Fuson, K. C., Smith, S. T., & Lo Cicero, A. (2002). Supporting Latino first graders' ten-structured thinking in urban classrooms. In J. Sowder & B. Schapelle (Eds.), Lessons Learned from Research (pp. 155-162). Reston, VA: NCTM.

CHAPTER 19

SUPPORTING LATINO FIRST GRADERS' TEN-STRUCTURED THINKING IN URBAN CLASSROOMS

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Abstract. Year-long classroom-teaching experiments in 2 predominantly Latino low-socioeconomic-status (SES) urban classrooms (1 English speaking and 1 Spanish speaking) were designed to support 1st-graders' thinking of 2-digit quantities as 10s and 1s. A model of a developmental sequence of conceptual structures for 2-digit numbers (the UDSSI triad model) is presented to describe children's thinking. By the end of the year, most of the children could accurately add and subtract 2-digit numbers that require trading (regrouping) by using drawings or objects and could give answers by using 10s and 1s on various tasks. Their performance was substantially above that reported in other studies for U.S. 1st graders of higher SES and for older U.S. children. Their responses looked more like those of East Asian children than those of U.S. children in other studies.

Our first purpose in this chapter is to describe two aspects of the research that may be particularly helpful to others: (a) a developmental sequence of conceptual structures for two-digit numbers that guided the instructional-design work and (b) the conceptual supports we used to assist children's construction of these conceptual structures. In particular, we describe our method for encouraging children to draw quantities organized by tens; using this method, we addressed major pragmatic and instructional-assessment issues and afforded multiple solution methods. Our second purpose in this chapter is to describe the learning of the children in the two classes as it compared with that of East Asian and other U.S. students. We first describe the developmental sequence of conceptual structures for two-digit numbers.

ANALYSIS OF THE MATHEMATICAL DOMAIN OF MULTIDIGIT NUMBERS

We developed a triad model of two-digit conceptions (shown in Figure 19.1). (The introduction to this section contains additional information that might help you understand various parts of this complex figure.) *Triad* refers to the relationships among quantities, number words, and written numerals. In this model we describe 5 two-digit conceptions.

All children begin with a *unitary conception* of two-digit numbers; this conception is a simple extension from a unitary conception of single-digit numbers (see the top-middle and top-right-hand parts of Figure 19.1). With this conception, the number words and the two digits do not each refer to quantities being counted. Rather, the entire number word (e.g., *fifty-three*) or numeral (53) refers to the whole quantity; that is, the child does not recognize tens and ones.

With time and experience, each number word and each digit do take on meaning as a decade or as the extra ones in the *decade-and-ones* conception.

This chapter is adapted from Fuson, K. C., Smith, S. T., & Lo Cicero, A. M. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28,* 738–766.

The research reported in this chapter was supported in part by the National Center for Research in Mathematical Sciences Education, which is funded by the Office of Educational Research and Improvement (OERI) by the U.S. Department of Education under Grant R117G10002, and in part by the National Science Foundation (NSF) under grant RED935373. The opinions expressed in this chapter are those of the authors and do not necessarily reflect the views of OERI or of NSF.

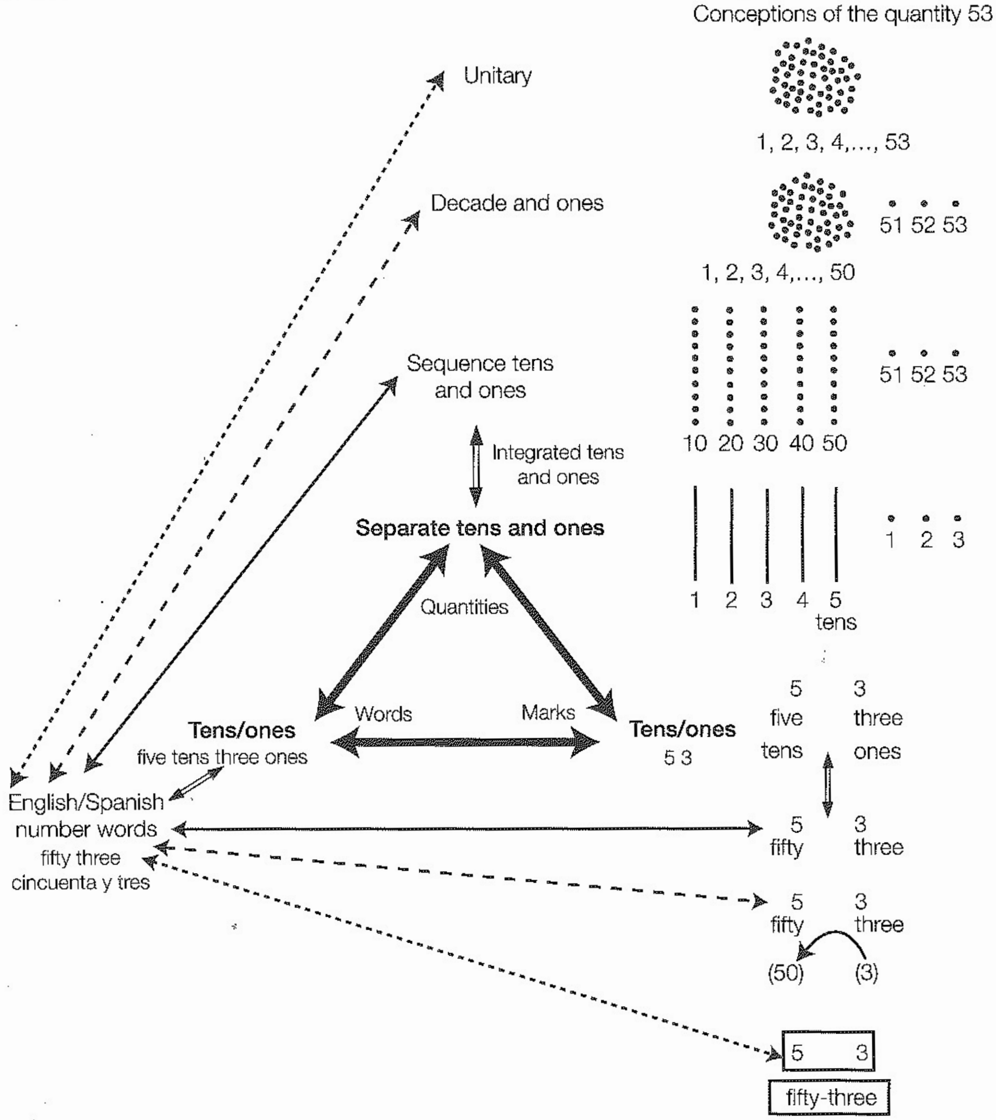

Figure 19.1. A developmental sequence of conceptual structures for two-digit numbers: The UDSSI Triad Model.

For example, in 53, the 5 means fifty and the 3 means three.

In the *sequence-tens-and-ones conception*, an extension of the decade-and-ones conception, units of ten single units are formed within the decade part of the quantity. These sequence-tens units are counted by tens (e.g., 10, 20, 30, 40, 50); then the ones are counted by continuing from 50 (51, 52, 53).

The *separate-tens-and-ones conception* is built through experiences in which a child comes to think of a two-digit number as comprising two separate kinds of units—units of ten and units of one. Both kinds of units are counted by ones (e.g., "1, 2, 3, 4, 5 tens and 1, 2, 3 ones"). In Figure 19.1, we show these units of ten as a single line to emphasize their (ten)-unitness, but the user of these units understands that each ten is composed of 10 ones and can switch to thinking of 10 ones if that approach becomes useful.

The first two conceptions are developmental, and children move from the first through the second. But children's construction of the sequence-tens-and-ones and separate-tens-and-ones conceptions depends heavily on their learning environments. Students in the same classroom may construct one or the other of these conceptions first: A child who focuses on the words will develop the sequence-tens-and-ones conception first; a child who focuses on the written numerals will develop the separate-tens-and-ones conception first.

Children may eventually relate the sequence-tens-and-ones and separate-tens-and-ones conceptions to each other in an *integrated sequence-separate conception* (these connections are shown in Figure 19.1 as the short double arrows). In the integrated conception, children connect *fifty* to 5 tens, and the written numeral 53 can take on either quantity meaning (fifty-three or 5 tens, 3 ones).

We call this model the *UDSSI triad model*, for the names of the five conceptions (unitary, decade, sequence, separate, integrated). We originally thought about each two-digit conceptual structure as a triangle (a triad) of six relations (each two-ended arrow represents two relations). However, we later realized that only the separate-tens-and-ones conception (in bold in Figure 19.1) has direct links between quantities and numerals. Such a direct link can occur only if the quantities of tens and ones are small enough to be subitized (immediately seen as a certain number of units) or are arranged in a pattern. In the other three conceptions, a person must, by counting, relate quantities to written numerals through the number words. Therefore, for these conceptions, the link between quantities and numerals is not drawn in Figure 19.1.

USE OF THE TRIAD MODEL IN OUR STUDY

Our analysis of the structure of Spanish words for two-digit numbers indicates that Figure 19.1 describes the main conceptual structures that Spanish-speaking children construct, albeit with some small advantages and disadvantages compared with English speakers. The relationship between the Spanish decade words (*diez, veinte, treinta, cuarenta, cincuenta, sesenta, setenta, ochenta, noventa*) and the corresponding words for 1 to 9 (*uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve*) is unclear. This lack of a clear relationship is similar to the rather opaque relations of the English words *twenty* and *two, thirty* and *three,* and *fifty* and *five.* The Spanish words for numbers from 21 onward use a construction (e.g., *cincuenta y tres* for *fifty and three*) that seems to support children's construction of the decade-and-ones conception. Spanish words in the teens are irregular at first and then, at 16, begin to name the ten: *once, doce, trece, catorce, quince, dieciséis, diecisiete, dieciocho, diecinueve.* A less common alternative spelling is conceptually clearer: *Diez y seis* means "ten and six," whereas *dieciséis* means "tens six." Thus, the conceptual contribution of the words for 16 through 19 may depend heavily on the emphasis given by the teacher to the "ten and six" meaning. Most European systems of number words for two-digit numbers have a decade structure, although with various irregularities. We believe, therefore, that the UDSSI triad model is relevant to these other European languages also.

Our goal in this study was to help children in both English-speaking and Spanish-speaking classrooms construct all the triad conceptual structures. But in what order and how were we to help them? The literature on East Asian children's multidigit learning contributed major elements to the design of the learning activities. Two conceptual structures in Figure 19.1 (the decade-and-ones and the sequence-tens-and-ones structures) come from the decade structure of the English number words. Some countries have number words with no separate list of decade words. For example, in East Asian countries in which the number words are based on ancient Chinese number words, children say 12 as "ten two" and 53 as "five ten three." These children need to construct only the unitary and the separate-tens-and-ones conceptual structures. The construction of the latter triad (shown in bold in Figure 19.1) is supported by the presence in children's lives of ten-structured cultural artifacts and experiences (e.g., the abacus, the metric system), by school instruction on ten-structured methods for adding and subtracting single-digit and multidigit numbers (Fuson & Kwon, 1992), and by the support of parents and teachers in demonstrating ten-structured quantities (Yang & Cobb, 1995). Cross-cultural work on East Asian children's numerical thinking indicates that children build highly effective ten-structured conceptions of numbers; use these very well in their addition and subtraction; and far outperform U.S. children on single-digit and multidigit addition, subtraction, and place-value tasks.

The simplicity of the inner triad indicated to us that for English-speaking children, constructing the separate-tens-and-ones conception might be easier than constructing the decade-and-ones and the sequence-tens-and-ones conceptions. Also, with the separate-tens-and-ones conception, children could participate in classroom activities involving ten-structured quantities before they learned the English (or Spanish) sequence to 100; learning the sequence can take months or even years. To facilitate the learning of the separate-tens-and-ones conception, we decided to use tens-and-ones words to describe quantities. We thought that children's construction of the decade-and-ones and the sequence-tens conceptions might be facilitated by activities that also simultaneously supported the construction of the separate-tens-and-ones conception; such activities include arranging quantities in groups of tens, counting them by tens and ones (a sequence-tens conception), and counting the units of ten and the units of one separately (a separate-tens-and-ones conception). Using both tens-and-ones words (*one ten, two ones*) and ordinary English (*twelve*) or Spanish (*doce*) words would help focus children's attention on both conceptions (i.e., on the separate-tens and on the sequence-tens conceptions).

THE INSTRUCTIONAL PHASE OF THE STUDY

The K–8 school in which the study took place is located in a predominantly Latino neighborhood; 87% of the students qualify for free or reduced-cost lunch. Each grade level from 1 through 5 has a Spanish-speaking and an English-speaking class. Mathematics classes and most other classes are carried out almost entirely in the specified language (English or Spanish). All first graders who had entered the school by mid-December were included in the study sample. The Spanish-speaking first-grade class had 17 children who were in the class from mid-December through June. The English-speaking class had 20 children who were in the class from mid-December through June.

We viewed addition and subtraction work as important settings for children's continued construction of place-value concepts, and thus instruction did not begin with place-value constructions. In solving problems that do not require regrouping, students do not need to use the trading-ten-for-10-ones strategy. Because they do not need to trade in such problems, they often make errors when they later encounter problems that do require regrouping. For example, in subtraction, if instruction begins with solving problems not requiring regrouping, students often make the common top-from-bottom error on problems for which they need to regroup. For these reasons, we began instruction with two-digit problems requiring regrouping.

Finally, our experience indicated that to construct robust ten-structured conceptions, children needed to explore two-digit addition and subtraction using quantities as referents, not using just numerals. Therefore we had to select and design such referents.

We faced various pragmatic constraints concerning our choice of quantity referents to use in the classroom, because we wanted them to be usable in any inner-city classroom. They had to be easy to manage, inexpensive, and require minimal teacher-preparation time. Penny-frames fit these criteria; children fitted pennies in rows of ten, and after a child put a penny into the frame, she or he wrote below that penny the new total number of pennies in the frame. Other referents, such as a hundreds chart, were also occasionally used.

For the initial triad activities, the teacher at first led simultaneous performance by all children; each child did the activity at his or her seat, following or with the teacher. We began with activities designed to help children (a) see objects grouped into tens and (b) relate these tens groupings and the leftover ones to number words and written numerals.

In all classroom activities, the teacher used each of the triad conceptual structures successively in various orders. For example, pennies were placed individually into the penny frame while the teacher counted each one unitarily. When the activity progressed over days, an increasing number of children became able to count with the teacher. As each ten row was filled, the rows of ten would be counted by tens (e.g., 10, 20, 30) and then by ones (1, 2, 3 groups of ten). All the pennies might then be counted again by ones to verify that 30 pennies were found in the three groups of ten. Written numerals would be read as tens-and-ones words, in English words in English-speaking classes and in Spanish words in the Spanish-speaking class: "So 36 is thirty-six pennies, three groups of ten pennies and six loose pennies left over. We write 3 tens here on the left and 6 ones here on the right." Gradually, individuals would learn to do the task alone, with informal help from peers and the teacher.

Base-ten blocks and other object quantities leave no records after class to help teachers assess their students' understanding. We therefore introduced a system of recording quantities as ten-sticks and dots, which children could count by using any of the conceptions. Initially, children made dots in columns of 10 to make a record of objects the class was collecting. They counted by ones while they made these columns of 10 dots. When they had fewer than 10 left, they made a horizontal row of dots, often with a space between the first five dots and the last four dots to facilitate seeing the number of dots. When many children could make such drawings confidently, the columns of 10 dots were connected; the children drew a line through them while the counting by tens or of tens was done. Some children had already spontaneously begun to do this step. Eventually only the vertical stick was drawn to show a ten. These activities occupied part of each class period for about 2 weeks.

DESIGN ISSUES

One research teacher worked in the English-speaking class, and the other research teacher worked in the Spanish-speaking class. When the regular classroom teacher became familiar with an activity, she took over some or all of the teaching. The research-team teachers were in the classrooms 5 days a week from late September through November and then on Mondays, Tuesdays, and Wednesdays for the remainder of the year. On Thursdays and Fridays the classroom teachers continued with their usual approaches to nonproject topics. During the fall, activities on single-digit addition and subtraction to 10 and word problems occupied much of the time.

After considerable discussion with the classroom teachers, the research team decided on, for example, "five tens and three ones" and "cinco dieces y tres unos" for the tens-and-ones words. Beginning in January we tried various activities, including vertical-number-line or number-bar activities. None seemed particularly powerful or interesting to the children. In mid-February, rows of dots were made for each addend; 10 dots were enclosed, if possible, and the answer was then recorded as "one ten and x ones." For a few minutes on many days during the winter and spring, children solved addition and subtraction problems with sums in the teens; they then demonstrated various finger methods they were using to solve the problems. Explicit practice activities focused on the prerequisites for the ten-structured methods (e.g., "How many more to make 10 [with a given number]?" and "10 plus x = ?"). By midyear most children were counting on or using fingers in other ways that could lead to ten-structured methods.

We decided to use the ten-sticks and dots for two-digit addition and subtraction for their advantages as written records and for cost and management reasons. Teachers used this activity periodically from late February through May. In the English-speaking class, the children did triad review activities for 2 days and then spent four classes on two-digit addition. Six classes on two-digit subtraction (in April and May) were followed by six sessions of mixed addition and subtraction problems with and without trading (in May). The time spent on these two-digit activities averaged 30 minutes for each class. The Spanish-speaking children spent about this same amount of time on triad activities, two-digit addition with trading, three-digit triad activities (drawing squares for hundreds), three-digit addition with no trading, and two-digit subtraction with no trading.

Addition and subtraction lessons typically began with instructional conversations in which the teacher elicited children's ideas about methods. Each method was then carried out by the whole class together. The children next worked alone, solving problems by any method they chose. Conversations to facilitate the children's reflections on, and comparisons of, methods were intermixed with periods during which the children worked either alone or spontaneously together. When errors arose, the teacher also identified and discussed them. The children explained why these examples were errors and how to correct them.

Individual interviews with all participating children were carried out in late May and early June. Interview items were selected mainly from other published studies to obtain data on the children in our classes to compare with the data reported in the other studies. The interviews included a large number of items. Therefore, some items were given to all children, and some were given only to a subgroup of the children drawn from across the whole achievement range of the clas's.

RESULTS

The classroom teachers reported that the children, especially the least advanced, found the regular tens-and-ones words easier to learn than the standard English or Spanish number words. The children did not seem to confuse the two kinds of words; no construction from one kind was carried into the other kind of words.

The children varied in the ways they drew the ten-sticks and dots, enclosed 10 dots, and showed their answers (see the methods in Figure 19.2). When we watched children solve a problem, we saw other differences in the children's methods. For finding the total, some children counted ones first, and others counted tens first. For finding the number of tens, some children counted all, some counted on from the first number of tens, and some used known facts (e.g., "3 tens and 2 tens make 5 tens") to add the tens. Some children integrated the new, enclosed ten (see new groups of 10 created in the addition problems in Figure 19.2) into a sequence count of the total, and others counted it as another ten in their count of the tens.

Many children made rapid progress in subtracting correctly. All children opened a ten by drawing the 10 ones (see two examples at the bottom of Figure 19.2). Correctly taking away the tens was particularly easy: After only two sessions, 17 of 22 children correctly crossed out the correct number of ten-sticks on the quizzes at the beginning of class.

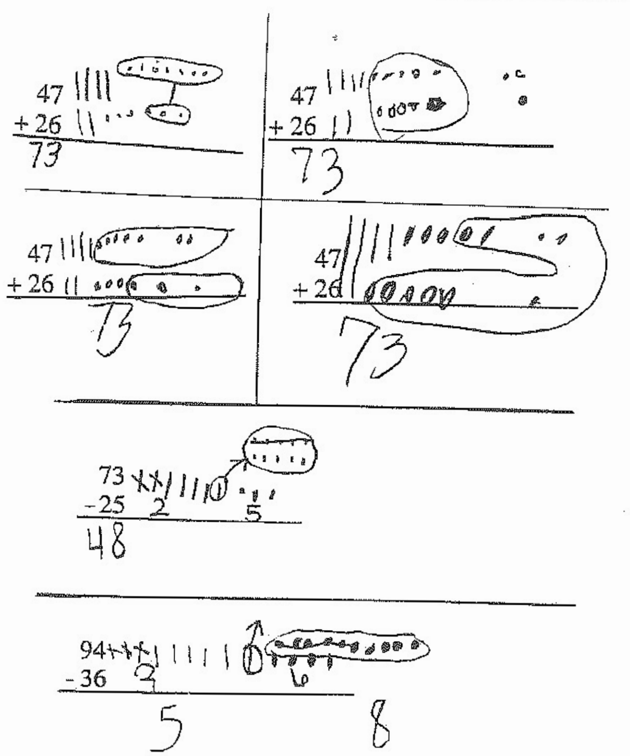

Figure 19.2. Addition and subtraction methods using ten-sticks and dots.

The children's performance on subtraction dropped somewhat when addition and subtraction were mixed. However, by the third class with mixed operations, many children were differentiating their addition and subtraction methods. The children made many more kinds of errors in subtraction than in addition. Nearly every error we could anticipate was made by some child at some time. In addition problems, most errors were in regrouping, and these mistakes were of only a few kinds. In contrast, the subtraction regrouping (opening a ten) seemed to be relatively easy to understand and to carry out with an overall correct approach. However, it was subject to many minor execution errors. Children almost always opened a ten and crossed out the correct number of ones. But miscounting occasionally occurred at each possible counting step, and children sometimes counted only part of the tens or part of the ones.

All children in both classes could do the tasks assessing all six relations for the inner separate-tens-and-ones triad. For the unitary conception, one child in each class could not count to 100 by ones. These children, and an additional child from the English-speaking class, could not count to 100 by tens. All other children did the sequence-tens-and-ones triad tasks correctly. Most children in the Spanish-speaking class did these tasks directly by counting by tens. Many children in the English-speaking class demonstrated the separate-tens-and-ones triad relations and then translated this result to English number words (e.g., counted "1, 2, 3, 4 tens and 1, 2 ones, so that's forty-two").

Many children were able to demonstrate triad relations in the cardinal ten-structured task ("There are 53 first graders at Esperanza School. How many teams of 10 can be made?"). Almost all Spanish-speaking children (94%) and half the English-speaking children answered correctly. Almost all these children (92%) knew the answer rapidly, without counting or drawing.

On various tasks that assess whether children are thinking unitarily or with tens and ones, our first graders from both classes predominantly demonstrated tens-and-ones thinking. Their performance thus looked more like that of East Asian children than that of U.S. children, who predominantly demonstrate unitary or concatenated single-digit conceptions (Miura, Kim, Chang, & Okamoto, 1988). The East Asian children were tested in the first half of the year, whereas ours were tested at the end of the year, so our students are still behind East Asian children in the timing of their use of tens and ones. On Miura and others' task of the cognitive representation of numbers (use blocks to represent a given number), 88% of our students made a ten-structured 42 using 4 tens blocks and 2 units blocks compared with a mean of 89% of children from the People's Republic of China, Japan, and Korea making a ten-structured display (a mean of 10% of the 89% made a noncanonical-ten version that had some tens and more than 9 ones). When asked to make a different block presentation for 42, only 7% of our students made a unitary presentation; a mean of 53% of the East Asian first graders made a unitary presentation. Most of the 74% of our students making a correct second presentation made a noncanonical-ten arrangement in which some ones were arranged in groups of ten.

Sixty-three percent of our first graders immediately said that the *1* in 16 was 10 chips. This percentage is considerably higher than either the 42% of M. Kamii's (1982) 9-year-olds who were correct or the 32% of the second and third graders in our project school before the project began. Our students' 63% correct is about the same as the 60% of C. Kamii's (1985) affluent suburban sixth graders.

The children were given three place-value-understanding tasks that assess children's quantity meanings for each digit in a two-digit number. All tasks and comparison samples are from Miura, Okamoto, Kim, Steere, and Fayol (1993). Our first graders did as well as East Asian children tested in the first half of the year and much better than U.S. first graders from a selective, academically rigorous school with monolingual middle- and upper-middle-class children (that sample and our students were tested at the end of the year).

On the two-digit addition problem (48 + 36) using ten-sticks and dots, 90% of our students' solutions were correct. On the two-digit subtraction problem using ten-sticks and dots, all the children in the English-speaking class correctly opened a ten-stick by drawing 10 enclosed dots and correctly took away the required ten-sticks and dots. Of these, 70% then wrote the correct answer. The Spanish-speaking first graders had not had opportunities in class to solve subtraction problems requiring trading.

Word-problem tasks with base-ten blocks were given to assess whether the ten-structured conceptions built by our students would generalize to these unfamiliar tools. On the addition problem, 100% of the Spanish-speaking children were correct, and 90% added using sequence-tens-and-ones counting of the total. Of the English-speaking children, 75% were correct; 84% of these children traded 10 units to make another ten bar, and most of these children counted the tens and ones separately.

On the subtraction problem, all the children showed correct digit correspondence by making 74 with 7 ten-bars and 4 units and trying to take away 3 ten-bars and 8 units. All but two children used a correct strategy for solving this problem and did see the ten-bars both as 1 ten (when making the 74) and as 10 ones (when taking away some or all of the 8 ones from it). Half of these children carried out their strategies correctly, and the rest made some error in executing their strategies.

DISCUSSION

By having the children use ten-stick-and-dot quantity drawing, we were able to support most children's construction of most elements of the conceptual structures shown in Figure 19.1. Furthermore, on a range of unfamiliar tasks, many children showed a robust preference for ten-structured conceptions, performing like children in China, Japan, and Korea rather than like agemates in the United States or like children in higher grades in the United States. Most children were also able to carry out a ten-structured solution to two-digit addition and subtraction problems and to explain their regrouping. First graders in the United States do not ordinarily learn to solve such problems with trades; these problems are usually not included in first-grade textbooks or appear in the final chapter, which many teachers do not reach. Our students' performance was considerably above that reported for U.S. children receiving traditional and reform instruction and was above that reported for Japanese and Taiwanese first graders on some tasks. This superiority is partly because the children's conceptual tool, the ten-sticks and dots, could be drawn on paper and thus could be used on homework or in an assessment whenever pencil and paper were available.

Through some reform approaches to primary school mathematics, many children have been successful in inventing accurate methods of two-digit addition and subtraction and in understanding them. But some children in those projects continue to use unitary methods into second, third, and even fourth grade. Our teacher-orchestrated activities designed to help children construct sequence-tens and separate-tens conceptions and then to use one or the other in two-digit addition and subtraction were quite successful: No first grader used a unitary method. This result shows the benefits in teachers' carrying out such activities with whatever conceptual supports for quantity are used in their classrooms or their introducing ten-sticks-and-dots activities as recordings of any quantities in problems.

In several tasks, the children in the Spanish-speaking class showed a preference for counting by sequence-tens over counting by separate-tens. This preference facilitated their solutions with the unfamiliar media of noncanonical sticks and dots and of the base-ten blocks because they did not have to explicitly make another ten: Counting by tens and then counting the ones took them up over the next decade to get the answer. In contrast, more children in the English-speaking class demonstrated separate-tens-and-ones conceptions in which they had to explicitly make another ten by grouping or adding or had to break a ten. In some new situations, fewer of these children were able to complete this activity accurately. The difference between the two classes in preferred conceptions illustrates how instructional emphases in the uses of a conceptual tool and the uses of different tools can support different conceptual constructions.

The results reported here clearly indicate that all U.S. children can do enormously better than they ordinarily do in primary school mathematics. Furthermore, the widely reported gap in performance and understanding between East Asian children and children in the United States can be narrowed or eliminated, even in poor, inner-city schools. Doing so requires a substantially more ambitious first-grade curriculum and active teaching that supports children's construction of a web of multiunit conceptions in which number words and written number marks (numerals) are related to ten-structured quantities. Drawn quantities, instead of objects, can serve as meaningful ten-structured quantities that support reflection, communication, assistance, and teachers' assessment of children's thinking.

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