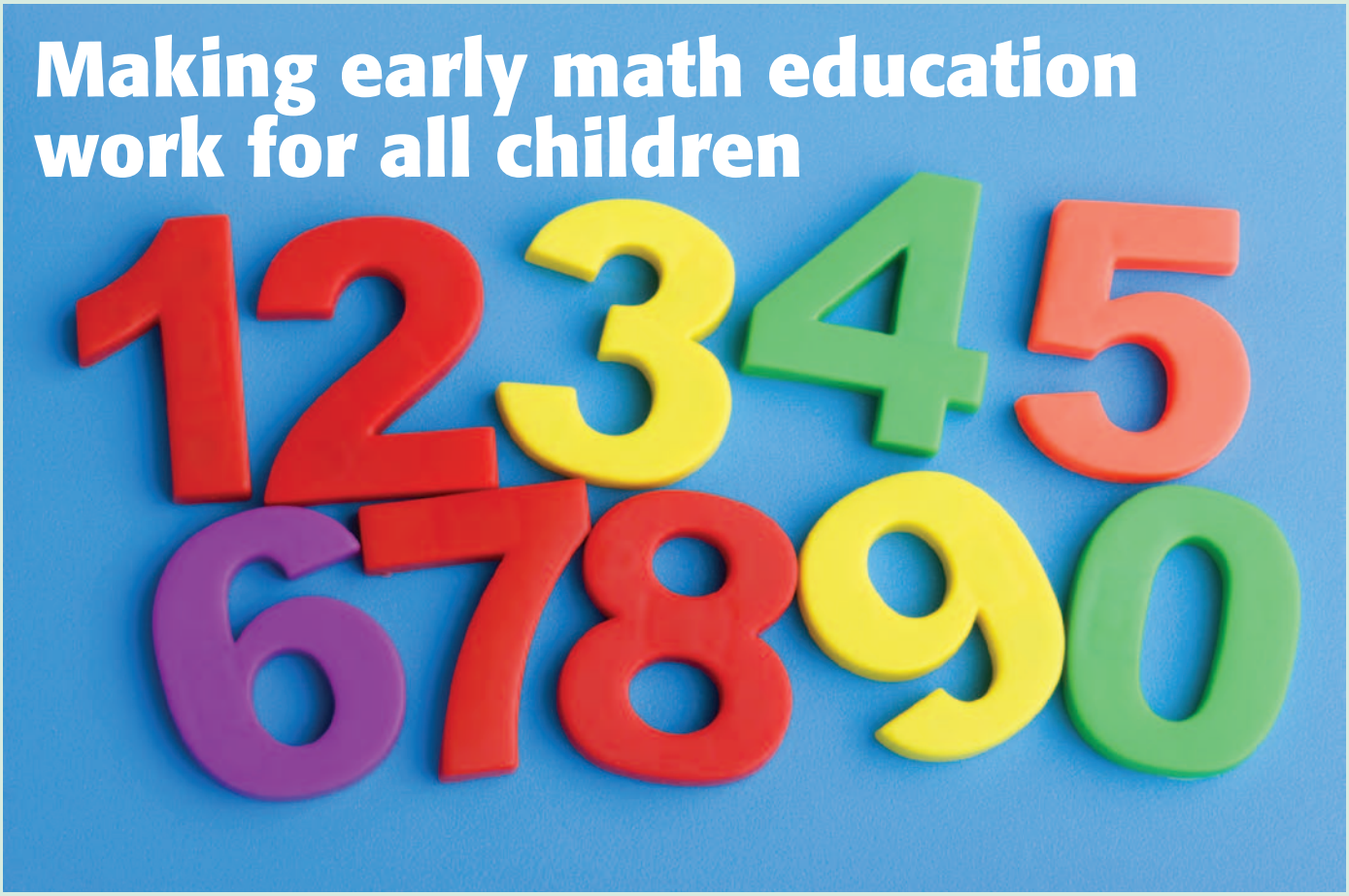
Making early math education work for all children



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Common Core

Prekindergarten teachers lay the foundation for later success in mathematics when they attend to the concepts that young children can and should learn.

By Karen C. Fuson, Douglas H. Clements, and Julie Sarama

Young children are learners. They learn in different situations about different things. They learn from parents and from other children. They learn by interacting with things. They learn by helping others. Now many of them are learning much in preschools and in kindergarten. What should they be learning, and how should they be learning these things?

A recent National Research Council report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Cross, Woods, & Schweingruber, 2009), identified math concepts that young children can and should learn. The report looked at four age groups: 2- and 3-year-olds, 4-year-olds (prekindergarten), kindergarten, and 1st grade (see Chapters 5 and 6, which you can view online at www.nap.edu/openbook.php?record_id=12519&page=R1).

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The report recommended that math experiences in early childhood settings concentrate on the identified foundational and achievable content goals in:

- Number, which includes whole number, relations, and operations (arithmetic); and
- Geometry, spatial relations, and measurement, with more math learning time devoted to number than to other topics.

These goals, along with possible teaching approaches, are summarized for teachers in two additional books, one focusing on prekindergarten (NCTM, 2010a) and the other on kindergarten (NCTM, 2010b). These books are published by the main professional organization of math educators and researchers, the National Council of Teachers of Mathematics (NCTM), and they also are endorsed by the main early childhood organization, the National Association for the Education of Young Children (NAEYC).

> Practice does not mean rote experiences. Practice involves repeated experiences that give children time and opportunity to build their ideas, develop understanding, and increase fluency.

Children enter kindergarten with a huge range of numerical knowledge and skills. Closing the income and race/ethnicity gaps in preK and kindergarten is crucial for improving math learning in this country, particularly because math knowledge at these ages predicts school achievement in math and in other topics, such as reading. Indeed, early math knowledge is one of the strongest predictors of math grades in high school, high school graduation, and college entry. Especially important are children's competence with quantity and number, as well as geometric and spatial reasoning. This is why the second part of the NRC report title is *Paths Toward Excellence and Equity*.

The NRC report summary of research and its recommendations for learning goals was used in developing the Common Core State Standards for kindergarten through 2nd grade. The Common Core State Standards for kindergarten include some goals that are preK goals in the NRC report because the preK goals in various states are limited and variable, and it's crucial that children are prepared to meet these goals. The huge gap in math understanding in preK and kindergarten children summarized in the NRC report makes it crucially important that preK classrooms do all they can to prepare children for kindergarten to decrease the gaps in learning. A summary of the involvement of early childhood professionals in the Common Core State Standards appears at www.achievethecore.org.

Many children don't learn the count words in their home language or how to count things accurately or the meanings of and words for written number symbols. They must see and hear and try these for months and years as they gradually extend their count word list, increase their counting accuracy, and learn more concepts about number. How can preschools and kindergartens support all of this extensive learning? The Common Core State Standards do not specify methods of teaching. But the NRC report does identify effective teaching-learning practices.

Destructive false dichotomies

Unfortunately, most of us learned math without much understanding. Our experience can limit our vision to rote teaching and learning, such as telling or showing, with little thinking by children. An alternative that is proposed especially in early childhood is that the child discovers concepts herself through interacting with objects or in play. These are oversimplified and destructive dichotomies: Play vs. learning/academics, adult-directed vs. child-directed, adult-initiated vs. child-initiated, and student-centered vs. teacher-centered/directed. The research reviewed in the NRC report found that learning is much more complex and interactive and that these simplifications are limiting and therefore damaging to children.

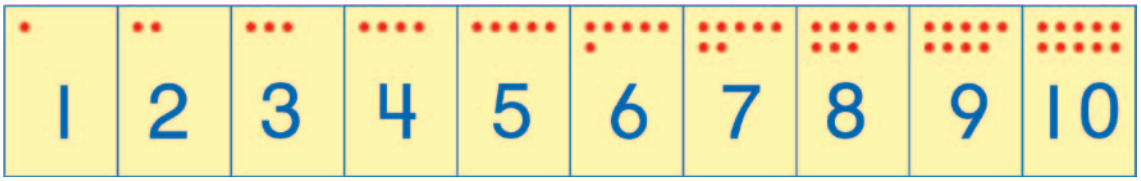
Such simplifications are especially problematic when they use strong language that is prejudicial about the opposite view. An example is a recent *Kappan* article, "Making play work for education" (Weisberg, Kittredge, Hirsh-Pasek, Golinkoff, & Klahr, 2015). These authors make a useful extension of the usual early childhood emphasis on play by including guided play that is initiated by an adult and therefore can support educational goals. But they use strong and limiting language about other types of activities in the preK classroom, for example, direct instruction or didactic situations instead of saying intentional teaching, or co-opted play instead of teaching on the fly. They seem to characterize most adult interactions with children who are playing as exerting harmful control that interferes with children's autonomy. And they seemingly ignore research that indicates that children are inherently driven to learn because of the feelings of competence that result; they seem to imply that children will only learn if the learning is at least disguised as play: "Even children are sensi-

tive to the difference between circumstances where they lead and those where they are given an educational experience disguised as play — what one might call "chocolate-covered broccoli" (pp. 9-10). These authors apparently have not seen the joy and deep engagement in mathematics itself displayed by very young children in optimal environments such as the NRC report describes. There is no need to "disguise" high-quality educational experiences. Summaries of relevant research about play that go beyond those in the Weisberg et al. article and more discussion of problems with the false dichotomies can be found at http://preschoolmatters.org/2014/03/03/play-mathematics-and-false-dichotomies/.

To help overcome these false dichotomies, the NRC report summarized effective teaching-learning practices that can support all children's paths toward excellence and equity. Consider an activity that supports learning of many of the number concepts and skills so that we can understand the complexities of such learning. The activities shown in Figure 1 have been done by one of us in many preK and kindergarten classrooms. In preK, they can be started in groups of four to six or even larger if children have some of the knowledge. As the steps in the activity become a known routine and children learn more, they can be done with larger groups. They can be done with a whole kindergarten class from the beginning of the year. Young children love routines in which they know what to do and can do this with other children. This builds feelings of belonging. The Counting Mat activity is designed for children who have a range of knowledge at the beginning because that is the reality of preK and kindergarten classrooms. Some children may never have seen numbers before, but they learn from others as they see what other children are doing. The Number Parade shows the number symbols in order and can be followed visually even without knowing number words or meanings. Children can count aloud together from 1 to 10 with a child pointing to each number on the Number Parade, either at the beginning of this activity or at another time. In

FIGURE 1.
Counting Mat activities for quantity-word-symbol and decomposing knowledge

Number Parade with 5-groups for 6 to 10



Numbers within 5

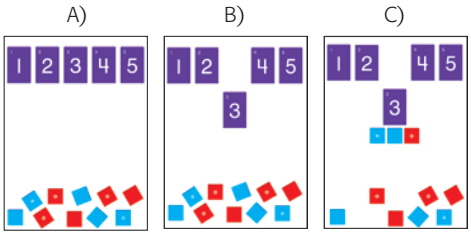
A) Put number tiles in order at top; can look at Number Parade.

Put red and blue tiles at bottom (5 red and 5 blue in a baggie; one side is plain and the other side has a white dot).

Point to the number tiles in order and say the number on that tile.

B) Pull down the number tile for the number said by teacher or a student.

C) Show that number of tiles.



D) Have a "math talk" discussion:

a. Relate the visual quantity 3 to 3 fingers, 3 sounds, and 3 body movements.

b. Practice visual imagery: Close your eyes. Visualize. (See the 3.)

c. Describe different arrangements by color, dot/no dot, spatial relationships (e.g., 3 = 2 + 1)

d. Change your arrangement and discuss why you still have 3.

e. Copy the arrangement of another person.

f. See and describe partners of 3 (decompositions of 3 into 2 numbers) and create new partners that make 3 using attributes listed in substep c.

g. Graph on a graph mat (2 rows/columns of 10 empty squares) by putting tiles dot side up on the graph mat and say which is more (later say which is less).

step B, as children pull down the number said by a child or the teacher, children who do not know which is 3 can watch others and pull down the same tile using different ideas (it is in the middle, it has curvy loops on it). As children do all of the remaining steps, those who cannot yet do them alone have models all around them that stimulate their thinking and let them participate. The teacher also creates a nurturing, helping community in which a child who knows something can actively help and explain to another child. Children with advanced knowledge get to do more complex things in step D's substeps of c, d, f, and g. They are relating different arrangements and quantities to build their flexibility. Before long, all children can begin to do this with these small numbers.

This activity can be done many times, and each time children advance in their own learning path about multiple math goals. Children contribute creatively and are thinking and learning all the time. Sometimes, the teacher may want to highlight certain issues to be sure children are thinking about them. At other times, the activity may unfold almost entirely from what children do and say. Both are needed to move all children along their learning path. This social activity in which children can see other math approaches and help and be helped is much richer than limited images within the false dichotomies.

Closing the income and race/ethnicity gaps in preK and kindergarten is crucial for improving math learning in this country, particularly because math knowledge at these ages predicts school achievement in math and in other topics, such as reading.

Turning to the domain of spatial thinking and geometry, Figure 2 illustrates an activity, Shape Step, which can develop several levels of thinking. The teacher makes shapes on the floor with painters' tape (no angry custodians!) or chalk shapes outdoors. The leader (the teacher at first) names a shape category such as "triangles." Children step on only the triangles. If more than a handful of children are playing, four or five can step and the others can watch carefully to make sure the group steps on all the triangles and only the triangles. Then they take turns. Children love playing on their own and taking turns being the leader.

Activities such as Shape Step can be done many times to develop children's geometric thinking at

FIGURE 2.
Participating in a Shape Step activity

progressively more sophisticated levels. For example, they might simply step on typical triangles vs. circles or squares to develop simple visual thinking. Then, the teacher can add triangles that are less typical (e.g., long "skinny" triangles) as well as shapes that are not mathematical triangles such as a deltoid (or "chevron," with four sides). Now children explain why the shape they stepped on was the correct shape ("How do you know that was a triangle?") and why they did not step on others. Finally, children might step on squares and not step on shapes that visually resemble squares but are not such as a non-square rhombus (does not have right angles,) or rectangles that are not squares (not all sides are equal in length). Impressively, even 4-year-olds learn and love to explain to each other, such as Julio's, "You missed that one! Look, it's tilted but all sides are the same and the angles are the same — it's a square for sure!"

Effective teaching-learning practices

The NCTM/NAEYC *Focus*. . . books emphasize the two ongoing vital roles of teachers:

- Expect and support children's ability to make meaning and mathematize the real world.
- Create a nurturing and helping Math Talk Community.

Teachers also need to lead the class through a research-based learning trajectory based on children's thinking, as outlined in the report. We saw both of these in action in the examples above. An important part of such learning paths is that they provide repeated meaningful experience with core concepts

so young children can truly learn in depth. Just as hearing a story repeatedly is interesting to young children, so too young children are interested in repeatedly reciting the list of number words, repeatedly counting collections of objects, and repeatedly putting shapes together to make new shapes. And just as hearing a story anew or dramatizing it gives children new insights and a deeper understanding of the story, so, too, repeated counting and repeated examination of shapes help children develop new insights and a deeper understanding of mathematical ideas.

Finally, teachers should help children relate work with quantities and objects in the real world to pictures of these in books and on other two-dimensional surfaces or pieces of paper. This is especially important for children coming from homes where such experiences are infrequent or nonexistent. In later preK and in kindergarten, it is also important to help children relate real three-dimensional objects to math drawings of these objects and to written mathematical symbols. At these ages, all children need both kinds of experiences: with actual objects and things and with math drawings and symbols of the things. At present, too many children have only one kind of experience, and this often is linked to children's background. Too many children in preK settings, many from backgrounds of poverty, experience only or primarily worksheets that support only rote learning, and too many children from backgrounds of affluence experience only or primarily work with objects not related to math drawings or symbols, reducing the meanings they can make. Worksheets (activities or drawings on paper) can support understanding and increase fluency if they are used after the experiences with objects, relate to the experiences, and are used by adults focusing on the two teaching-learning practices: meaning making and mathematizing and a math talk community. Worksheets can stimulate discussions of mathematizing in the real world and of visual patterns that are important mathematically. They also can provide practice, reflection, and experience with seeing things and drawing on paper. The real issue is not worksheets versus objects. The vital issue is to support meaning making, mathematizing,

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and making connections among mathematical language, symbols, and quantities and shapes.

Mathematics Learning in Early Childhood specified that math process goals such as representing, problem solving, reasoning, connecting, and communicating should be integrated into the teaching of the foundational and achievable goals. These were extended in the Common Core State Standards to eight standards for mathematical practice. These eight can be paired and named to provide a summary of effective math teaching-learning in action: The teacher and the children are doing math sensemaking about math structure using math drawings (visual models) to support math explaining.

Focused math teaching in preK

Mathematics Learning in Early Childhood found that targeted learning path time for specific math goals is crucial. Children need time and support to consolidate thinking at one step and to move along the learning path to the next step. Weaving math into other activities is not enough although integrating math can be part of many learning opportunities. Children need enough time focused on the foundational and achievable math goals to gain understanding and fluency in them. Even children who learn mathematical ideas at home will benefit from a consistent high-quality program experience in preschool and kindergarten. Time must be allocated for more formal parts of mathematics instruction and discussions that occur in whole group or in small groups and for plenty of follow-up practice. Practice does not mean rote experiences. Practice involves repeated experiences that give children time and opportunity to build their ideas, develop understanding, and increase fluency. Children also need time to elaborate and extend their mathematical thinking by exploring and sharing their own methods.

Research reviewed in *Mathematics Learning in Early Childhood* indicates that children need at least 20 to 30 minutes a day to learn the identified goals. More is helpful, especially for children who enter with relatively little mathematical knowledge. Appropriate learning experiences in math also support language development and real-life knowledge, so some of this math time is accomplishing language goals.

Mathematics Learning in Early Childhood recommended that home, childcare, preschool, and school environments need to support children in the process of becoming self-initiating and self-guiding learners. When children have opportunities to move along the learning paths described, they become interested in consolidating and extending their knowledge. They begin to practice by themselves. They learn by asking questions and varying what they do. Understanding the math content requires repeated experiences with the same numbers or same shapes and related similar tasks. All children must have sustained and frequent times in which they engage in important mathematical ideas and talk about what they're doing and why they're doing it. These repeated experiences can be in structured activities as in our examples and in individual activities as long as there are some opportunities for feedback about the math involved.

PreK math education works for all children

In math learning, effort creates ability. In the U.S., many of us think of math as a subject at which a person is either good or not. And it is acceptable to be not good at math and even better to be not good at math in some groups. People do not think like this in all countries. They think that one needs to work at learning math just as one works at learning anything else. This view of math falls within what Carol Dweck has called a growth mindset: the understanding that talents and abilities can be developed through effort, good teaching, and persistence. We have described examples of preK math teaching-learning opportunities that engage children and let them develop the knowledge they need to succeed in kindergarten. When preK teachers have opportunities to teach to these goals using effective teaching-learning practices, they always report amazement about how much young children can learn about math.

Our children can learn much more math than most of them do now, and it empowers them. Let us all leave behind the damaging false dichotomies about math teaching and learning for young children including language that is simplistic and prejudicial to one view. Let us work together to help all children learn the math that they can learn and that makes them feel and be competent. Let us close the math gap in preK and in kindergarten so that all children move to 1st grade ready to engage with 1st-grade math goals. K

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