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14

Blending the Best of the Twentieth Century to Achieve a Mathematics Equity Pedagogy in the Twenty-first Century

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As we enter the twenty-first century we carry some unsolved problems with us. Many children from all backgrounds do not understand mathematics enough to use it or cannot even do many tasks accurately. This problem is

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especially acute for children of poverty or from homes where standard English is not spoken or is not the main language (e.g., Secada 1992). Many children leave third grade hopelessly behind, especially in urban schools. We share in this article an approach to teaching and learning that was developed in urban first- and second-grade classrooms in English and in Spanish. This approach did result in much higher levels of performance and understanding by our urban children (Fuson, Smith, and Lo Cicero 1997; Fuson 1996). This approach has more recently been extended successfully to third grade and to middle- and high-income children. We highlight some of the less familiar aspects of our pedagogy in the hope that it can help us enter the new century with approaches to overcome these problems. Some modifications are necessary for children older than grade 3, especially some source of extra support for homework, but the major elements apply to all grade levels.

OVERVIEW

We combined aspects of traditional pedagogical approaches with the most powerful elements of reformed teaching based on meaning-making to create our Toward a Mathematics Equity Pedagogy (TMEP) program. It uses the research on children's thinking as a basis for designing learning trajectories (Simon 1995) of increasingly advanced solution methods and then helps children move through these learning trajectories. We have on one page (see Table 14.1) summarized the attributes that we found to be effective in helping our children from urban backgrounds learn high levels of mathematics. The top three categories in the table focus on the reform aspects, and the bottom three summarize more traditional elements.

TABLE 14.1
Six aspects of TMEP

I. Start where children are and keep learning meaningful. Use many meaning-focused classroom activities:

Referential classroom and math modeling: Mathematical words and symbols linked to meaningful referents

The teacher leads mathematizing (seeing math in children's worlds) and rich language use by all.

Everyone's use of Meaningful Drawn-Models (MDMs) facilitates the reflection, discussion, and analysis of everyone's thinking; MDMing enables teachers to analyze children's thinking and children's errors.

All children produce their own MDMs, language, questions, and problems (not just answers).

Connected teaching: Cocreation of an inclusive and participatory classroom culture

Table 14.1 (cont.)

Class coconstructs emerging related understandings and builds on individual and shared MDMs.

Curriculum and teacher connect math topics to children's lives and to their imaginations

Mathematizing, MDMing, and rich language use

a) make problems accessible to all by providing multiple levels of access (everyone can participate),

b) validate and use all children's language and experiences while connecting them to standard language and symbols,

c) facilitate listening, speaking, writing, and helping competencies.

The teacher establishes coherence within a class, through the year, and across subject-matter areas.

II. Set high-level mathematical goals and expectations for all. Bring children up to the higher mathematics.

Use a few ambitious core grade-level topics and cumulative experiencing and peer helping throughout the year.

Construct ladders to mathematical concepts, symbols, methods, thinking, and discourse through

a) meaning-focused classroom activities that help children build knowledge skills,

b) work on prerequisite competencies to bring all children to mastery,

c) activities to help children move through developmental progressions to more efficient and general methods.

Teachers use assessment results to adapt their teaching and to help children focus their learning.

III. Develop a collaborative math talk culture of understanding, explaining, and helping.

Connected knowing: Learning occurs within a supportive environment in which help is available from peers and from the teacher; emotional as well as cognitive needs are addressed.

Separate knowing: All listeners can separate an idea from the person having the idea, so an idea can be respectfully modified or improved, or errors fixed, without diminishing its originator.

All teachers are learners, and all learners (students) are teachers of themselves and of others (peer helping).

Everyone expects that each child will understand his or her mathematical actions and use of MDMs and with practice will come to verbalize these understandings.

Math talk connects to referents (e.g., MDMs, story situations) so that participants and listeners can understand.

Debugging errors helps everyone learn more deeply; everyone including the teacher sometimes makes errors in math; no laughing or making fun about errors; errors deserve respectful and sensitive help from everyone.

Table 14.1 (cont.)

IV. Build on the best of traditional instruction.

Teacher and children demonstrate and explain.

Teacher assists children in learning productive roles in each classroom activity structure.

Children practice and memorize after repetitive meaning-making experiences for concept building.

Individuals solve problems with help available from a peer or teacher.

Children do worksheets. (Ours facilitate the use of MDMs related to standard mathematical symbols.)

Homework is somewhat similar to parental experiences. Mobilize a home helper to support homework.

V. Facilitate the learning of general school competencies. Increase self-regulatory actions so that children:

Become more organized, including learning to do work neatly and regularly returning completed homework;

Understand when they do not understand, and seek appropriate help when they need it;

Are involved in setting some learning goals, making a plan for meeting them and carrying out the plan;

Are helped to reflect on their own progress to affirm how much they have learned.

VI. Mobilize learning help in the home. Actualize home-school links focused on students' learning by:

Designating a home helper to monitor and help with homework (daily homework is expected; much of the homework is familiar to parents, and most is similar to that experienced by the student in the classroom);

Giving home games and activities to learn and practice prerequisite competencies and knowledge skills;

Conducting home-helper learning sessions to improve their helping and understanding of new aspects of the curriculum.

Equity, to us, means a classroom in which each child is included and affirmed as an individual and in which access to mathematical competencies valued by the culture is provided to all children. Given that educational resources are always finite, equity means balancing the needs of various individuals and trying to organize socially to maximize the learning of all. Central to equity is setting high-level goals for all and then using various methods of learning support so that individual children can go as far as they can. In an analysis of our urban school experience (Fuson et al. 1999), we summarize how the frequent reduction of goals to accommodate the realities of some or many underprepared children prevent all children in urban schools from reaching high-level goals. Such a reduction of goals is common in bilingual settings (Moll 1992).

TMEP sets high-level goals for learning with understanding, for high-level oral language competencies, and for the sophisticated use of mathematical modeling and mathematical symbolizing. These high-level goals are achieved by enabling all children to enter the mathematical activity at their own level. Teachers accomplish this by using rich and varied language about a given problem so that all children come to understand the problem situation, by mathematizing (focusing on the mathematical features of) a situation to which all children can relate (and that may be generated by a child), and by having children draw models of the problem situation. Cumulative experiencing and practicing of important knowledge skills helps children move through developmental trajectories to more advanced methods. Peer helping provides targeted assistance when necessary. The knowledge of the helper also increases. Assessment provides feedback to all and permits realistic adjustments of proximal learning goals by children and by the teacher.

We found that we needed to consider affective, social, motivational, self-regulatory, and self-image aspects of learning and not just focus on building mathematical conceptions. Thus, children need to be helped to see themselves as included in the world of mathematics. They need to be taken seriously as learners so that they can begin to see themselves as learners. Mathematization (starting with children's experiences), the coconstruction of understandings in a collaborative classroom, and involvement in learning-goal setting and evaluation all contribute to students' growth. Children and their home experiences and families are valued and affirmed by their inclusion in word problems stemming from mathematics storytelling. We emphasize effort rather than "ability," which is often misdiagnosed in schools, and we have created many examples of what Resnick (1995) discussed as ability created by effort.

Traditional and reform practices are usually posed as alternatives. We found many elements of value in both perspectives, especially in the context of urban schools. We wove these elements into a fabric of teaching-learning activities that would support teachers' and children's construction and use of mathematical meanings linked to the traditional mathematical symbols and words of the culture. In each mathematics domain the teacher connects children's experiences, words, meanings, object and drawn representations, and methods to the traditional mathematical symbols, words, and methods of that domain. These meaning connections define what we call the referential classroom: referents for mathematical symbols and words are used pervasively within the classroom. The referential classroom provides the links between what Vygotsky called spontaneous concepts and scientific concepts (Vygotsky 1962, 1986). The attributes of such a referential classroom and their relationships to Vygotsky's theory are discussed in more detail in Fuson, Lo Cicero, et al. (1999) and Hiebert et al. (1997).

We do not have space here to discuss all aspects of TMEP. We will instead provide an overview of the least familiar but core aspects of our approach.

AFFECTIVE SUPPORTS

Our affective supports are described by the terms *connected teaching, connected knowing,* and *separate knowing* (from Belenky, Clinchy, Goldberger, and Tarule 1986). By connected teaching, we mean helping children in a classroom cocreate a culture within which each child can be affirmed, can steadily grow in a range of social and conceptual competencies, and can teach (assist others) as well as learn. Coherence is a key to emotional, social, and mathematical growth because urban young children frequently experience too little coherence in their lives. Our focus on the conduct of the whole child is consistent with the Latino broader meaning of *educación* (education) as *formación* (formation): forming the child to know proper ways to behave in many different settings. Our focus on active listening and helping stems from the Latino experience in family-centered, rather than individually-focused, living. Sharing and helping is a pervasive fact of everyday life for some children, thus making it relatively easy for them to function in these ways in a TMEP classroom. For children who have had less such experience at home, the focus on listening and helping in our classrooms provides a valuable learning opportunity.

FOUR PHASES BUILDING TO HIGH-LEVEL MATHEMATICAL GOALS

In the initial phase, discussion and activities begin where the children are. Teachers find out and build on what children already know and understand. In the second phase, teachers introduce activities that emphasize and support the learning of the mathematical structure of a given mathematics domain. If the domain is initially accessible to most children, as with many word-problem situations involving single-digit numbers, this phase merges with the first phase. Activities elicit individual children's own views of the structure through engaging children in making their own meaningful drawn models (MDMs) of the situation. If the domain has a lot of specialized cultural knowledge, such as multidigit numeration in words and in written symbols, the activities initially provide structured conceptual experience with this cultural mathematical knowledge so that children can learn these structures and the meanings of the symbols and words.

The third phase is the longest. Cumulative experiencing (cf. repetitive experiencing, Cooper 1991) of the mathematical structure enables children to construct robust conceptions of those structures that can be generalized, abbreviated, internalized, and used flexibly in problem solving. For many major mathematical concepts, this generalization, abbreviation, and internalization follows a developmental progression of identifiable levels through which children need to be supported. A great deal has been learned about these developmental progressions in the past 20 years, especially for the addition and subtraction of small whole numbers (see Fuson 1992a, 1992b for summaries of this literature and the chapters in Leinhardt, Putnam, and Hattrup 1992 for summaries of the literature in other areas). Teachers' understandings of these learning trajectories (Simon 1995) enable them to recognize where children are and then to help children move through these learning trajectories to more advanced and efficient methods. Helping children move through these progressions may take a long time—weeks and even months for some concepts and some children. Cumulative mathematical experiencing at each of these developmental levels is required.

The final learning phase has many characteristics of traditional drill and practice, although we think of it as consolidation because of the frequent connotation of practice as a rote activity. Practice is necessary to bring certain prerequisite understandings to a rapid and accurate level of competence. However, such final-phase practice must rest on a base of meaning that can be accessed during problem solving and used in the construction of higher-level concepts. For example, learning to count by tens is not just a rote, meaningless activity; it can be surrounded with various kinds of meaning supports (Fuson, Smith, and Lo Cicero 1997; Fuson and Smith 1995, 1997). But after the introduction of these meaning supports, counting by tens needs to be practiced in various ways until everyone in the class can count by tens rapidly and accurately. Multiplication and division concepts need to be introduced meaningfully, but later specific facts also need to be practiced so that they can be used in multidigit computation.

In TMEP, such meaningful practice is the culmination of extensive earlier activities in the first three learning phases. It is conceptually-based drill and practice. We call the outcomes of all these learning phases *knowledge skills*—skills based on knowledge—to distinguish them from the rote skills that are the goals of much of traditional mathematics teaching. Knowledge skills are rapid and flexible, but they rest on a system of knowledge that can be used in deployment of the skill. This helps children use these knowledge skills in different situations. An important aspect of this final phase is that teachers attempt to structure such practice to facilitate children's learning of self-regulatory actions. Children grow in monitoring their own progress and in exercising some choice in what they attempt. For example, third graders choose which count-bys (7, 14, 21, 28, and so on) and multiplication facts they work on using feedback from checkups.

MATHEMATIZING AND RICH LANGUAGE USE

The core of the initial learning phase is mathematizing children's stories of their experiences and using rich language about these experiences (Hiebert et al. 1997; Lo Cicero, De La Cruz, and Fuson 1999; Lo Cicero, Fuson, and Allexsaht-Snider 1999; Ron 1999). The teacher elicits stories and helps children focus on, or extends the story to, potential mathematical elements that are the focus of that lesson. Children pose questions about the situation. They retell the situation in their own words. Children then pose mathematics problems from the situation. They frequently solve problems, and some children describe their solution methods. All this mathematizing and rich language use is done as a coconstructed process by the class as a whole orchestrated by the teacher as leader. The story is initially that of one child, but the participation of everyone as a contributor or as an active listener turns it into a shared class experience. The process attends to affective, social, and cognitive needs. Immigrant children can be connected to both their new and their old homes by telling stories about their grandparents or other family members they left in Mexico or elsewhere as well as by telling stories about their lives here.

Some stories are revisited throughout the year to provide continuity, coherence, and shared coconstructed referents. Everyone gains additional extended family feelings as they experience everyone's stories about their own lives and families. For example, one year a child told a story about a dog he had to leave with his grandfather in Mexico, and the teacher mathematized this story to focus on how many bones the dog ate. Related problems for the class to make up as the year went on concerned how far the dog ran, how many packages of food were needed for specified times if the food was packaged in boxes of ten, and how much food cost when it was purchased versus being leftovers. In other years repeated story scenarios concerned candy made and sold by one child's mother and apples grown on a farm.

USING MEANINGFUL DRAWN MODELS (MDMs)

The core of the second and third phases is extensive use of children's MDMs progressively linked to standard mathematical symbols and words. For example, children make drawings of a word-problem situation using circles and any other means of their choice to portray the mathematical relations in the situation (see fig. 14.1). These later become related to equations and to tables. This route permits early algebratizing of children's problem solving (Fuson, Hudson, and Ron in press). Children draw quantities to add and subtract multidigit numbers (see fig. 14.2 and Fuson, Smith, and Lo Cicero 1997; and Fuson and Smith 1995 for more details). These quantities are linked to, and over time are replaced by, mental and written numeral methods. Using such models resolves many management problems created by extensive use of manipulated objects as well as supporting reflection and discussion.

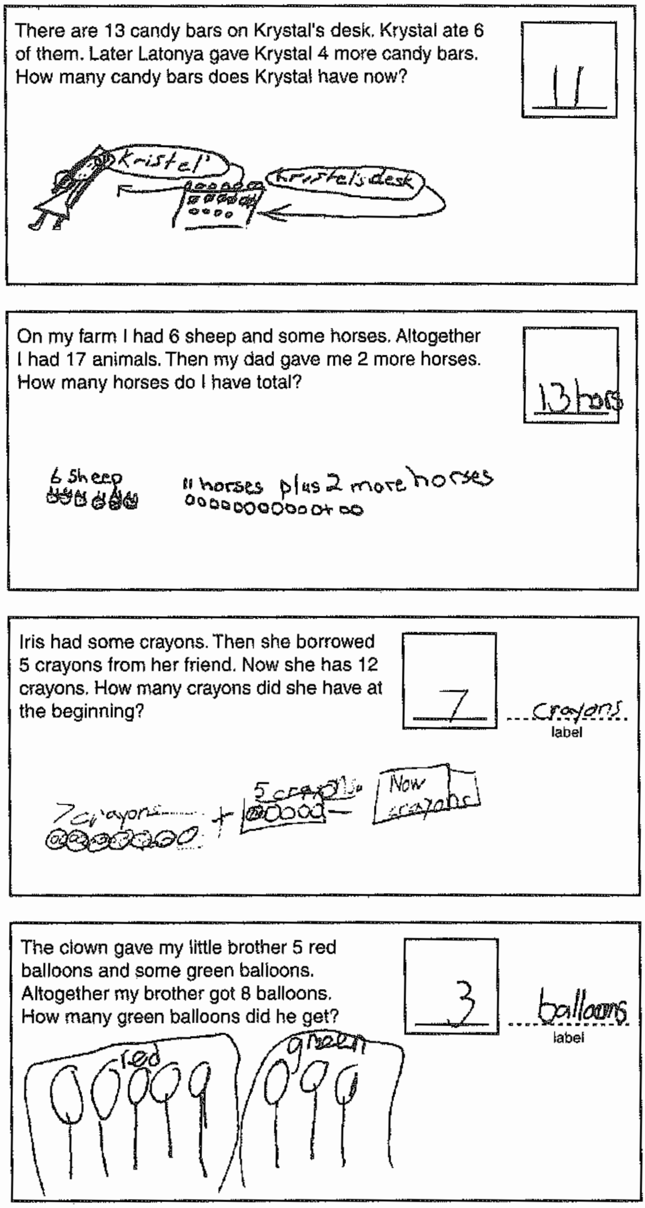

Fig. 14.1. Examples of children's labeled math drawings

MDMs give children experience in mathematizing situations to show their own conception of the mathematical structure. Children symbolize in different ways, so they can become aware of the goal of symbolizing in mathematics as well as see different kinds of symbolizing. This range of experiences with two-dimensional symbols drawn on paper is especially important for those children from low-literacy families who have comparatively little experience with symbolizing. Furthermore, MDMs leave records of thinking, thus facilitating reflection by their drawer and assessment of the drawer's thinking. Therefore MDMs facilitate the reform discussion styles of math talk—describing and justifying solution methods—by providing visual referents for such discussions. This enables everyone to look at the explainer's MDM during the discussion. Discussions that are merely verbal can lose many lower-achieving children (Murphy 1997). MDMs also allow the teacher to look at and think about a child's work outside class when there is time to reflect on and analyze it. Manipulatives leave no such records, and it is difficult for a teacher to get around to everyone during class to see each person's thinking.

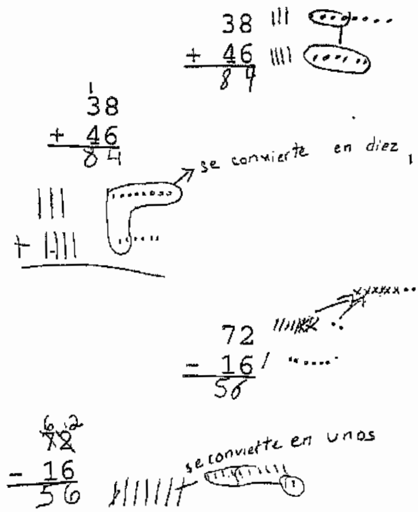

Fig. 14.2. Children's "ten-stick and dots" solution methods for two-digit addition and subtraction

LANGUAGE ISSUES

The issues of language learning and language use are particularly important for Latino children, many of whom face schooling in a language that is not their native language and may not be spoken at home. We developed teaching-learning activities that directly support language learning and rich language use. These result in rich learning for all children, including the upper-middle-class children with whom some of our teachers have been working more recently. TMEP stimulates children to produce their own words and spatial drawn models of the mathematical aspects of a situation. Teachers elicit multiple ways to discuss and view the same math situation. Children make up problems and pose various questions. Children describe their models and their solution methods in a variety of representations.

A typical school approach to students who do not have strong English-language skills is to view mathematics as consisting solely of mathematical symbols and to reduce both the levels of mathematics taught and the use of language in such teaching and learning (Moll 1992; Secada 1992). We have found it more productive to increase the amount of language use, but to situate this use in referential situations that can provide active meanings for such language. For example, in the situations from children's lives described earlier that were returned to during the year, children would generate story problems of varied types using that situation. Other children would ask the mathematical question in varied ways to practice producing mathematical language in a context.

WORKING WITH FAMILIES

We found it to be crucial to involve families in their children's mathematical education. An overview of our project work with families is presented in De La Cruz (1999). We have concentrated on efforts that will help families support their child's daily learning in the mathematics classroom, their practicing of needed skills at home in family games, and their experiencing and noticing mathematics in the real world outside of school. We found that when efforts were made to establish home-school connections, human resources can be found within the homes of almost all children to support their mathematics learning. Teachers found it to be important to appreciate and affirm all of the efforts parents were making in raising their children. The building of home-school links also needs to be viewed by all participants as informationsharing, as building mutual adaptations between the school and home settings, and as involving joint working toward the common goal—mathematical learning by the family's child.

To achieve the high level of mathematics understanding and skill that was our goal, children needed to complete daily homework. Our urban children and families were very supportive of such homework. It made the children feel grown-up, and most of them enjoyed doing the homework. Families were involved by identifying a "math helper" in each home to be responsible for monitoring the child's homework completion and to help if necessary. Identifying a particular person for this responsibility was helpful in homes where many pressures create difficult ongoing and changing life demands. When families understood that it was important to do so, almost all would organize themselves to identify such a helper. This might be a parent, an older sibling, an uncle or grandparent who lived with the family or nearby, or a neighbor. Because of language differences, no phone in the home, and distance from the school, teachers had to be resourceful and persistent in order to communicate the need for a math helper to some families. Older siblings or children who lived nearby sometimes carried messages or communicated the need to meet with the teacher and also provided translating help at any meetings.

Most homework involved mathematical situations that placed few demands on family helpers. New elements of TMEP (e.g., the use of MDMs) were explained in notes sent home, at parent nights, and by the children themselves. Wherever possible, the homework was designed to be not too different from homework the home helper might have had. In some reformed mathematics curricula, parents are asked to do many different kinds of activities with their child. The reading level is very high, too high for many of our families. Teachers did have to work hard with some children to establish patterns of effective return of homework. But most teachers who worked at it were able to achieve high rates of such return—rates that were surprising to them and considerably higher than they had experienced in the past.

Because many Latino families are family-centered, we designed games that family members could play to help their child practice important math competencies. Some families and some children had no previous experience in playing board games or games with cards, so teachers had family nights in which they taught families to play the games. These were generally well attended and enjoyed by all. One school librarian made fancier versions of the games and made them available to family members in the school library so that they could learn the games there. The feedback from families involved in using the games was quite positive. We also suggested activities parents could do to help children see math in their everyday lives. Some teachers also brought in family or community members to illustrate math links to the real world (Civil 1992, 1993; and Moll et al. 1992 also did this).

USING THE EQUITY PEDAGOGY

Although our teachers had the support of teaching materials we were developing in the classroom, teachers can implement many aspects of TMEP while using many different kinds of teaching materials. We have three major suggestions for teachers. First, try to collaborate with at least one other teacher in this effort. This collaboration can stimulate ideas and provide support for the different kinds of changes each teacher will be making.

Second, concentrate efforts on a few central grade-level goals. Try to help the classroom become a place where everyone explicitly is helping everyone learn those central goals. For these major goals, use all six aspects of TMEP (see Table 14.1).

Third, devise learning trajectories for these central goals by paying attention to children's different solution methods and to their errors. Everyone in the class can become a solution collector and an error detective who finds and helps correct errors. This approach removes the total responsibility from the teacher to identify, understand, and correct errors. It also provides more-advanced children important opportunities for rich mathematical learning as they compare and contrast different methods.

CONCLUSION AND A VISION OF THE FUTURE

Equity requires a balance between rights and responsibilities, between the needs of less-advanced and more-advanced children, and between the needs of less-advantaged and more-advantaged children. Equity guarantees access to mathematical learning, but requires and supports continuous effort by all participants. Meaning-making and problem solving are major foci in an equity classroom. These are facilitated by children's use of MDMs, situations linked to mathematics language and symbols, and rich language use. Practice focused on vital prerequisite topics also plays a crucial role. We offer our vision of a combination of the best elements of the twentieth century in the hope that it can stimulate productive dialogue and action that can move us toward achieving more substantial mathematics learning by all in the twenty-first century.

Such productive dialogue and action could focus on many elements of the equity pedagogy. The ease of communicating through e-mail and the Web, combined with technological advances that could make sharing videotaped segments of classroom interactions inexpensive and widely available, could enable teachers and mathematics educators all over the country, and indeed all over the world, to share productive conceptual supports. Ways to improve classroom mathematical discussions could be shared, and participants could share, critique, and suggest improvements for a given videotaped segment of classroom discussion. Children's language and solution methods could be shared, as could different traditional methods coming from homes, especially those of families from other countries. Errors children make could be shared, and other teachers could participate in debugging those that are not obvious to a given teacher. Clearly, any of these kinds of interactions could also take place in face-to-face teacher groups.

TMEP focuses on meaning-making, high expectations, and a collaborative culture for supporting understanding of children. Teacher development efforts for both in-service and preservice teachers require the same features. Curriculum developers, administrators, in-service educators, and teachers themselves working alone or in groups need to start where teachers are and keep learning meaningful. They must work to develop a collaborative math talk culture of understanding, explaining, and helping about teaching math with understanding. We have found that much teacher learning occurs in a classroom that uses TMEP through mutually stimulating cycles of teacher and child mathematical learning (Drake, Spillane, and Hufferd-Ackles in press; Hufferd-Ackles 1998; Hufferd-Ackles and Fuson 1999). Teaching using TMEP leads to teacher growth that is stimulating and fulfilling as well as to increased understanding by all children.

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