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LEARNING PATHS TO 5- AND 10- STRUCTURED UNDERSTANDING OF QUANTITY: ADDITION AND SUBTRACTION SOLUTION STRATEGIES OF JAPANESE CHILDREN

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Abstract: This study examined the strategies used by Japanese first- and second-graders as they solved simple addition and subtraction numerical problems. For problems with totals ≤ 10, most children initially used recomposition strategies using five as a base or recall strategies, and then increasingly used recall. For problems with totals > ten, children increasingly used recomposition strategies involving 10. Count-based strategy use decreased. Overall, children's strategies relied on the flexible use of embedded numbers that supported their fluency in recomposition, and five and ten were used as bases for these recompositions.

There have been numerous studies that examined the addition and subtraction strategies of young children (e.g., Carpenter and Moser, 1984; Fuson, 1992a, 1992b; Fuson and Kwon, 1992; Siegler, and Shrager, 1984). Table 1 shows the well-documented developmental/experiential paths for conceptions of quantities and strategies of U.S./European and Korean/Taiwanese children. They typically move from Level I: perceptual unit items (single presentation of the addend or the sum) to Level II: sequence unit items (simultaneous presentation of each addend within the sum) and then to Level III: ideal chunkable unit items (simultaneous embedded mental representation of both addend and the sum). Recall strategies develop increasingly through the levels, varying with individual children. At the Level III, U.S. and European children usually use doubles $\pm$ 1 or known addition strategies, while Fuson and Kwon (1992) and Lee (2000) found that Korean and Taiwanese children used recomposition strategies using tens.

This study examined the strategies used by Japanese first- and second-graders as they solved simple addition and subtraction numerical problems. It contributes to an effort to describe the developmental sequences used around the world as children build their understanding of 5- and 10- structured numerical relationships. When examining different strategies of children, it is also necessary to consider how and why their strategies are developed. Although this paper focuses on the results of the strategy interviews of Japanese children, this is a part of a larger study that also explores how the strategies are supported by the culture as appropriate ways to consider numerical quantities by examining children's experiences in and out of school. This study also

Table 1. Conceptual Developmental Levels of Addition and Subtraction Solution Methods (Adapted from Fuson, 1992a, 1992b; Fuson and Kwon, 1992; Lee, 2000).

Levels	Conception of Quantities	Addition Solution Methods	Subtraction Solution Methods
I	Perceptual Unit Items: count things	Count All	Count Take Away
II	Sequence Unit Items: number words are the things	Count On	Count Down
III	Ideal Chunkable Unit Items: numbers are things	*Derived Facts/Recomposing: U.S.*	
		Doubles ± 1	Doubles ± 1
		Make a ten from one number: Korean and Taiwanese	
		Make a Ten	Take From Ten
		Up Over Ten	Down Over Ten
		Add Fives and Then Add Amounts Over Five	

relates students' strategy sequence to their learning opportunities in the classroom. The culture supports and influences the way school mathematics is taught and also the ways children experience numbers outside of school. This view was adapted from the Vygotsky's sociocultural perspective (1978, 1986) that asserts that understanding the formation of minds requires the study of sociocultural contexts and of particular semiotic cultural supports and practices as reflections of the culture itself.

Methods

Fifty ($n = 50$) Japanese first- and second-graders participated in the study. The setting of our study is an official full-day Japanese School in the U.S. It follows the official Japanese national course of study, and all the teachers at the school are certified Japanese teachers who are sent from Japan directly through the Ministry of Education. All of the participating children and their families are temporary residents in the U.S., living here for business purposes for a few years. Because they planned to return to Japan, the parents tried to preserve Japanese cultural ways in children's lives, and the members of the Japanese community in the area generally interacted among themselves rather than with Americans.

Out of the fifty children, twenty-eight (n = 28) children were first-graders and twenty-two (n = 22) children were second-graders at the time of the study. The 28 first-graders were from two different classrooms, and the second-graders were from a single classroom. Two different strategy-interview sessions were held: one in the middle of the school year in October, and another at the end of the school year in March. Because the second-grade classroom was fairly large (38 children), randomly-selected children were interviewed from the classroom; therefore the samples for the October interview differed from the one from March. For the first-graders, all children were interviewed for both sessions from both first-grade classrooms.

The individual interviews were conducted in Japanese. At the beginning of the interviews, it was explained to the children that the interviewer was interested in studying their thinking as they solved the problems and not just the answers. Counters, papers, and pencils/markers were placed within children' reach, and it was also explained that they were allowed to use them if they wanted to while solving and describing their thinking. Children were then presented the following ten problems: 5 + 2 or 6 + 2, 4 + 4, 7 + 5 or 8 + 9, 6 + 9, 7 + 7, 7 – 2 or 8 – 2, 8 – 4, 12 – 5 or 17 – 8, 15 – 9, 14 – 7, one at a time written on a 7" x 5" index card. When the strategy the child used was not observable (> 96%), the interviewer asked the child to explain how he/she solved the problem after he/she stated the answer. Because Japanese mathematics lessons often focus on problem solving with multiple solutions, the participating children sometimes attempted to explain the solution in more than one way. In that case, a standard follow-up probe was used to identify the strategy the child had just used. The interviews were tape-recorded, transcribed, and translated. The interviewer also took interview notes during interviews.

Response data were first differentiated as immediate (≤ 2 sec. after Siegler and Shrager, 1984)) or non-immediate responses. Observable counting strategies were then differentiated according to the coding framework (Table 1) of Fuson and Kwon (1992). For non-observable strategies, children' explanations were analyzed and differentiated according to the Conceptual of Quantities Levels and then to the different strategies. When a strategy did not fit the previously identified methods, new categories were created. Strategies were then further differentiated according to the kinds of quantities used with explanations. Follow-up interviews were conducted for unclear responses. The data were then separated into the problems whose totals were smaller than ten (≤ 10) and larger than ten (> 10) for analysis.

Results and Discussion

Japanese first- and second-graders who participated in the study used all of the strategies identified previously by other researchers in Table 1, and they used five new strategies: *Make-five* strategy for addition, *take-from-five* strategy for subtraction, *break-apart-to-make-ten* strategy for addition, *count-on-to-ten-and-add-over-ten* strategy for addition, and *count-up-to-ten-and-add-over-ten* strategy for subtraction (see Table 2 and the later text).

The percentage of correct answers overall was 97%. Addition and subtraction answers were both 100% correct at mid Grade 1 for ≤ 10, at mid Grade 2 for ≤ 10, and end Grade 2 for ≤ 10 and > 10. The only cases with less than 95% correct answers were mid Grade 1 addition > 10 (94%) and end Grade 1 subtraction > 10 (92%). The percentage of strategy use for the correct answers is shown in Table 3.

For the addition and subtraction problems with totals smaller than ten (≤ 10), almost half of the children (46% overall; 31% for addition and 65% for subtraction) initially used a *make-five* recomposition strategy involving fives (see Table 3). This *make-five* strategy is an addition strategy similar to ten-based recomposition strategies, but uses five as a unit instead of ten. For example, for the problem 4 + 4, a child explained;

> "4 + 4 is 8, because 4 and 1 more is 5, so bring 1 from the other 4, and 3 is left. Then, 5 and 3 is 8."

Children used such 5- structured thinking for subtraction also in the *take-from-five* strategy. For example, one child explained:

> "8 – 4 is 4. Because 8 is 5 and 3, take 4 from 5, then 1 left, plus 3 would be 4."

The children's understanding of numbers in relation to five is reflected by these strategies. At the beginning of the first-grade, children are provided much experience in their mathematics lessons breaking-apart and putting-together numbers between 5 and 10 using five as a base (e.g., 6 is 5 + 1, 7 is 5 + 2). With repeated exposure to seeing and experiencing numbers in relation to five, many children used 5- structured understanding of quantities by the middle of the first-grade when the interviews were conducted.

As the children gained more experience with the problems with totals ≤ 10, they increasingly used recall strategies (by the end of the second grade, 95% for addition and 100% for subtraction). Children typically explained that they "remembered" and "knew" the answer or "did not think" while solved the problem. In all three classrooms, children regularly practiced their "calculation" by using calculation cards (small flash cards put together by a ring). Fluency and speed were emphasized in the classrooms, and by the end of the second grade, children quickly recalled answers.

Table 2. New Strategies Japanese Children Used.

Levels	Conception of Quantities	Addition Solution Methods	Subtraction Solution Methods
II	Sequence Unit Items		
II-III	(Transitional Level)	Count On to Ten and Add Over Ten	Count Up to Ten and Add Over Ten
III	Ideal Chunkable Unit Items	Make-Five (total < 10)	Take-From-Five (total < 10)
		Break Apart to Make Ten	

Table 3. Percent Correct Responses by Strategy for Addition and Subtraction.

Strategy	Mid Grade 1	End Grade 1	Mid Grade 2	End Grade 2
		Totals ≤10		
Count-based strategies:				
Addition	12	14	5	5
Subtraction	12	14	5	0
Recomposition strategies:				
Addition	31	4	10	0
Subtraction	62	30	35	0
Recall strategies:				
Addition	57	82	85	95
Subtraction	26	55	60	100
		Totals in Teens		
Count-based strategies:				
Addition	24	20	3	0
Subtraction	40	18	10	0
Recomposition strategies:				
Addition	65	58	73	70
Subtraction	54	65	80	97
Recall strategies:				
Addition	11	21	23	30
Subtraction	6	15	10	3

Note: The problems for totals ≤ 10 were 5 + 2 or 6 + 2, 4 + 4, 7 – 2 or 8 – 2, 8 – 4), for totals in teens were 7 + 7, 6 + 9, 5 + 7 or 8 + 9, 14 – 7, 15 – 9, 12 – 7 or 17 – 8.

For problems with totals >10, Japanese children predominantly used recomposition strategies involving tens, and they did so increasingly as they gained more experience (at the middle of the first-grade, 65% for addition and 54% for subtraction; at the end of the second-grade, 70% for addition and 97% for subtraction). Unlike the ten-based recomposition strategy *up-over-ten* identified by Fuson and Kwon (1992) for Korean children, Japanese children's explanation for addition solutions using ten lacked directionality but indicated instead that they were breaking a number into two different parts. Therefore we called the strategy *break-apart-to-make-ten* strategy. A typical explanation is as follows:

> "7 + 7 is 14. 7 needs 3 more to make 10, so I *split* the other 7 into 3 and 4, then 10 and 4 is 14."

For subtraction, children also did not indicate directionality but used a more neutral *Take-from-Ten* solution. For example, one child explained her thinking for 15 – 9;

> "15 – 9 is 6. That's because 15 can be separated into 10 and 5, then 10 minus 9 is 1, then add the left-over 5 will be 6."

For all problems, children used fewer count-based strategies as they gained more experience. Relatively small percentages of children used count-based strategies in the middle of the first-grade (addition $\leq$ 10, 12 %; subtraction $\leq$ 10; 12%; addition >10, 24%; subtraction >10, 40%), and almost none used them at the end of the second-grade (See Table 3).

However, a group of children (first-graders, 5% in first interview, 7% in spring interview) did combine counting with a recomposition strategy to form transitional strategies. For an addition problem, they counted from one addend up to ten, recognized the number of times counted, and then separated the other addend into that number and a second number, and added the second number to ten. As one child explained this *count-on-to-ten-and-add-over-ten* strategy:

> "5 + 7 is 12. From 7 . . 8, 9, 10 . . so 3 take from 5 is 2, then 10 plus 2 is 12."

Children used this hybrid of counting and recomposition strategies for subtraction also. As one child explained the *count-up-to-ten-and-add-over-ten* strategy::

> "14 – 7 is 7. 7, and, 8, 9, 10 . . . is 3 more, then 4 will be 7."

These strategies are useful transitional strategies for children who are moving from seeing quantities as embedded sequences (Level II) to embedded chunks (Level III).

This paper does not allow space to present the rich variety of children's own words, terms, and references to describe their strategies. This considerable variation in children's descriptions of their thinking suggest that their understanding was developed through individual learning experiences and was not due to memorization.

At the beginning of the first-grade, children spent many classroom periods practicing decomposing and recomposing the numbers smaller than 10 before they learned addition and subtraction. Once addition and subtraction concepts were introduced, children were encouraged to use what they knew about embedded numbers within a number and discouraged to count unitarily. Classroom discussions often focused on seeing relationships between different numbers using 5 and 10 as bases. As they gained more experience, children practiced calculations for speed, fluency, and accuracy. This calculation practice always occurred in connection to what they knew about numbers conceptually, and teachers combined the fluency practices with check-ups of answers using semi-concrete manipulatives, such as small flower-shaped counters or counting sticks. Various calculation practices of this sort continued in to their second-grade year as they built on to what they knew with multi-digit addition and subtraction, and later learned about multiplication and its relationship to addition and subtraction.

Overall, children's strategies seemed to rely on the flexible use of embedded numbers that supported their fluency in recomposition. With the curricular emphasis on chunking of numbers in relation to five and ten from early on and de-emphasis on unitary counting, Japanese children seemed to have developed the 5- and 10- structured understanding of quantities well by the end of the second-grade and even very substantially by the end of first-grade. This 10-structured ways of looking at numbers should be of advantage as they work with multi-digit numbers. Future studies should address broader cultural influences upon children's thinking of quantities and identify interacting factors in their lives.

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