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SUPPORTS FOR LEARNING MULTI-DIGIT ADDITION AND SUBTRACTION: A STUDY OF TAIWANESE SECOND-GRADE LOW-MATH ACHIEVERS

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Abstract: Taiwanese second-graders were randomly chosen from ten classes and three schools in Taiwan and tested to compose a low-math-achiever (LMA) and high-math-achiever (HMA) group (each n =37). All children were given an IQ test (Raven), a place-value task, a written test of 2- and 3-digit addition and subtraction problems, and a strategy interview of mental math problems. LMAs compared to HMAs had a less mature place-value concept, used less sophisticated and slower procedures in solving the addition but not the subtraction mental math problems, and performed more poorly on the multi-digit addition but not subtraction problems. The base-10-structured methods were favored by both groups when interviewed about their strategies. A "Linguistic and Visual Support" Model was proposed to explain the unusual findings of subtraction easier than addition.

Rationale

In the past two decades, Asian students have been the "winners" of various international studies in math. These results sometimes lead to the stereotype that all Asian students are math wizards who do not need to struggle with math. However, little is known about Asian low-math achievers. The results of most current cross-national studies have failed to explain why Asian low-math achievers do not do as well as their normally-achieving peers, given the facts as most cross-cultural studies have suggested (e.g., Stevenson & Stigler, 1992) that they all learn from a more centralized math curriculum, use a more regular number-word system, and live in a society where effort is stressed more and parental supports are more available to a child's education. This study initiates an examination of characteristics of Chinese low-math achievers.

The theoretical framework was based on a Vygotskiian socio-cultural perspective (1978, 1986) which postulates that the formation of minds requires study of the sociocultural setting in which activities take place. Thus, solution strategies were examined to ascertain how well all children could use the semiotic tools used in their culture in the context of math problem solving: the regular Chinese number words that name the ten (e.g., 12 is said as "ten two" and 32 is said as "three ten two") and the 10-structured methods of adding and subtracting taught in the classroom. "Make-a-ten" methods were predominantly used in the class to solve addition problems. These methods varied in which number made a 10: making the big number to 10 [e.g., 7+8 = (8+2)+5=15] or making the small number to 10 [e.g., 7+8 =5+(3+7)=15]. For subtraction problems, the "make-a-ten" method was taught in class. This method splits the teen number into ten and some [e.g., for 15-8: ten five (15) = ten and five, and the 8 is taken from the 10 leaving 2 (many students just know 10-partners of all numbers) which is combined with 5, the other part of 15, to make 7]. Korean students (Fuson & Kwon, 1992a) use two related methods that involve going up over ten or down over ten. Other methods that have been proposed by American researchers (e.g., Fuson, 1988) such as doubles, or counting on were not emphasized in the instruction either.

Methods and Data Sources

Second-grade Chinese children from three schools in Taiwan chosen to span a range of typical schools (one in a city and two in rural areas) were followed throughout their second-grade year. The subjects were randomly chosen from ten classes randomly chosen from the three schools. Children from the top and bottom 10% of each class based on a composite score of their math screening test given in the beginning of second-grade and their first-grade math GPA made up the high-math-achiever (HMA: n=37) and low-math-achiever (LMA: n=37) groups. A nonverbal IQ test (Raven) was used to choose children with IQs in a normal range. Children included in this study were also required to have no hearing or visual impairment and no emotional problems. This information was gathered from students' psychological reports and class teachers' reports.

The math tasks included a place-value task (the Kamii task), a written test of 2- and 3-digit addition and subtraction problems (for details, see Fuson & Kwon, 1992b), and an interview about their strategies for solving two single-digit mental math problems (8+7, 14-6). The Kamii task required children to show how many objects the number "1" means in 16. Students were given the 3-digit problems before they had studied the topic in school. The goal was to examine whether they could transfer their understanding and procedures in solving 2-digit to 3-digit problems. The 2-digit addition problems required a trade from the ones (27+57 and 54+19), and the 3-digit problems required a trade from the tens (571+293 and 284+681); the subtraction problems were the inverse of the addition problems.

Results

Fewer LMAs than HMAs had a solid understanding of place value. In the fall, only 26 (70%) LMAs answered that "1" in 16 was worth "ten" instead of one, whereas 36 (97%) HMAs answered correctly without a prompt. Five LMAs needed further prompts by the researcher ("Is this a 10 or a 1?") to be able to come up with the right answer. However, six LMAs were too adamant to change their answer even after the prompt. In the winter, still one LMA answered incorrectly and two LMAs needed a prompt, while all HMAs answered correctly.

LMAs were as accurate as HMAs in solving mental math problems with totals in the teens when there was no time constraint. However, more LMAs adopted a less efficient strategy to solve the problems, such as finger counting or "counting on" methods. For the addition problem (8+7), there was a significant difference in the strategy use for the two groups (see Table 1).

Table 1. Frequencies of Use of Different Strategies for Solving 8+7.

Category	I	II					III			I	II	III
Methods	1	2	3	4	5	6	7	8	9	Subtotal 1**	Subtotal 2-6**	Subtotal 7-9**
HMAs N= 37	7	16	4	4	1	2	3	0	0	7	27	3
LMAs N= 37	2	15	0	0	0	1	16	2	1	2	16	19
Total	9	31	5	3	1	3	19	2	1	9	43	22

Notes

Category I means "Automatic Procedures". 1) direct retrieval

Category II means "More Sophisticated and Faster Procedures" 2) making the big number ten; 3) making the small number ten; 4) Chinese imaginary abacus method; 5) finger abacus method, 6) doubles

Category III means "Less Sophisticated and Slower Procedures" 7) finger counting on from the big number, 8) finger counting on from the small number, 9) counting all.

** means a significant difference between HMAs and LMAs on a x^2 test at $p < 0.01$.

Nineteen (26%) LMAs, but only three (4%) HMAs used the less sophisticated and slower procedures, especially counting. More HMAs than LMAs (27 vs. 16) adopted more sophisticated and faster procedures (many used the "make-a-ten" methods). Methods 4 to 6 were not emphasized in class and more HMAs than LMAs used these methods (7 vs. 1). Additionally, fewer LMAs than HMAs rapidly retrieved the answer rather than using a solution method (2 vs. 7). However, the groups did not show significant differences in solving the subtraction problem (14-6) (see Table 2). Interestingly, most students in both groups commonly used the "make-a-ten" method to solve this problem. For both 2-digit and 3-digit problems, LMAs did significantly worse than the HMAs in the addition but not in subtraction problems (see Table 3).

Discussion

First, Chinese LMAs' place-value concept was less mature and took longer to develop than did that of the HMAs. Although they did read any 2-digit numbers with the Chinese words "ten something" (42 as "four ten two"), the verbal label "ten" did not necessarily create an automatic "magic" for all Chinese children to be aware of the meaning of the word in relation to the place value of that number and understand the number sense (e.g., embeddedness of number relations). The result supports

Table 2. Frequencies of Use of Different Strategies for Solving 14-6.

Category	I			II		III	IV	I	II	III	IV
Methods	1	2	3	4	5	6	7	Subtotal 1	Subtotal 2-5	Subtotal 6	**Subtotal 7**
HMAs N= 37	0	27	3	4	2	1	**0**	0	36	1	**0**
LMAs N= 37	1	30	4	0	0	1	1	1	34	1	1
Subtotal	1	57	7	4	2	2	1	1	70	2	1

Notes

Category I means "Automatic Procedures" 1) direct retrieval

Category II means "More Sophisticated and Faster Procedures" 2) up over ten, 3) down to ten, 4) Chinese imaginary abacus method, 5) finger abacus method

Category III means "Less Sophisticated and Slower Procedures" 6) counting down.

Category IV means "Others" 7) don't know

Table 3. Percentage Correct for Written Multi-Digit Problems for the Two Achievement Groups.

Addition	HMAs	LMAs	Subtraction	HMAs	LMAs
26+57*	100 (0)	87 (35)	83-57	97 (16)	89 (32)
54+19#	100 (0)	89 (32)	73-19	97 (16)	92 (28)
571+293**	97 (16)	76 (44)	864-571	89 (32)	78 (42)
284+681**	95 (92)	73 (45)	965-284	87 (35)	76 (44)
Total**	98 (9)	81 (31)	Total	93 (20)	84 (32)

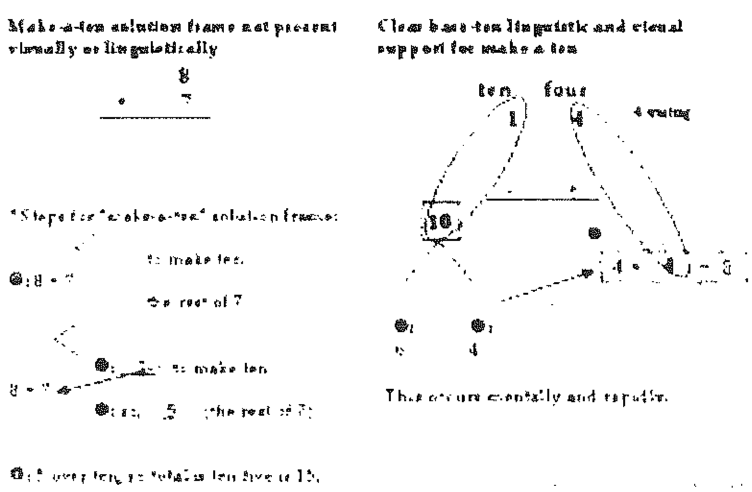
Table 3 Notes: # means a marginal difference on a Fisher's exact test at $0.05 < p < 0.10$;
*means a significant difference on a t-test at $p < 0.05$;
** $p < 0.01$. Numbers are in percentage and the numbers in the parentheses are standard deviations.

the notion proposed by Vygotsky that psychological functioning occurs first in the interpersonal level and then gradually shifts to the intra-personal state. Although the Chinese number words correspond well with the Arabic number system, it is only through the enculturation process that Chinese children gradually come to understand the link between the language (number words) spoken and the object meaning of the base-10 place value of the number system used in the society. It is a gradual process, and LMAs seem to need more time than their high-achieving peers to develop this concept. Thus, the gap Ho and Fuson (1998) identified in Chinese kindergarten children between those who understood the ten in teen numbers and those who did not is not closed by grade 2. This suggests that LMAs may need to be provided more and longer explicit teaching before their place-value concept is consolidated.

The subtraction-superior-performance findings seem to contradict the common assumption that subtraction is more difficult and error-prone (e.g., Fuson, 1984). Based on the analyses of strategy data, I propose a "linguistic and visual support" model to justify this unusual finding (see Table 4). In the process of solving the subtraction problem, the base-10 structure is explicitly accessible via both visual and linguistic supports. In 14-6, the thinking procedure would be: ten minus six, four; four plus four, eight. "Ten" is first seen within and read for the ten four (14), that then sustains and connects linguistically and visually with the make-a-ten method. However, such supports are less transparent in the addition problem. If 8+7 is solved by a "ma e-a-ten" method, the procedure will be as: eight plus two, ten; ten plus five, ten five. The base-10 linguistic support is less clear (it must be generated during the solution) along with no visual base-10 support in this case.

The educational implications of this study are as follows. First, given adequate supports, especially in both linguistic and visual domains, it is possible for Chinese LMAs to do as well as their HMA peers. This finding is encouraging because it suggests that given the right scaffolding, children can learn to SEE and HEAR and

Table 4. Contrast of Base-Ten Linguistic and Visual Support for Problems of 8 + 7 and 14 - 6.



BE AWARE of patterns, structures, and relationships, and then use these as tools to solve problems. This echoes the *Principles and Standards for School Mathematics* of NCTM (2000) for how to learn and teach children mathematics. Second, following this paradigm, the question of whether children use a less regular number system or not seems not as important as asking the question: "How do we use (linguistic) support to help our children learn and do math?" Finally, as Vygotskiian theory indicates, "...we are empowered as well as constrained in specific ways by the mediational means of a sociocultural setting" (Wertsch, 1992, p.42). Thus, teachers should be aware of potential means of support when teaching children in math or any subject.

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