Fuson, K. C., Sherin, B., & Smith, S. T. (1998). A Vygotskiian action-research model for developing and assessing conceptual models and instructional materials inter-actively. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 541-548). Raleigh, NC: North Carolina State University.

A VYGOTSKIIAN ACTION-RESEARCH MODEL FOR DEVELOPING AND ASSESSING CONCEPTUAL MODELS AND INSTRUCTIONAL MATERIALS INTER-ACTIVELY

Karen C. Fuson, Bruce Sherin, & Steven T. Smith
Northwestern University
fuson@nwu.edu, bsherin@nwu.edu, ssmith@nwu.edu

U.S. mathematics curricula have serious design limitations. This “underachieving curriculum” that is “a mile wide and an inch deep” dramatically underestimates what most children can learn. In prior work, we have described more ambitious classroom interventions for K-3 mathematics that build on individual experiences, interests, and the practical math knowledge of children. In this paper we describe not a particular curriculum, but a general model for the research process that we call “Classroom Conceptual Research.” The central feature of this model is that it involves a tight interaction among model building, design work, and classroom-based action research, with a strongly “conceptual” emphasis. The work is carried out collaboratively by an interdisciplinary design team of university faculty, teachers, and research staff. We believe that the wider use of this model by the research community can lead to significant improvements in U.S. mathematics curricula.

U.S. mathematics curricula have serious design limitations that limit student learning. The grade placement of topics is delayed relative to other countries, and excessive spiraling (returning to topics every year) leads to too much reviewing at the cost of learning time on new topics. This “underachieving curriculum” that is “a mile wide and an inch deep” dramatically underestimates what most children can learn because curricular placement allows so little time for any one topic (Fuson, Stigler, & Bartsch, 1986; McKnight et al., 1989; Peak, 1996; Stigler, 1997). Deep development of student and teacher understanding of a given topic requires time. This time can be obtained by identifying core grade-level topics and concentrating deeply on these.

Significant theoretical and empirical research on mathematics learning and teaching is also severely limited by these curricular issues and by the fact that research and the development of instructional materials are often independent endeavors with relatively little coordination. NSF funding is separate, so projects must focus on research or on materials development. Traditional textbooks in the past rarely used much research on student’s conceptual structures, developmental progressions of concepts or methods, types of word problems, or conceptual supports for learning (e.g., see the analyses in the chapters in Leinhardt, Putnam, & Hattrup, 1992). Ascertaining what kinds of learning are really possible, and identifying progressions of understandings and solution methods within a domain, require designing teaching-learning materials that will potentially support students through a more ambitious learning trajectory (Simon, 1995) of activities. Undertaking such a coordinated effort is complex but can be very productive. Understanding and naming this kind of work would seem to advance our current way of thinking about both research and development. Toward this end, we describe here a preliminary model in which the development of conceptual models and empirical action-research are tightly interwoven with the on-going design of teaching-learning activities.

Thus, the purpose of this paper is not to describe a particular curriculum, but instead to argue for a general model for the research process. We should note that we are not alone in advocating programs that integrate fundamental research, design, and enactment (e.g., Brown, 1992). However, we believe that articulating the particular features of our approach will facilitate dialogue about this class of approaches and thus begin to sharpen this paradigm. We use the term “Classroom Conceptual Research” to describe our model, to highlight what we broadly refer to as a “conceptual” emphasis in our classroom research. This emphasis is manifested in a focus on the following four kinds of issues: (1) discovering, enabling, and articulating learning trajectories of conceptual structures children use for certain kinds of problems, (2) creating conceptual learning supports of various kinds, (3) uncovering and creating models of affective, conceptual, cultural, and social aspects of classroom interactions, and (4) developing pedagogical models of ways in which teachers and peers can support children’s constructions of concepts in a given mathematical domain. In what follows, we briefly overview the success of the Classroom Conceptual Research approach in prior work. We then describe the research model in more detail.

The Classroom Conceptual Research model was developed and used in a 6-year action-research project directed toward designing a conceptually complex and challenging K-3 math curriculum that builds on the individual experiences, interests, and practical math knowledge that diverse children bring to our classrooms. In order to ensure that our work generalizes across socioeconomic boundaries, our collaborative research project is carried out in urban schools of underrepresented minorities, schools in which most students are Latino English-speaking and Latino Spanish-speaking children, as well as in English-speaking upper-middle-class schools. We have higher grade-level expectations than in most present U.S. curricula, and have had success across the populations studied. In our formal assessments, we used a range of whole-class and interview items assessing single-digit and multi-digit numerical, word problem, and place-value competence (Fuson, Smith, & Lo Cicero, 1997; Fuson, 1996; Fuson, 1998). The items were taken from other studies to provide comparison data. Highlights of outcomes include:

- Though over 90% of our urban children meet federal guidelines for the free-lunch program, they considerably outperformed heterogeneous and middle-class samples of U.S. children who received traditional mathematics instruction. On many items, they outperformed children from Taiwan, U.S. children using the reform curriculum Everyday Mathematics, and, on some tasks, equaled or exceeded the performance of Japanese children.

- On standardized tests, 90% of the first- and second-grade urban children were at grade-level on computation and 65% on word problem solving. Class means on standardized overall math scores were above grade level, some children were 3 years above grade level, and no child was more than one year below grade level.
- No child used a unitary strategy in multi-digit subtraction problems in contrast to children using the reform curriculum Everyday Mathematics, where 45% of average- and low-achieving children still used unitary methods for subtraction (Drueck, 1996) or did not have effective multi-digit addition or subtraction methods (Murphy, 1997).
- Results for suburban children were even stronger, though on many items the typical urban-suburban gap was decreased.

The activities of our project are similar to the developmental research process described by Gravemeijer (1994) that was used in developing the Realistic Mathematics instructional materials in the Netherlands. That effort began with a substantial theoretical base of the work of Freudenthal and participants in the Freudanthal Institute. It then contributed to more articulated and detailed theoretical perspectives in several areas as well as to the initial and the newer instructional materials. These instructional materials have had remarkable commercial success in the Netherlands, holding a considerable amount of the market. The long period of development, and the intertwining of the theoretical model-building and design of instructional materials, produced a coherent product whose pedagogy, domain analyses, and developmental progressions in children's thinking could be described for and used by teachers and by other researchers. In describing our research model we have not borrowed the Dutch term "developmental research" because, in this country, that term implies "research about children's development."

Our Vygotskiian Model of Classroom Conceptual Research

Our Vygotskiian Model of Classroom Conceptual Research is shown as Figure 1. Although our purpose here is to present a generally applicable model, we will mix high-level description with details of our own implementation in order to provide a more grounded account, and to give a feel for the scope and depth of our approach. The model shown in Figure 1 provides an overview of how we concurrently integrate the design of instructional materials, enactment in the classroom (empirical action research), and the development of conceptual models. Our project efforts in these areas are interwoven in continuing cycles of mutually adapting reflection and revision of the model building, instructional materials design, and classroom implementation work.

In the first of these three areas, our development of models focuses on four major activities. First, we undertake domain analyses of real-world situations that can help children build meanings for and uses of mathematical concepts. Second, we create Full Quantity Conceptual Support Nets of learning supports for particular concepts (Fuson & Smith, 1997). These use physical quantity referents (e.g., penny strips of ten pennies on one side and one dime on the other), drawn quantity referents (e.g., ten-sticks and one dots), meaningful language (e.g., 3/4 said as “out of 4 parts, take 3”), and meaningful math notation connected to ordinary (5 dimes 3 pennies) and to math meanings (5 tens 3 ones). Third, we articulate our pedagogical approach in a model of an Equity Pedagogy (Fuson, De La Cruz, et al., 1997). This model, which builds on prior Vygotskiian work (Fuson, Lo Cicero, et al., 1997), outlines ladders of support to help students build on their initial personal meanings and experiences to create advanced and ambitious mathematical concepts, notations, and methods. Finally, the fourth model-building activity is to specify Learning Trajectories for students and for teachers that describe developmental progressions through which learners advance. Our emphasis in these models on inter-psychological phenomena (Equity Pedagogy) and semiotic tools (Full Quantity Conceptual Support Nets, Domain Analyses of Real-World Situations) reflects our Vygotskiian perspective.

Moving to the second part of our three-part model, the instructional materials design work focuses primarily on four aspects of classroom learning, as well as on the design of home learning-support materials. We identify problem situations that both occur frequently in the real world and are mathematically clear and generative (e.g., using money calculations with dimes and pennies to extend understanding of place-value concepts). We design worksheet-based activities that facilitate children’s approach to a learning activity. We consider features of classroom discourse (questions, language) that will support understanding and clear communication in a co-constructing environment. We create participant structures with attention to which students might be marginalized by each structure. All of these features are subject to modification in action in the classroom.

The empirical action-research work in the classroom begins from the initial instructional materials, with enactment in the classroom informed by the theoretical models. These models suggest adaptations to unfolding student thinking that extend and modify the initial teaching plan. More generally, the mutual adaptations that occur among enactment, the developing models, and the design of instructional materials operate at multiple time-scales: repeatedly during the design work, several times while teaching a lesson, daily in revising tomorrow’s lesson, several times yearly as new teachers try the newly designed unit, and over years as full conceptual support models and full developmental learning trajectories of student thinking are developed and adapted. These many different kinds of feedback loops, and the sustained prolonged efforts, facilitate the development of coherent and powerful theoretical models and teaching-learning units and curricula based on these models.

It is worth emphasizing that, at its highest level, enacting this research model can be seen largely as a project in orchestrating a complex process of collaboration. What we propose is a broad social design that includes not only what happens in the classroom, but also a set of interactions that includes the design team, the school, and the classroom. Our interdisciplinary team includes people with strengths in teaching, mathematics education, developmental psychology, and linguistics. Individuals lead design efforts in a particular area and grade level, with repeated consultation from two to five other people. A unit is sometimes taught by a staff teacher-researcher, often in active collaboration with a classroom teacher. One or more team members may be present at any of the teaching efforts to gather empirical data on classroom activity. In addition, interview data are gathered from children and teachers, frequently during initial development and summatively for more final versions of units.

Particular team members also assume intellectual and management leadership roles in articulating and directing the theoretical model-building and writing. These then are adapted to the thinking of team members in successive reflective discussion cycles. This collaborative research method stimulates a continuing flow of good ideas while enacting our units in the classroom (a productive interaction of teacher, researchers, and students), during project meetings, and in individual work (through voices and perspectives of our fellow collaborators).

Conclusion

In conclusion, we have described a model for research that we call “Classroom Conceptual Research.” We have introduced and named this model in order to initiate a dialogue in the field about new research methods focused on conceptual teaching and learning. We believe that the adoption of this model can enhance the quality of research-based curricular reform efforts, as well as the usefulness of the research on which it is based.

References

Brown, A. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Science, 2,* 141-178.

Drueck, J. V. (1996, April). *Progression of multi-digit addition and subtraction methods in high-, average-, and low-math-achieving second graders in a reform curriculum.* Paper presented at the Annual Meeting of the American Educational Research Association, New York, NY.

Fuson, K. C. (1996, April). *Latino children's construction of arithmetic understanding in urban classrooms that support thinking*. Paper presented at the Annual Meeting of the American Educational Research Association, New York, NY.

Fuson, K. C. (1998). [*Performance of suburban CMW children*] Unpublished data.

Fuson, K. C., De La Cruz, Y., Smith, S. B., Lo Cicero, A., Hudson, K., Ron, P., & Steeby, R. S. (1997). *Toward a Mathematics Equity Pedagogy: Creating ladders to understanding and skill for children and teachers*. Manuscript in final preparation.

Fuson, K. C., Lo Cicero, A. M., Ron, P., De La Cruz, Y., & Zecker, L. (1997). *El Mercado: A fruitful narrative for the development of mathematical thinking in a Latino first- and second-grade classroom*. Manuscript under revision.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738-760.

Fuson, K. C., & Smith, S. T. (1997). *Pedagogical, mathematical, and real-world conceptual-support nets: A model for building children's mathematical domain knowledge*. Manuscript under review.

Fuson, K. C., Stigler, J., Bartsch, K. (1988). Grade placement of addition and subtraction topics in China, Japan, the Soviet Union, Taiwan, and the United States. *Journal for Research in Mathematics Education, 19,* 449-458.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. *Journal for Research in Mathematics Education, 25,* 443-471.

Leinhardt, G., Putnam, R. T., Hattrup, R. A (Eds.) (1992). *The analysis of arithemetic for mathematics teaching*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E. Swafford, J. O., Travers, K. T., & Cooney, T. J. (1989). *The underachieving curriculum: Assessing U. S. school mathematics from an international perspective*. Champaign, IL: Stipes Publishing Company.

Murphy, L. (1997). *Learning and affective issues among higher- and lower-achieving third-grade students in math reform classrooms: Perspectives of children, parents, and teachers*. Northwestern University, unpublished Ph.D. dissertation.

Peak, L. (1996). *Pursuing excellence: A study of the U.S. eighth-grade mathematics and science teaching, learning, curriculum, and achievement in an international context*. Washington, D.C.: National Center for Educational Statistics.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,* 114-145.