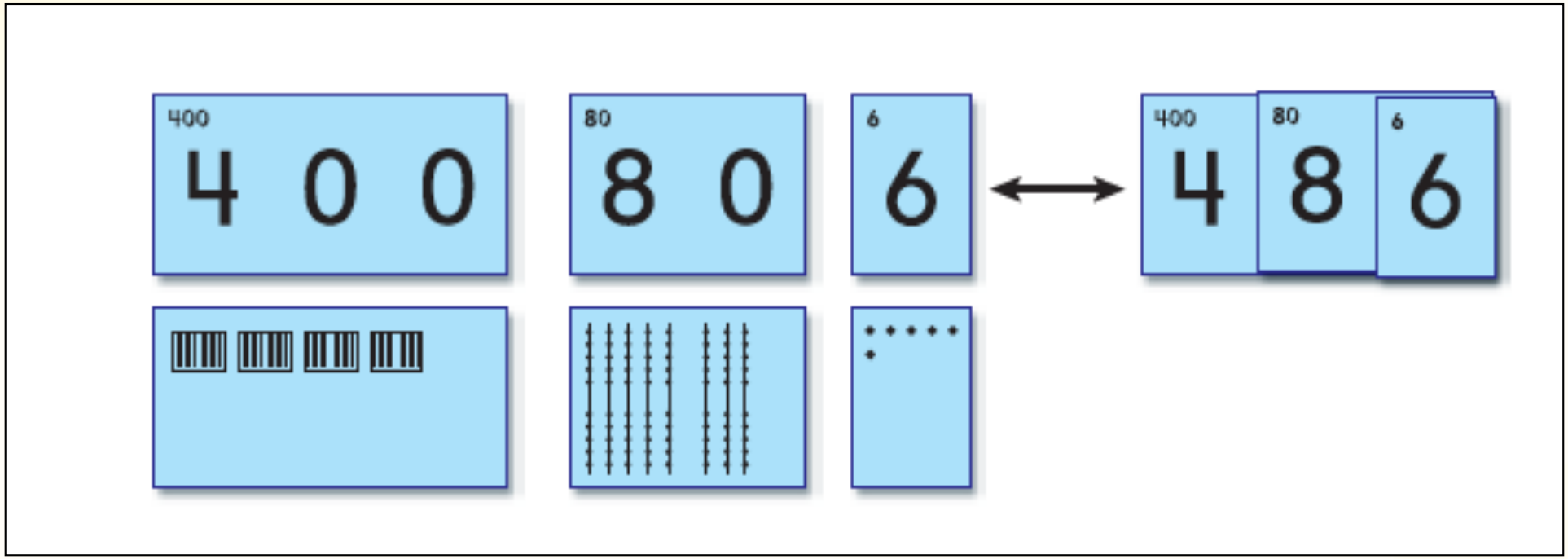
$$\begin{array}{r} 58 \\ +\ 36 \\ \hline {\scriptstyle 1} \\ \hline 94 \end{array}$$

Percentage Correct on 2-Digit Addition With Regrouping for Grade 1 Japanese, Chinese, and U.S. Children and Math Expressions Grade 1 Children

	Stigler, Lee, & Stephenson (1990)			
Task	Japanese Grade 1 $n = 120$	Chinese Grade 1 $n = 120$	U.S. Grade 1 $n = 240$	*Math Expressions* Grade 1 $n = 24$
2-digit word problem: Chris has 26 toy cars. Mary has 19. How many do they have in all?	29*	25*	13*	79

Note. The Stigler et al. (1990) sample U.S. Grade 1 children were from 20 public and private schools from a range of economic levels in the Chicago area (urban and suburban). The Japanese and Chinese children were from comparable backgrounds. Grade 1 *Math Expressions* children were randomly selected from 2 classrooms in School A. All of the children qualified for free or reduced lunch, and all were native Spanish speakers but were in English-speaking classrooms in Grade 1. Chi-square analyses on the number of children answering correctly found that significantly more *Math Expressions* first graders did so than did the Stigler, Lee, and Stephenson (1990) Japanese, Chinese, and U.S. children, each $p<0.001$. This task falls centrally within the following CCSS: 1.NBT.4 and 1.OA.1 knowledge.

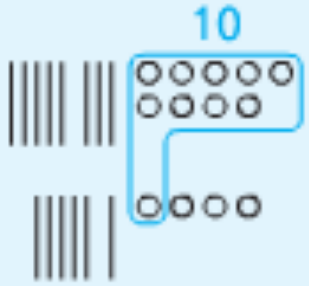
G2 Secret Code Cards for 486
2.NBT.1 and 3



G2 Mrs. Green's Method

Step 1: Add the Ones

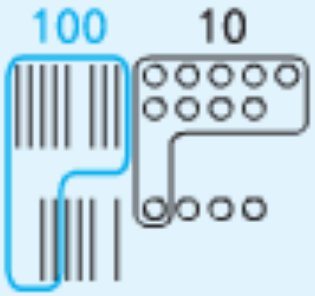
$$\begin{array}{r} 89 \\ +\ 64 \\ \hline 3 \end{array}$$



1 ten + 3 ones = 13 ones

Step 2: Add the Tens

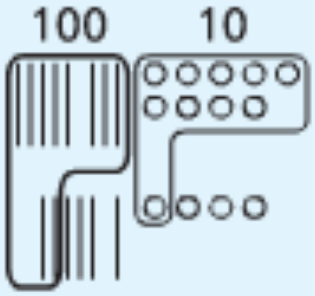
$$\begin{array}{r} 89 \\ +\ 64 \\ \hline 53 \end{array}$$



8 tens + 6 tens + 1 ten = 15 tens =
1 hundred 5 tens

Step 3: Add the Hundreds

$$\begin{array}{r} 89 \\ +\ 64 \\ \hline 153 \end{array}$$



0 hundreds plus 1 hundred = 1 hundred
The answer is 153.

Discuss how the New Groups Below method solves the problem of the "mysterious new hundred."

Concepts of Numbers over Ten

Unitary: a collection of single units
Kindergarten and Grade 1: Count these units by ones using English count words

Tens-Total and Ones: Separate the ones from the tens-total:
Count by ones using English count words but notice the total of the separated tens-total
Kindergarten: for *11 to 19: ten and eight is eighteen* 18 = 10 + 8
Grade 1: for *20 to 99: seventy and nine is seventy nine* 64 = 60 + 4

Sequence-Count-by-Tens and by Ones: count each group of ten ones by English tens words
Kindergarten: *ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety*
Grade 1: Count by tens and then count on by ones from the final tens word:
ten, twenty, thirty, forty, fifty, sixty SHIFT THE COUNTING UNIT
sixty-one, sixty-two, sixty-three, sixty-four

Separate-Count-the-Tens and Ones: count or see each tens quantity as 1 ten and use tens and ones words to give the multiunit value: *one ten, two tens, three tens, four tens, five tens six tens* and *one, two, three, four ones* to make *six tens four ones*

See these NCTM books for more information about NBT concepts:

Focus in kindergarten:
Teaching with curriculum focal points

Focus in grade 1:
Teaching with curriculum focal points

Focus in grade 2:
Teaching with curriculum focal points

Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings
that students can make and use in their own ways
in problem solving and explaining of thinking.

We want classrooms to be using the mathematical practices in the Math Talk Community:
Students focus on math sense-making about math structure using math drawings (visual models) to support math explaining.

Kindergarten and Grade 1 Children Living in Poverty Can Learn the CCSS NBT Concepts

Professor Emerita Karen C. Fuson

Northwestern University

For more details about NBT concepts and visual supports for the learning progression, or for details about other CCSS domains, please see the 13 hours of audio-visual Teaching Progressions I have made. You can find links to these and to papers and other presentations at karenfusonmath.net

This presentation is also posted there.