Teaching Area in the Common Core through Decomposing and Composing

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Outline

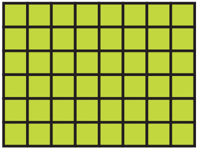
- What is area?
  Composing and decomposing unit squares in Grades 3 - 5
- Addressing Grade 6 Common Core area standards through decomposing and composing
  - developing area formulas
  - avoiding errors in applying formulas
  - using equivalent expressions
  - distinguishing surface area from volume
- The roots in PK – Grade 2 for understanding length, area, volume, and shapes
- A few connections to other math and later math
- Discussion

Area of rectangles: composing unit squares

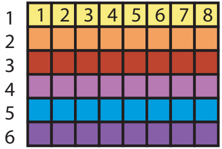
1 square unit

What is the area of this rectangle in square units?

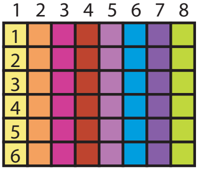
Cover the rectangle with squares. How many squares?



Is there a quicker way to find the area than counting all the squares one by one?

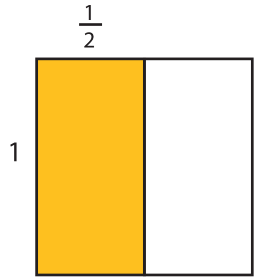


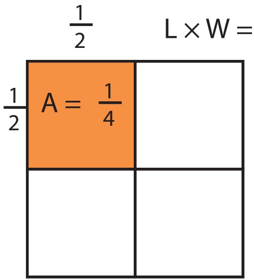
View as 6 groups of 8 squares.
$6 \times 8 = 48$ square units



View as 8 groups of 6 squares.
$8 \times 6 = 48$ square units

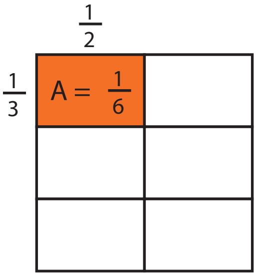
Fractional side lengths: decomposing a unit square

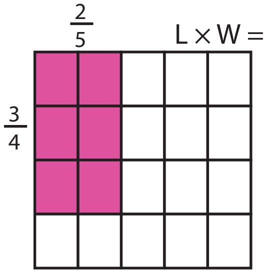




$L \times W = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2}$ of $\frac{1}{2}$ of a square unit





$L \times W = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$

$\frac{3}{4}$ of $\frac{2}{5}$ of a square unit

shaded $A = \frac{6}{20}$

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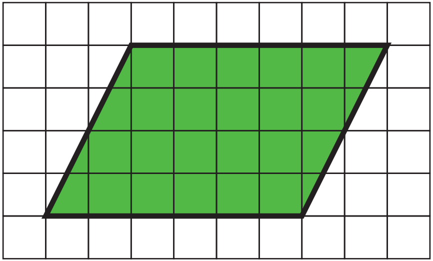
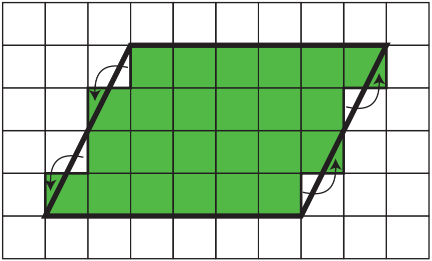
Areas of parallelograms

What is the area of the shaded parallelogram?



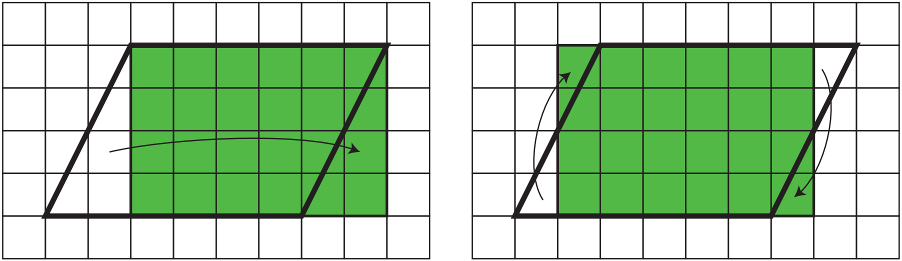
Most primitive method:



Move and combine small pieces; count the number of squares.

Areas of parallelograms

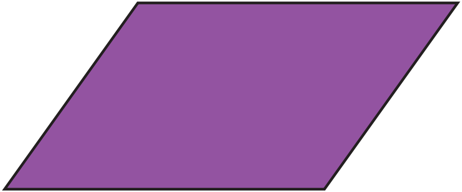
More advanced methods that will generalize to develop area formulas:



Move chunks to create a rectangle of the same area.

Areas of parallelograms

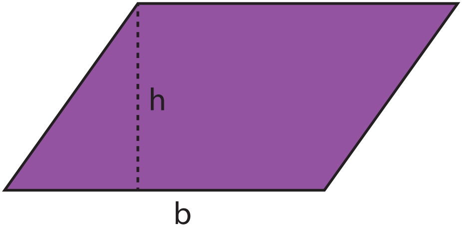
What information do we need to find the area of a parallelogram?



Common error: multiply side lengths.

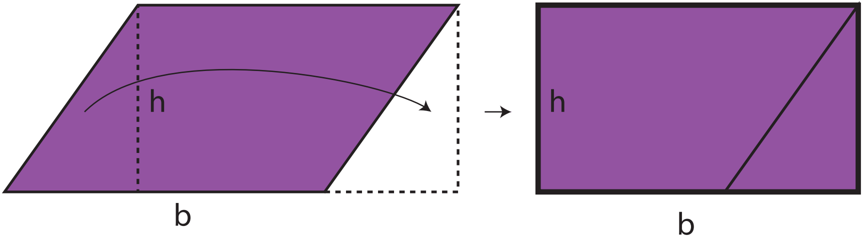
Developing a parallelogram area formula

What is a formula for the area, A, of a parallelogram, in terms of b and h, and why is the formula valid?



Developing a parallelogram area formula

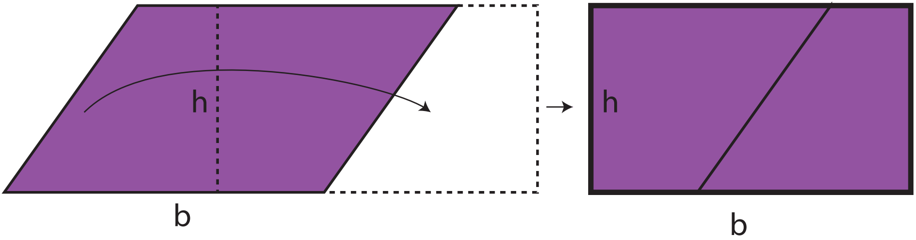
Decompose the parallelogram and compose into a rectangle of the same base, b, height, h, and area, A.



$$A = b \cdot h$$

Developing a parallelogram area formula

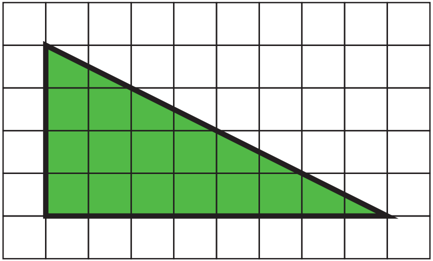
Another way to decompose the parallelogram and compose into a rectangle of the same base, b, height, h, and area, A.



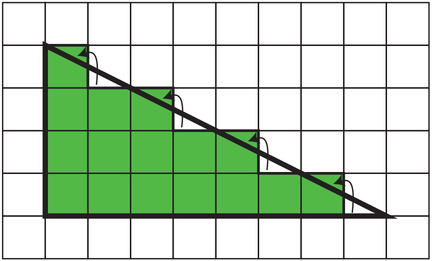
$$A = b \cdot h$$

Areas of triangles

What is the area of the shaded triangle?



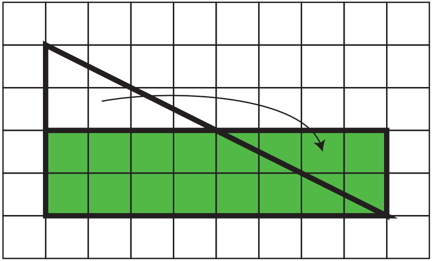
Most primitive method:



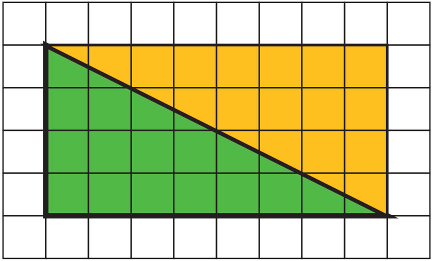
Move and combine small pieces; count the number of squares.

Areas of triangles

More advanced methods that will generalize to develop area formulas:



Move a chunk to create a rectangle of the same area.



Combine two copies to make a rectangle of twice the area.

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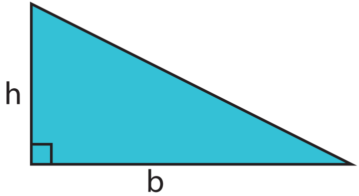
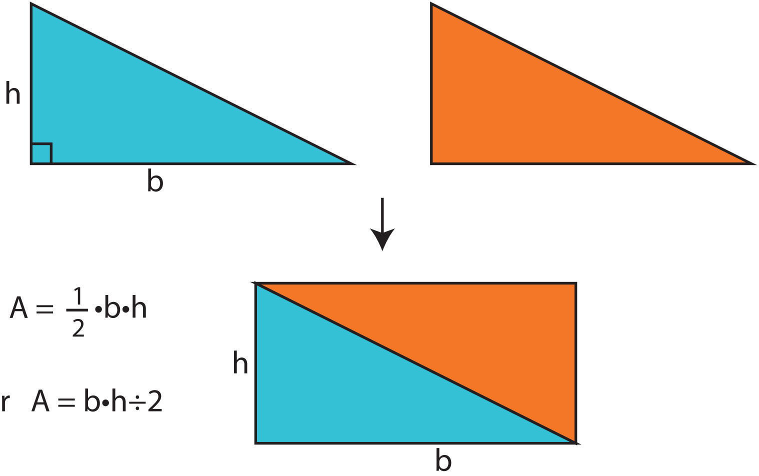
Developing area formulas for right triangles

How can we express the area of the triangle in terms of b and h?



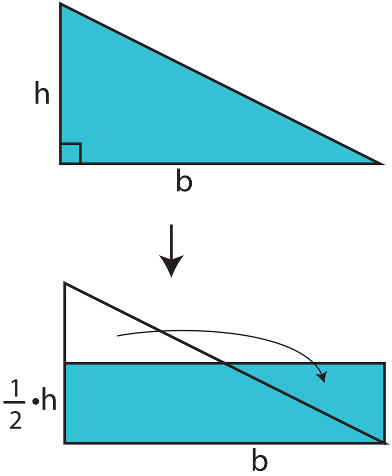


$$A = \frac{1}{2} \cdot b \cdot h$$

or $A = b \cdot h \div 2$

The right triangle is *half* of a rectangle of the same base and height.

Developing area formulas for right triangles



$$A = b \cdot \frac{1}{2} \cdot h$$

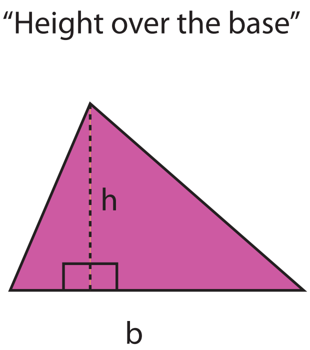
$$A = \frac{1}{2} \cdot b \cdot h$$

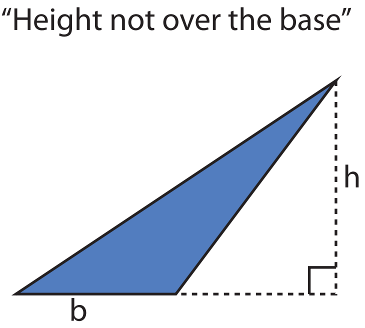
or $A = b \cdot h \div 2$

The rectangle has the same base but *half* the height.

Areas of non-right triangles

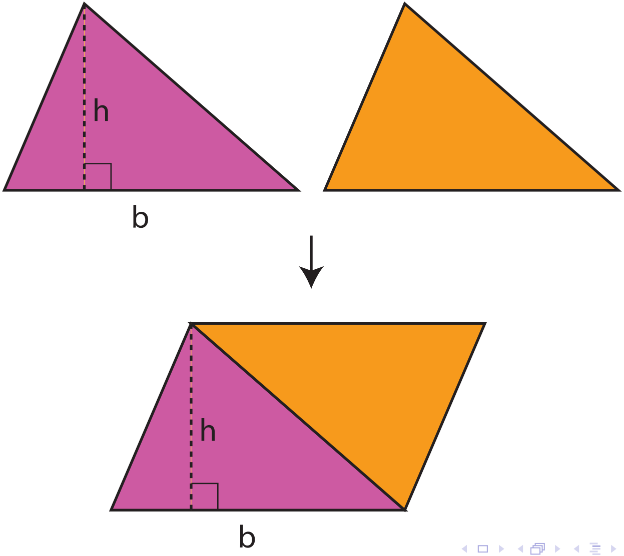
Why does the area formula still work for triangles like these?





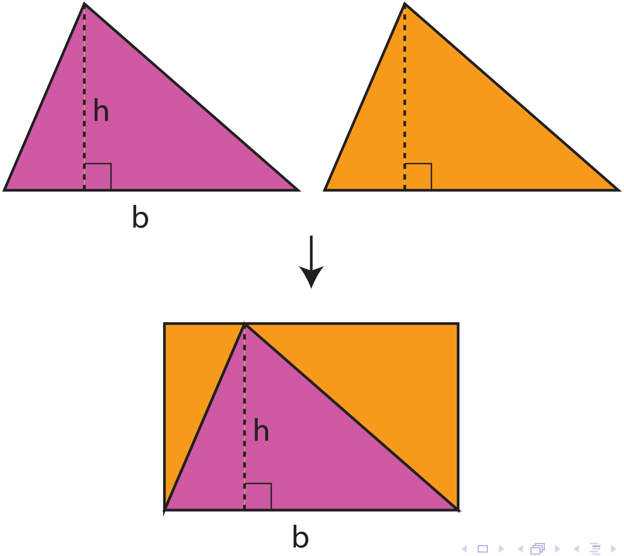
Areas of non-right triangles

The triangle is *half* of a parallelogram of the same base and height.



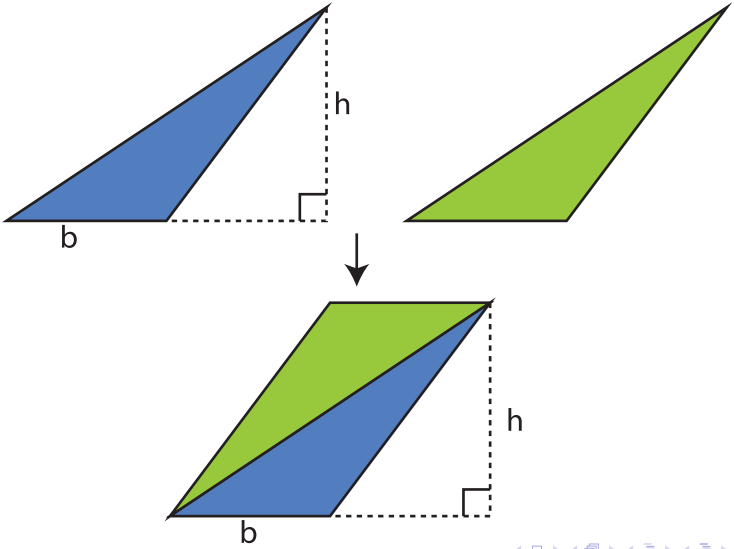
Areas of non-right triangles

The triangle is *half* of a rectangle of the same base and height.



Areas of non-right triangles

The triangle is *half* of a parallelogram of the same base and height.



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Areas of non-right triangles

Why do we need the “height not over the base” case?

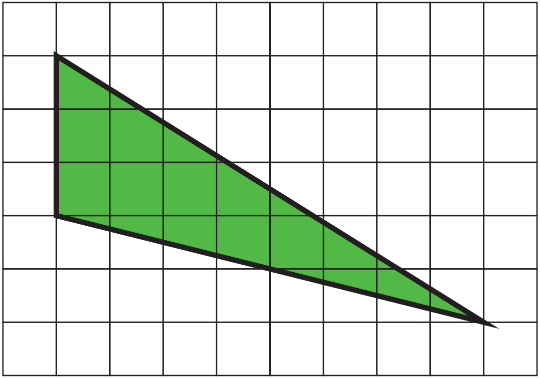
Couldn’t we just use a different base in that case?

Does the base have to be on the bottom?

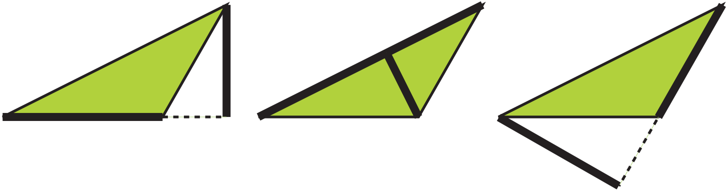
Areas of non-right triangles

We need to be able to use bases that are

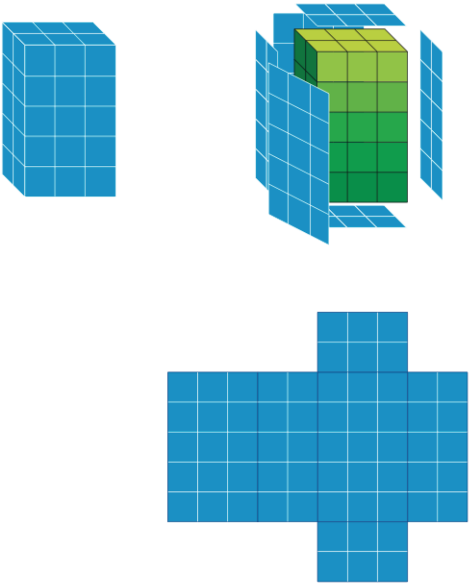
- not on the bottom
- result in the height not over the base.



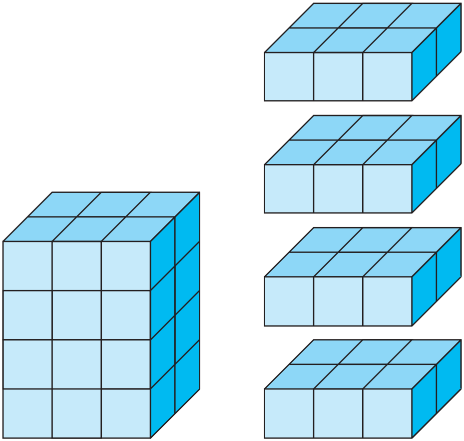
Any side can be chosen to be the base of a triangle



Decomposing the surface to find surface area



Decomposing prisms into layers to explain the volume formula



volume = (height)×(area of base)

PK through Grade 2

Prerequisite experiences for understanding

length, area, volume

and shapes

NRC Early Childhood Report
NCTM Focal Point Books PK, K, G1, G2
Consistent with CCSS progression

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Extensive grounding in
core geometric seeing frames

Shapes with right angles
(pre-area)

Horizontal/vertical axes
(pre-area and pre-graphing)

Unit lengths, unit squares
(square area grid), unit cubes
Units for measuring

Later (G1)
Parallel lines/sides
parallelograms

3 experiential worlds

The right angle world: rectangles, square rectangles, right triangles, isosceles triangles (make from 2 congruent right triangles)

The equilateral triangle world: one rhombus, one trapezoid, hexagon

The parallelogram and acute/obtuse triangle world: non-rectangular parallelograms and the acute and obtuse triangles made by their diagonals

Lots of composing/decomposing with objects from these worlds

Also from PK on children need to see varied examples of many shapes and in different orientations

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Grade level foci:

PK right angle world

K right angle and equilateral triangle world with some units of units

G1 all three worlds with extensive units of units and some units of units of units

G2 formal length units and lengths
Draw shapes from right angle world

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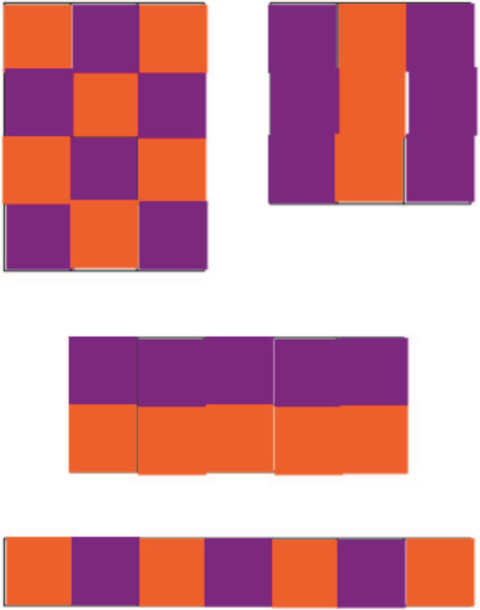
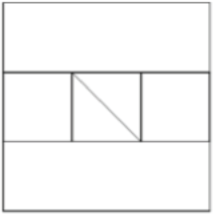
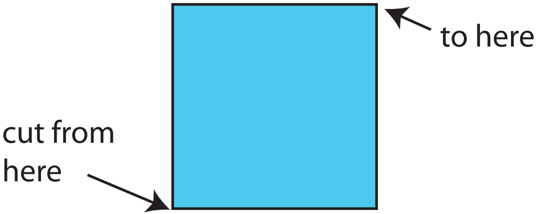




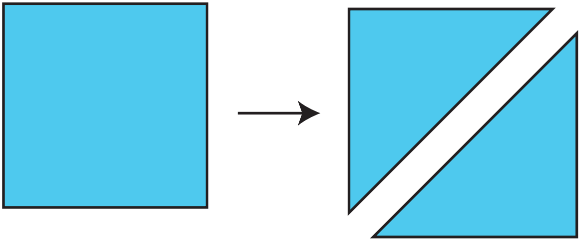
Decomposing a square and recomposing

What if we cut the square from one corner to the opposite corner? What shapes will we get?



Decomposing a square and recomposing

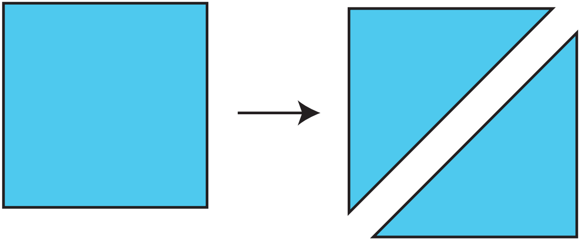
We get two triangles!



Can we put the triangles together in other ways?

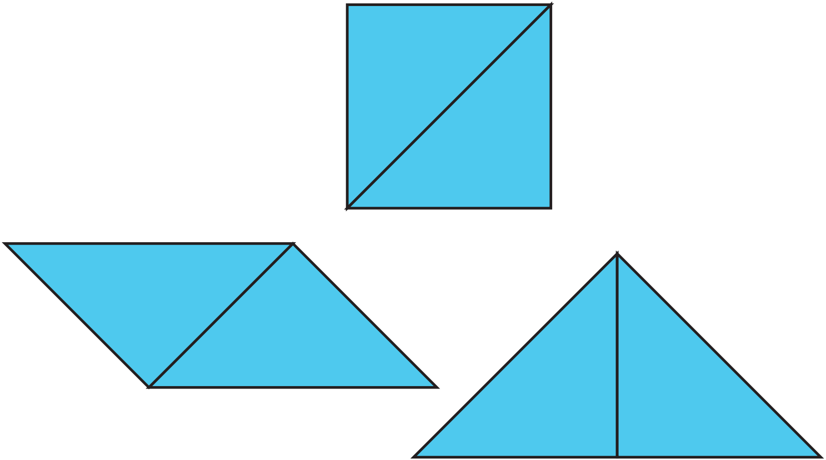
Decomposing a square and recomposing

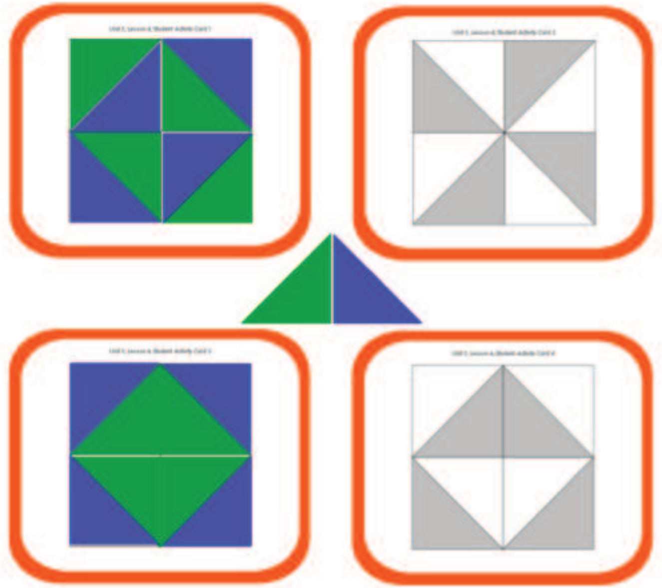
We get two triangles!



Can we put the triangles together in other ways?

Decomposing a square and recomposing





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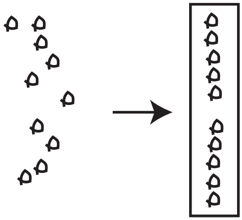
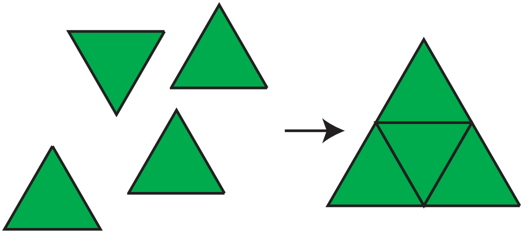
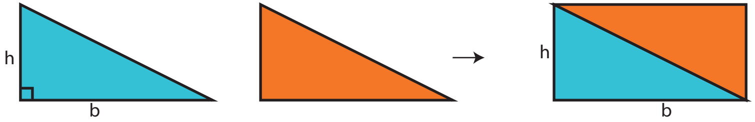
Composing and decomposing across math

Composing a unit, decomposing a unit



10 ones are grouped
to form one ten





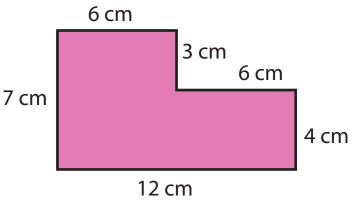
$$\updownarrow$$

$$5 \times 86 = \frac{1}{2}(10 \times 86)$$

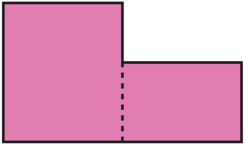
Composing and decomposing across math

Area and algebraic expressions

What is the area of the shaded shape?

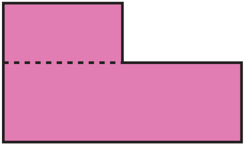


Method 1



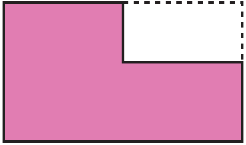
$7 \times 6 + 4 \times 6$

Method 2



$3 \times 6 + 4 \times 12$

Method 3

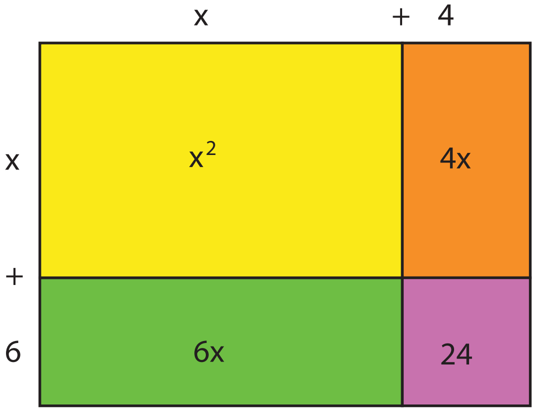


$7 \times 12 - 3 \times 6$

Composing and decomposing across math

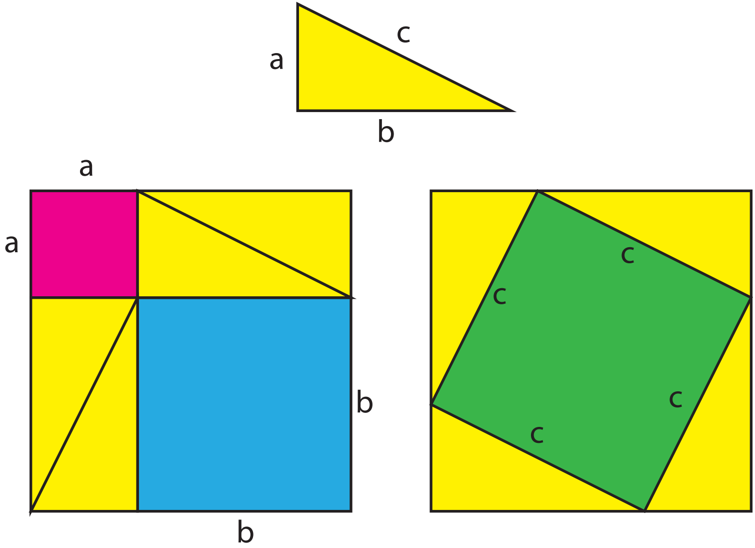
Area and algebraic equations

$$(x + 6)(x + 4) = x^2 + 4x + 6x + 24$$

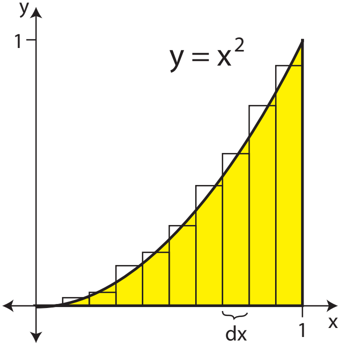


Composing and decomposing across math

The Pythagorean Theorem



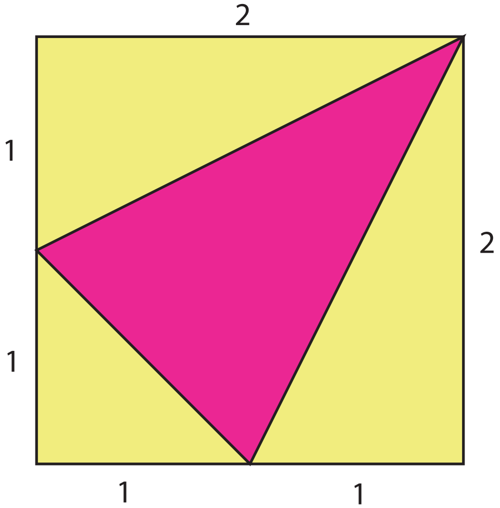
Composing and decomposing across math – calculus



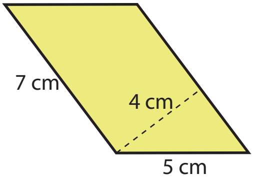
$$\text{area under curve} = \int_0^1 x^2 dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}$$

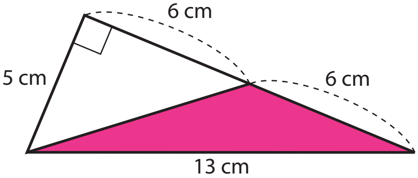
Area problem solving

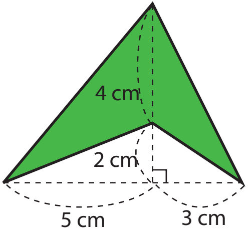
What is the area of the pink triangle inside the square?



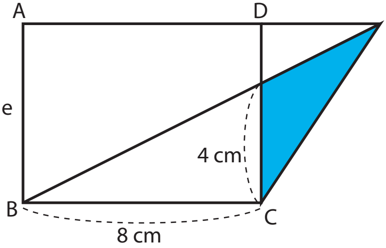
Area problem solving







ABCD is a rectangle of area 48 cm²



Area problem solving