Supporting Teachers to Systematically Analyze Students' Strengths and Struggles and Identify Next Instructional Moves

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For more details about problem solving with visual supports to bolster the mathematical practices, or for details about CCSS domains, please see the 13 hours of Teaching Progressions Fuson has made and posted at

karenfusonmath.net

This presentation is also posted there.

Background and Focus of Session

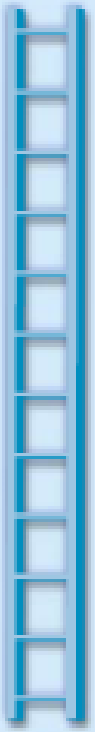
- This work grew out of Fuson-led *Children's Math World's* project (now called *Math Expressions*), 20+ years of research on math teaching, children's thinking, teacher learning, and curriculum development.
- Today we present one of the tools we used during our research, that has become a powerful tool used in effective coaching and teacher support.

Effective Coaching and Supporting

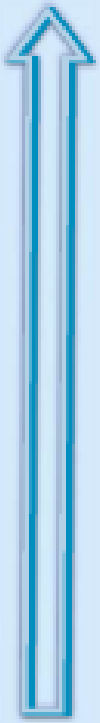
- is collaborative and thought-provoking
- involves a caring but critical friend relationship
- aims to strengthen a teacher's knowledge of **core mathematical concepts**; as well as her use of **core mathematical practices** in the service of building a **productive math talk community** where everyone explains and discusses mathematical ideas

Math Talk Community

Bridging for teachers and students by coherent learning supports



Learning Path



Phase 3 Formal math methods, fluency

Math Sense-Making
Math Structure

Math Drawings
Math Explaining

Phase 2 Research-based mathematically desirable and accessible methods, understanding and growing fluency

Math Sense-Making
Math Structure

Math Drawings
Math Explaining

Phase 1 Student-generated methods, exploring and growing understanding

Common Core Mathematical Practices Used in a Math Talk Community

Math Sense-Making: Make sense and use appropriate precision 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings: Model and use tools** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure: See structure and generalize 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining: Reason, explain, and question** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.

The Math Practices in Action

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

How can I do some part of this better tomorrow?

Visual models are core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings that students make and use in their own ways to problem-solve and explain their thinking.

Visual models help teachers and students understand what a student is thinking.

Recording errors in visual models or in numerical methods can help a teacher see student struggles and make decisions about how to overcome them.

Class Error Analysis Sheet

History and Uses (Then and Now)

- Used during research to focus content, design problem-sequences, collect information about student thinking and common errors, analyze and organize for learning progressions
- Used to build assessments and supports for all
- Used to know who and what to re-teach, whole-class, small-group, and individual learners
- Easy to use, robust tool gives teachers big picture of mathematical thinking of all students on unit tests, a one- or two-item quiz or other assessment work

Class Error Analysis Sheet

Grade 3 Unit 2		Put Together part?	Ch– start?	Comp big? misleading	Comp diff?	Ch+ chng?	Ch– start?	Comp small?	Comp Big? leading	Ch+ start?	Comp Big? misleading
		9 Robins and some Cardinals. 17 birds. C? 9 + C = 17 8	Some P. Gave 8. 6 left. P? P – 8 = 6 14	P had 5. 8 fewer than R. R? R – 8 = 5 or 5 + 8 = R 13	T has 14. K has 7. How many more T than K? 14 = 7 + ? 7	**Monday 127. Some more Tues. 205 altogether. Tues?** 127 + ? = 205 78	**Some. 122 eaten. 374 still. How many To start with?** S – 122 = 374 496	**R has 589. Y has 345 fewer: Y?** Y = 589 - 345 or Y + 345=589 244	**Z had 238. T 496 more than Z. How many did T have?** 238 + 496 = T 734	Some. 7 more. Then 15. How many to start? S + 7 = 15 8	F painted 12. 5 fewer than D. D? 12 + 5 = D or D – 5 = 12 **17**
		Problem #1	#2	#3	#4	#5	#6	#7	#8	#9	#10
	Score										
Victoria	100%	-	-	-	-	-	-	-	-	-	-
Ana	100	-	-	-	-	-	-	-	-	-	-
Madison	100	-	-	-	-	-	-	-	-	-	-
Samantha	100	-	-	-	-	-	-	-	-	-	-
Ryan	100	-	-	-	-	-	-	-	-	-	-
Keltyn	90	-	-	-	-	-	252	-	-	-	-
Tyler	90	-	-	-	-	127+205 or 332	-	-	-	-	-
Amelia	90	-	-	8-5=3	-	-	-	-	-	-	-
Brei	80	12	-	-	-	-	Cor/miscopy	-	-	-	-
Sierra	60	-	6	3	-	-	252-122=374	-	-	-	12-5=7
Edison	50	-	Off 1	16 miscopy	-	127-22=205	-	-	724	15 + 7 = ?	-
Kallie	50	26	-	3	-	88 (regrp err)	-	-	634	-	12-5=7
Chris	50	-	8-6	3	-	76	-	224 (cor/misc)	-	-	12-5=7
Nancy	10	17 9+8=17	6 8-2=6	3	21	127+205 or 332	122-374=348	-	735	22	12-5=7
Tests 14											
Errors		3	4	6	1	5	4	1	3	2	4

Zooming in on Problems 5, 6, 7, and 8

Grade 3 Unit 2	Ch+ chng?	Ch– start?	Comp small?	Comp Big? leading
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	Problem #5	#6	#7	#8
Victoria	-	-	-	-
Ana	-	-	-	-
Madison	-	-	-	-
Samantha	-	-	-	-
Ryan	-	-	-	-
Keltyn	-	252	-	-
Tyler	127+205 or 332	-	-	-
Amelia	-	-	-	-
Brei	-	Cor/miscopy	-	-
Sierra	-	252-122=374	-	-
Edison	127-22=205	-	-	724
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Chris	76	-	224 (cor/misc)	-
Nancy	127+205 or 332	122-374=348	-	735
Tests 14				
Errors	5	4	1	3

Nancy Problems 1 and 2

		Situation Equation	Answer
1. In my yard were 9 robins and some cardinals. Altogether there were 17 birds. How many cardinals were there?		$9 + \square = 17$	8
	Nancy	$9 + 8 = 17$	17

		Situation Equation	Answer
2. Josina had some plums. She gave 8 to her brother. She had 6 left. How many did she have at the beginning?		$\square - 8 = 6$	14
	Nancy	$8 - 2 = 6$	6

Nancy Problems 3 and 10
Compare Misleading Language

3. Pauline has 5 library books.
Pauline has 8 fewer than Roberto.
How many library books does Roberto have?

5 + 8 = R or R – 8 = 5 13

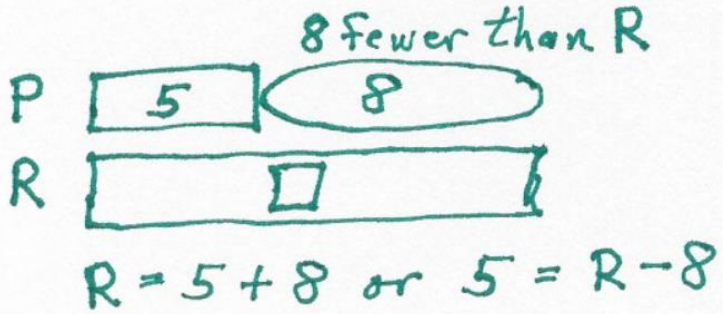
Nancy 3

10. Frances painted 12 pictures.
This was 5 fewer than Dan painted.
How many did Dan paint?

12 + 5 = D or D – 5 = 12 17

Nancy 12 – 5 = 7 7

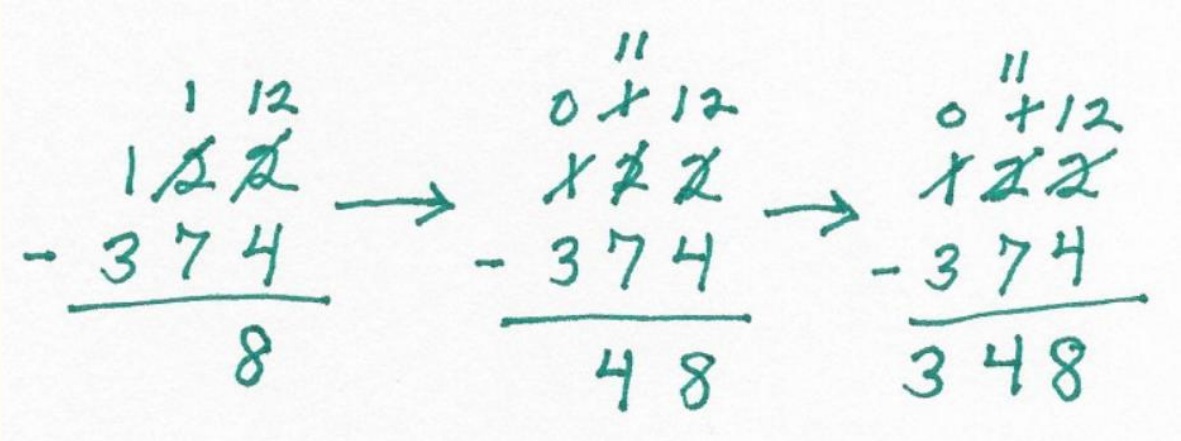
How do you know who painted more pictures?



Nancy Problem 6
122 – 374 = 348

6. There were some oranges in the school lunchroom. 122 were eaten at lunch. There are still 374 oranges in the school lunchroom. How many oranges were there to start with?

S – 122 = 374 496

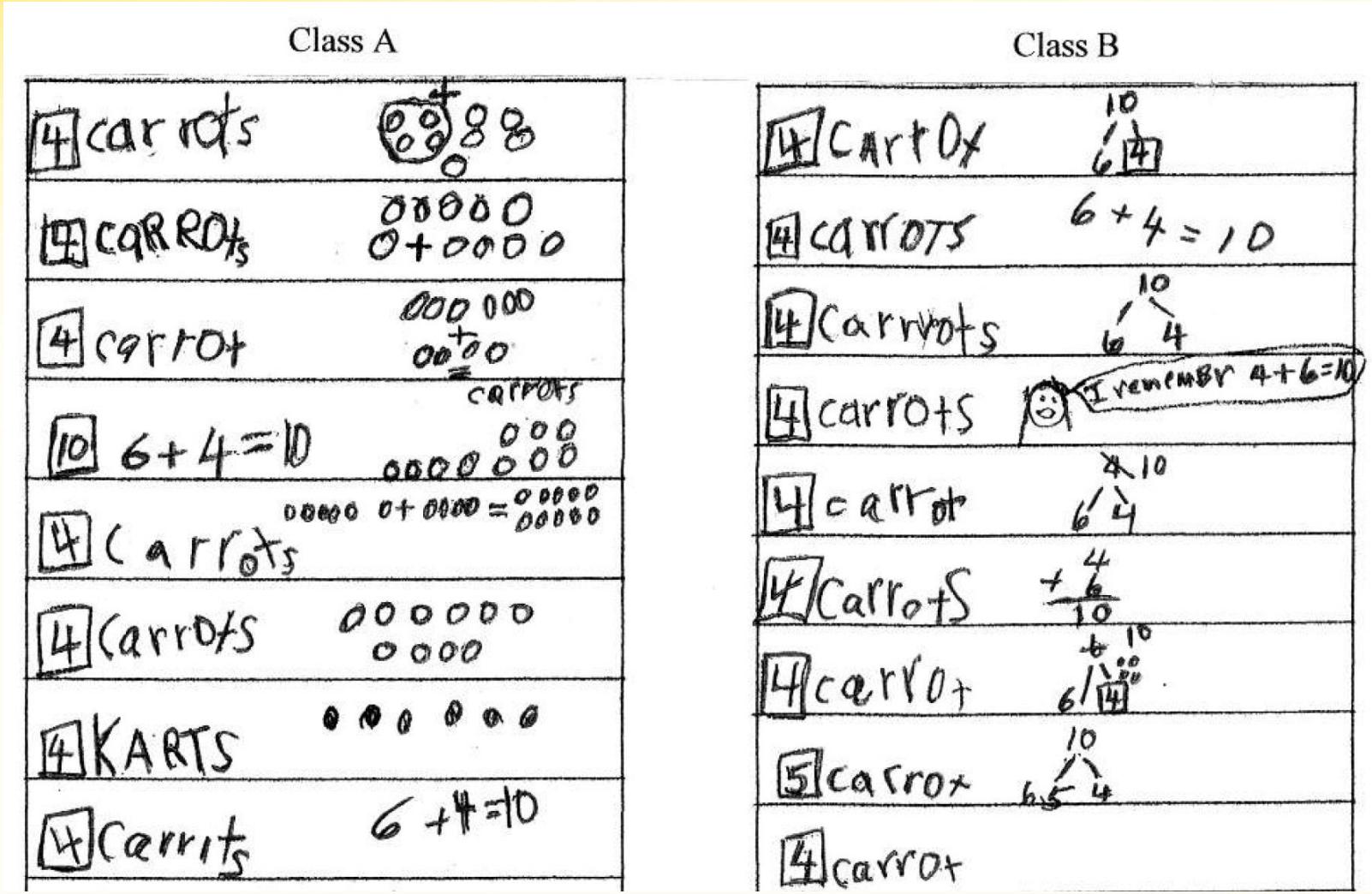


Grade 1

Rosa picked 6 carrots. Her sister picked some too. Together they picked 10 carrots. How many did Rosa's sister pick?

5-group

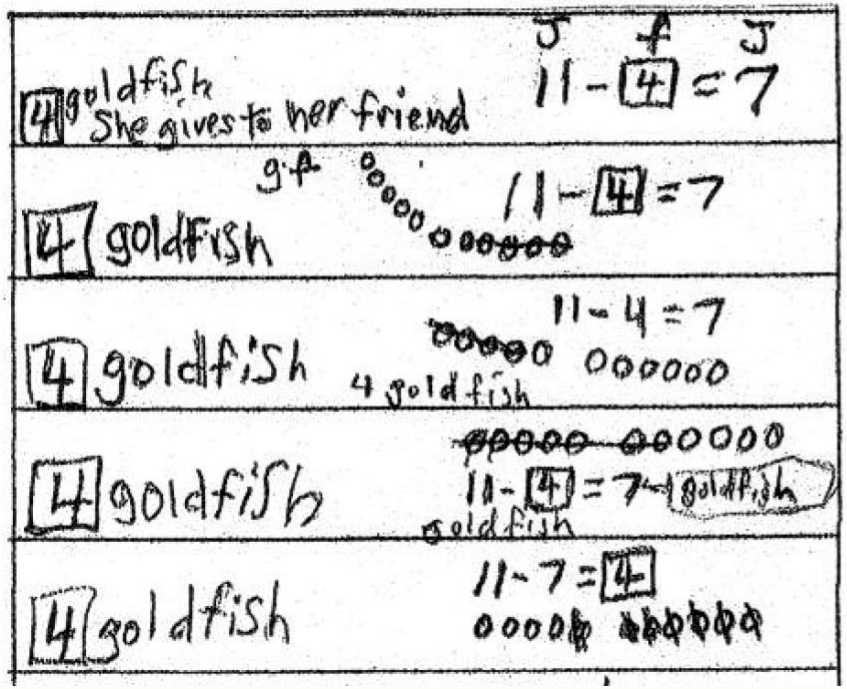
5-group



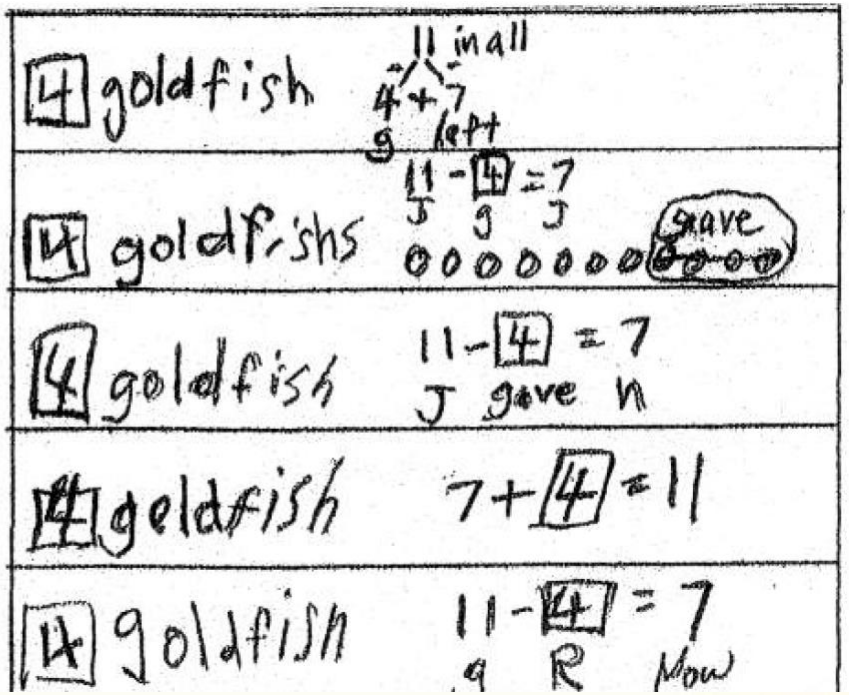
Grade 2

Jenna has 11 goldfish. She gives some to her friend. Now she only has 7 goldfish. How many goldfish did she give to her friend?

Class A



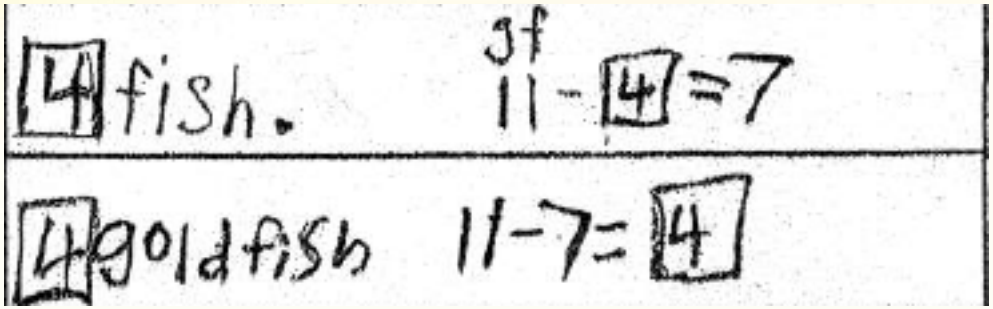
Class B



5-group

situation equation or solution equation?

Situation Equation



Solution Equation

The 6 Situations

K — Add To / Take From

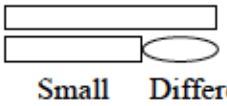
Start + Change = Result
Start – Change = Result

(becomes)

K — Put Together / Take Apart

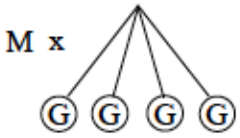
T = A + A
$\overset{i}{=}$ (identical)

Gr1 — Additive Comparison



Small + Difference = Big
Big – Difference = Small
Big – Small = Difference
$\overset{n}{=}$ (same number)

Gr3 — Equal Groups



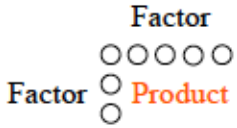
G + G + G + G = P
M times

M x G = P

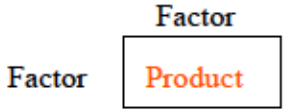
(becomes)

Rectangular Everything Times Everything

Gr3 — Array



(Long Division Format)

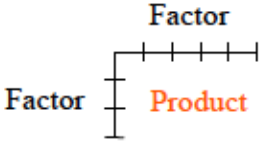


F x F = P
P ÷ F = F

(table)

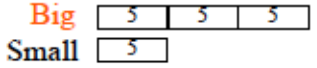
$\overset{i}{=}$ (identical)

Gr3 — Area



Gr4 — Multiplicative Comparison

Big = 3 x Small



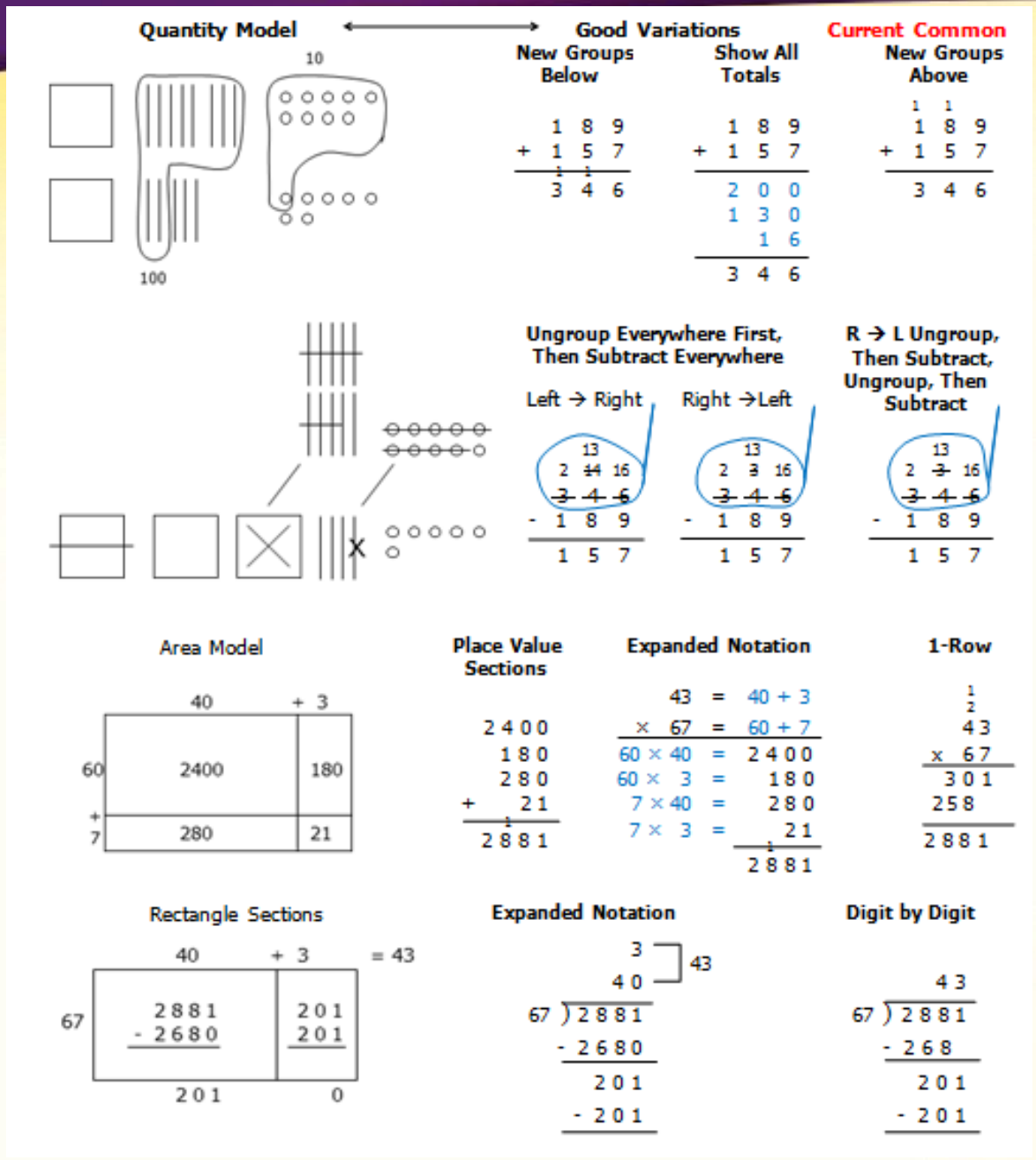
Small = $\frac{1}{3}$ x Big

Big ÷ 3 = Small

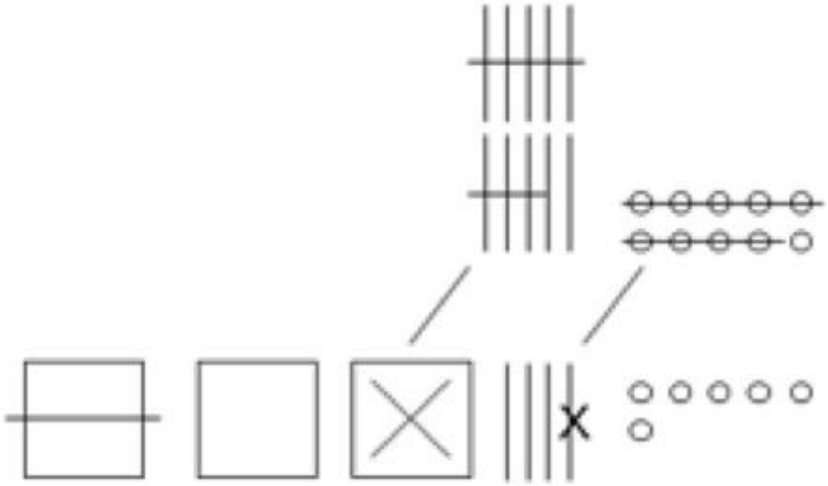
$\overset{n}{=}$ (same number)

CCSS Addition (top row) and Multiplication (bottom row) Word Problem Situations and *Math Expressions* Diagrams for Each

Drawings and Written Variations of Standard Algorithms



First write the total.
Then draw the magnifying glass around the total
with enough space above to write the ungroupings.



Ungroup Everywhere First, Then Subtract Everywhere

Left → Right

$$\begin{array}{r} 13 \\ 2\ \ \cancel{14}\ \ 16 \\ \cancel{3}\ \ \cancel{4}\ \ \cancel{6} \\ -\ 1\ \ 8\ \ 9 \\ \hline 1\ \ 5\ \ 7 \end{array}$$

Right → Left

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R → L Ungroup, Then Subtract, Ungroup, Then Subtract

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Visual models are core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings that students make and use in their own ways to problem-solve and explain their thinking.

We want classrooms to be using the mathematical practices in the Math Talk Community:
Students focus on math sense-making about math structure using math drawings (visual models) to support math explaining.

Effective Coaches

- have detailed mathematical content knowledge; they should have more time to pursue this deeply; it is their major responsibility
- support teachers in their roles as learner and instructional leader; offering questions, comments and suggestions for improving math interactions
- provoke shifts in teacher knowledge and practice by working collaboratively in classroom and school-based context, on the math those children are producing

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Presentation handout is slide 24

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1. In my yard were 9 robins and some cardinals. Altogether there were 17 birds. How many cardinals were there?
2. Josina had some plums. She gave 8 to her brother. She had 6 left. How many did she have at the beginning?
3. Pauline has 5 library books. Pauline has 8 fewer than Roberto. How many library books does Roberto have?
4. Tomas has 14 blocks to walk to get home. Kemal has 7 blocks to walk. How many more blocks does Tomas have to walk than Kemal?
5. **Monday 127 people saw the school play. Some more saw the play on Tuesday. Altogether 205 people saw the play. How many people saw the play on Tuesday?**
6. **There were some oranges in the school lunchroom. 122 were eaten at lunch. There are still 374 oranges in the school lunchroom. How many oranges were there to start with?**
7. **Ridgeway School has 589 students. Yardley School has 345 fewer students than Ridgeway. How many students does Yardley School have?**
8. **The Z Bakery baked 238 donuts to sell. The T Bakery baked 496 more donuts to sell than the Z Bakery baked. How many donuts did the T Bakery have to sell?**
9. Some children were playing in the park. 7 more children came. Then there were 15 children playing. How many children were playing to start with?
10. Frances painted 12 pictures. This was 5 fewer than Dan painted. How many did Dan paint? How do you know who painted more pictures?