Supporting the CCSS NF Learning Progression and Avoiding Errors Known in Research by Using the Mathematical Practices

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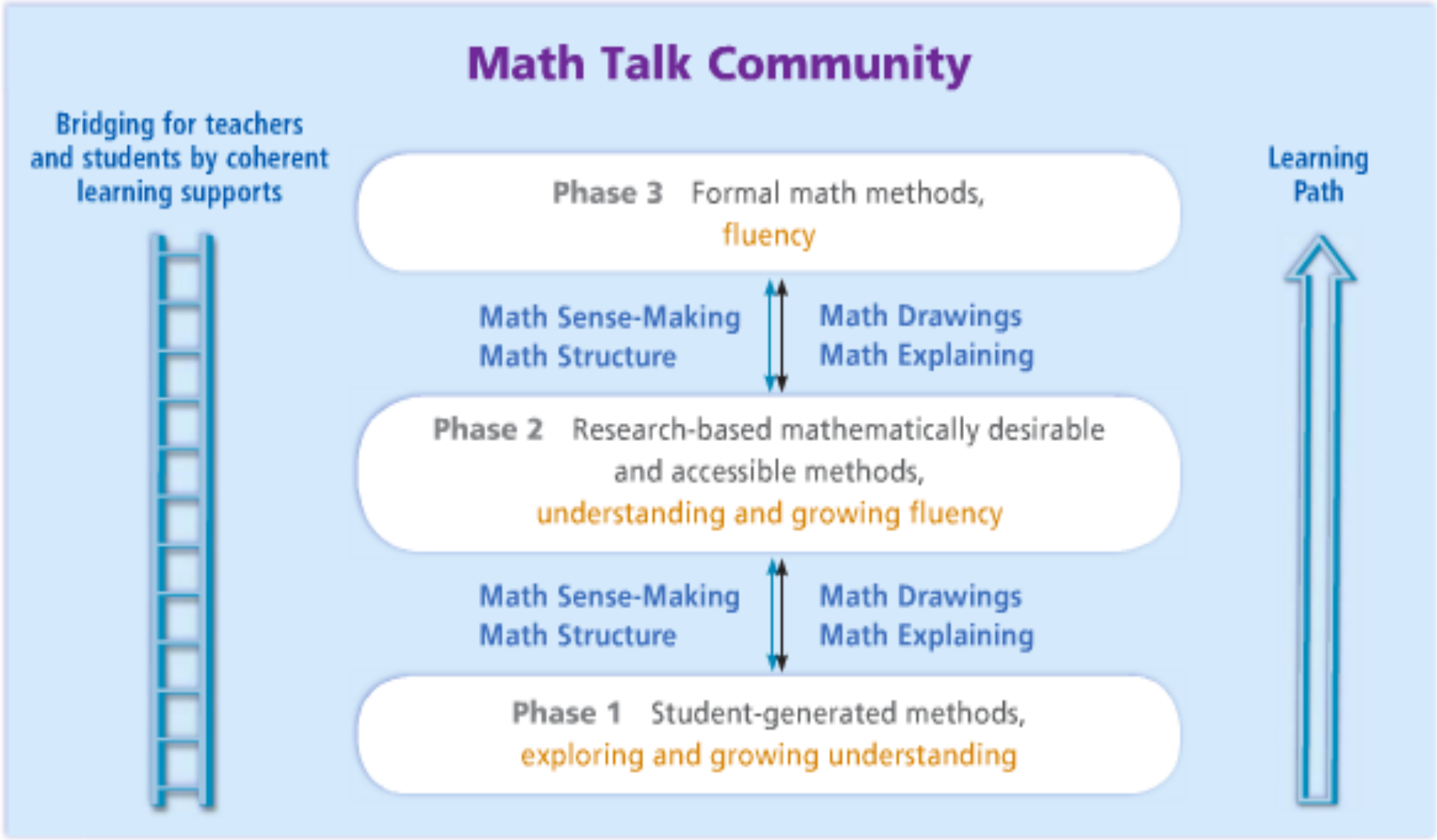
For more details about fraction errors and how to avoid them, see the Teaching Progression on Number Fraction at karenfuson.com. Teaching Progressions on other math topics are also at that website.


Math Talk Community
Bridging for teachers and students by coherent learning supports
Phase 3 Formal math methods, fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 2 Research-based mathematically desirable and accessible methods, understanding and growing fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 1 Student-generated methods, exploring and growing understanding
Learning Path

Common Core Mathematical Practices Used in a Math Talk Community

Math Sense-Making: Make sense and use appropriate precision 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings: Model and use tools** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure: See structure and generalize 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining: Reason, explain, and question** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.

Figure 2

The Math Practices in action

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

Can I do some part of this better tomorrow?

K	1	2	3	4	5
NF Number and Operations–Fractions: 3 to 5					
		General:	Unit fractions	Compare any frs find eq frs	+- any fr eq frs fr x WN, fr [G6 fr ÷ fr]
		Special cases:	Compare like n or d	+- like denom WN x fr	n ÷ d = fr WN÷fr fr÷WN

Highlights of NF Standards by Grade

Grade 3

Fractions as numbers composed of unit fractions 1/d

Find simple equivalent fractions and compare for like numerators or like denominators

Grade 4

General fraction equivalence and general compare fractions

Add and subtract like denominators

Whole number times a fraction

Grade 5

General addition and subtraction (find equivalent fractions if needed)

General multiplication of fractions; multiplication as scaling (resizing)

Division of whole numbers results in a fraction; division with unit fractions

Grade 6

General division of fractions

[**Grade 7** Fluency with all fraction operations]

G2 Related Geometry Standard 2.G.3

Grade 2: Reason with shapes and their attributes.

3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths.

Recognize that equal shares of identical wholes need not have the same shape.

G3 NF Standards: 3.NF.1, 2, 3

Grade 3: Develop understanding of fractions as numbers.

1. Understand a unit fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3. Explain equivalence of fractions in special cases [small numbers], and compare fractions by reasoning about their size [same denominator or same numerator].

G3 Difficulty Seeing the Parts Within the Total

Most fraction drawings are intended to be seen as the total with the part embedded within it. Seeing the embedded part is difficult.

Error: See the first part and the second part and write those numbers:

3/2 instead of 3/5

The *Children's Math Worlds* solution:

A. Initially make fraction drawings in two steps:

1. Divide the whole into unit fractions.
2. Circle/shade the number of unit fractions you want.

B. Write unit fractions numerically with each part to see a numerical unit fraction and to emphasize the meaning of the denominator as the number of equal parts of the whole.

First see all the unit fractions visually and in fraction notation.

Then shade or circle the unit fractions you need.

▶ Build Fractions from Unit Fractions

Write the unit fractions for each whole. Next, shade the correct number of parts. Then show each shaded fraction as a sum of unit fractions.

9.  Shade 2 parts.

Divide the whole into 5 equal parts.

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$

10.  Shade 2 parts.

Divide the whole into 3 equal parts.

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

11.  Shade 5 parts.

Divide the whole into 7 equal parts.

$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$ $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{5}{7}$

12.  Shade 7 parts.

Divide the whole into 8 equal parts.

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{7}{8}$

13.  Shade 3 parts.

Divide the whole into 6 equal parts.

$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

14.  Shade 8 parts.

Divide the whole into 8 equal parts.

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{8}{8}$

G3 Number Line Standard 3.NF.2

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

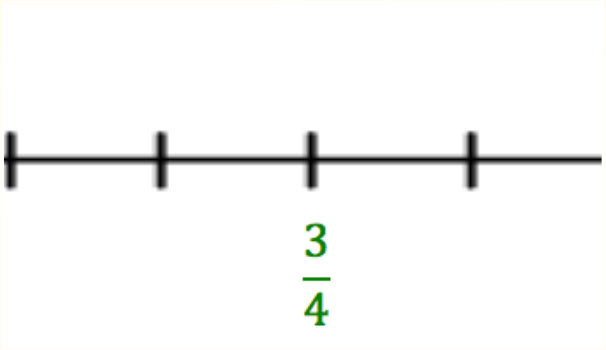
a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

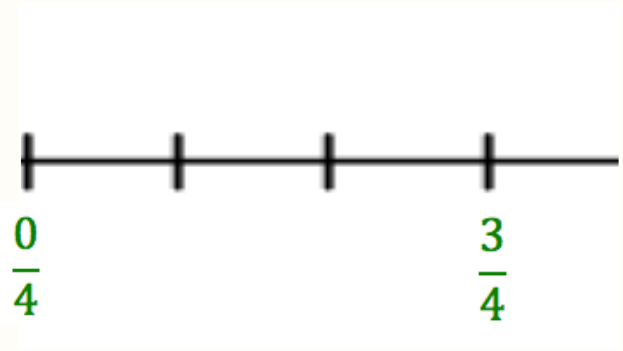
Not Enough Unit Lengths

Errors when drawing or using number lines

Error: Not enough unit lengths-
student counts marks rather than lengths.



Correct: Count 3 unit lengths



G3 Too Many Unit Lengths

Makes marks instead of lengths

Error: Makes 4 marks instead of 4 lengths.

Given:  , do 

$\frac{3}{4}$

Correct: Count 3 new marks to create 4 unit lengths.

Given:  , do 

$\frac{0}{4}$ $\frac{3}{4}$

G3 See Lengths on Number Lines

You can help students see the lengths on number lines by

a) relating unit lengths on number lines to lengths more easily seen in fraction bars,

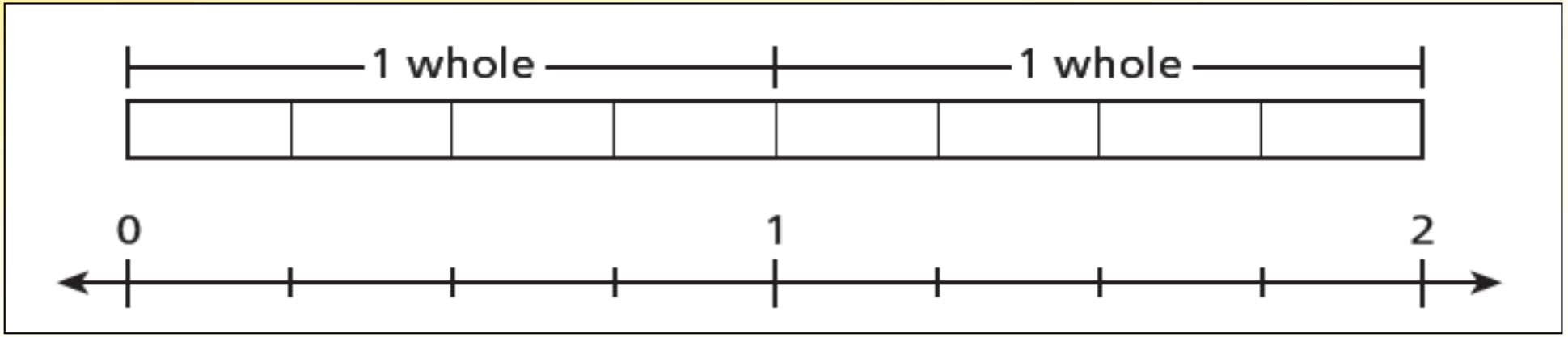
b) encircling the lengths on number lines,

c) having students run a finger along each length to count the unit fractions on the number line.

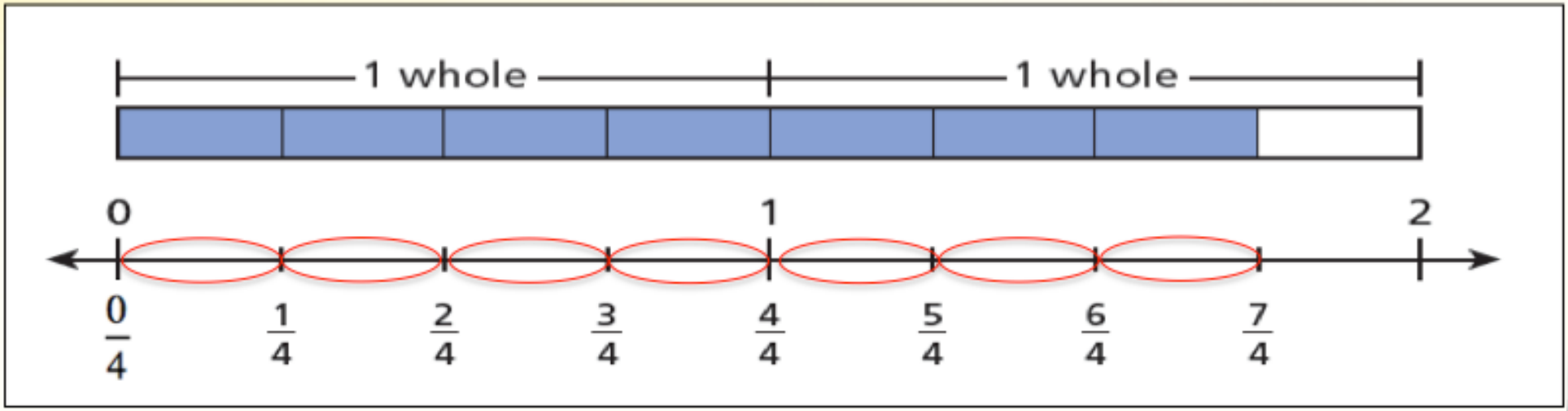
It is vital for the teacher to emphasize the lengths in all work with number lines.

Seeing the Fraction Lengths

Step 1: Make the 4 unit fractions ¼ within each 1 whole.



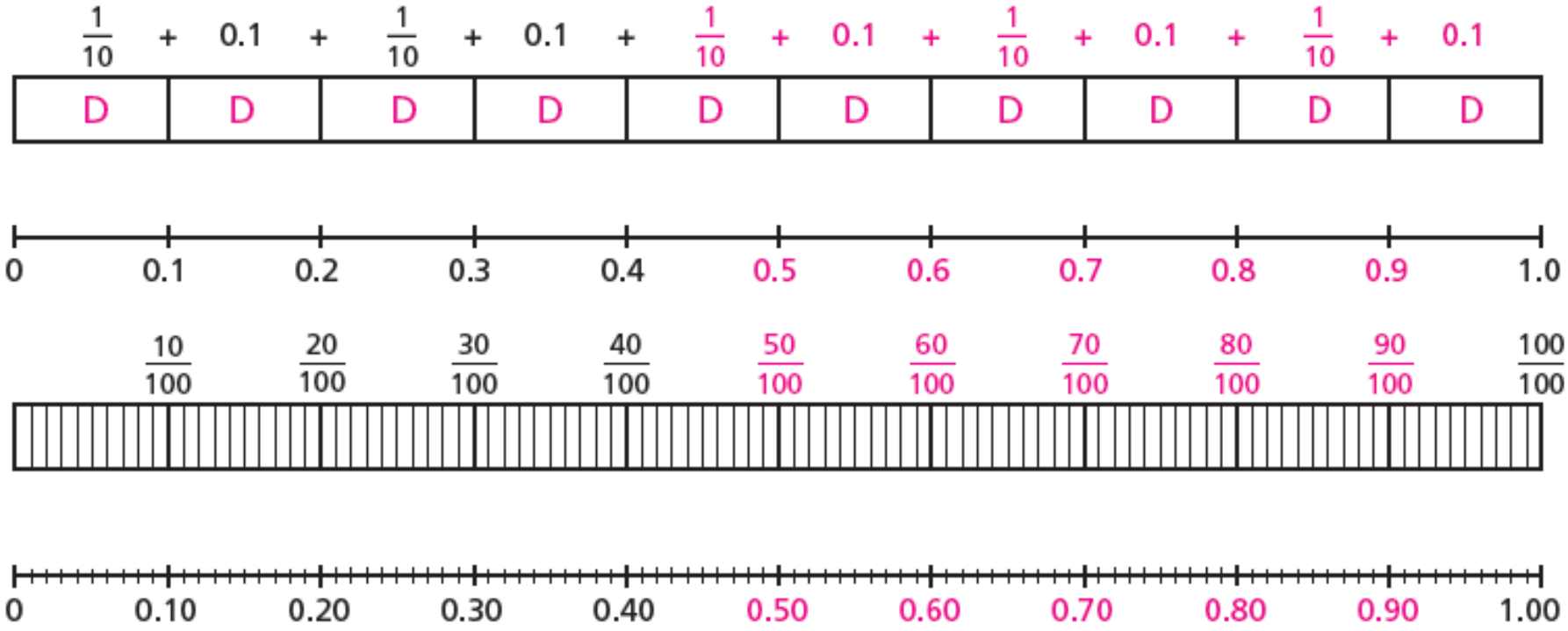
Step 2: Shade or encircle 7 unit fractions and label the number line.



The Number Line Numbers Tell the Number of Units So Far

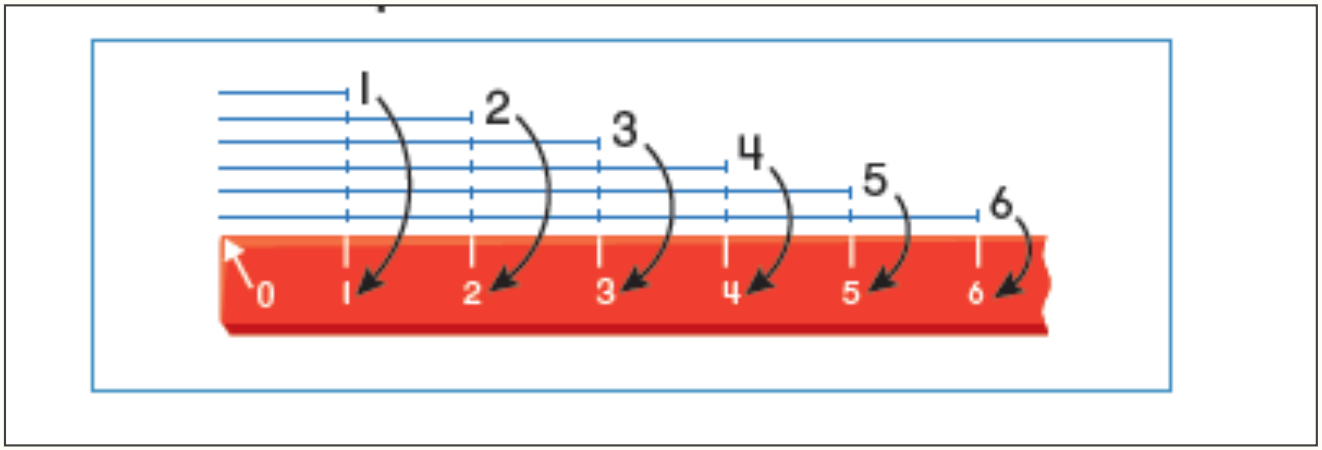
▶ Understand Tenths and Hundredths

Answer the questions about the bars and number lines below.



Grade 2: Students have to see the length units on a ruler and number line diagram and on bar graph and line plot scales

Students need to see how length units are composed to make a ruler



G3 and G4 Comparing Fractions

3.NF.3d. Compare two fractions with the same numerator or the same denominator by reasoning about their size.

4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.

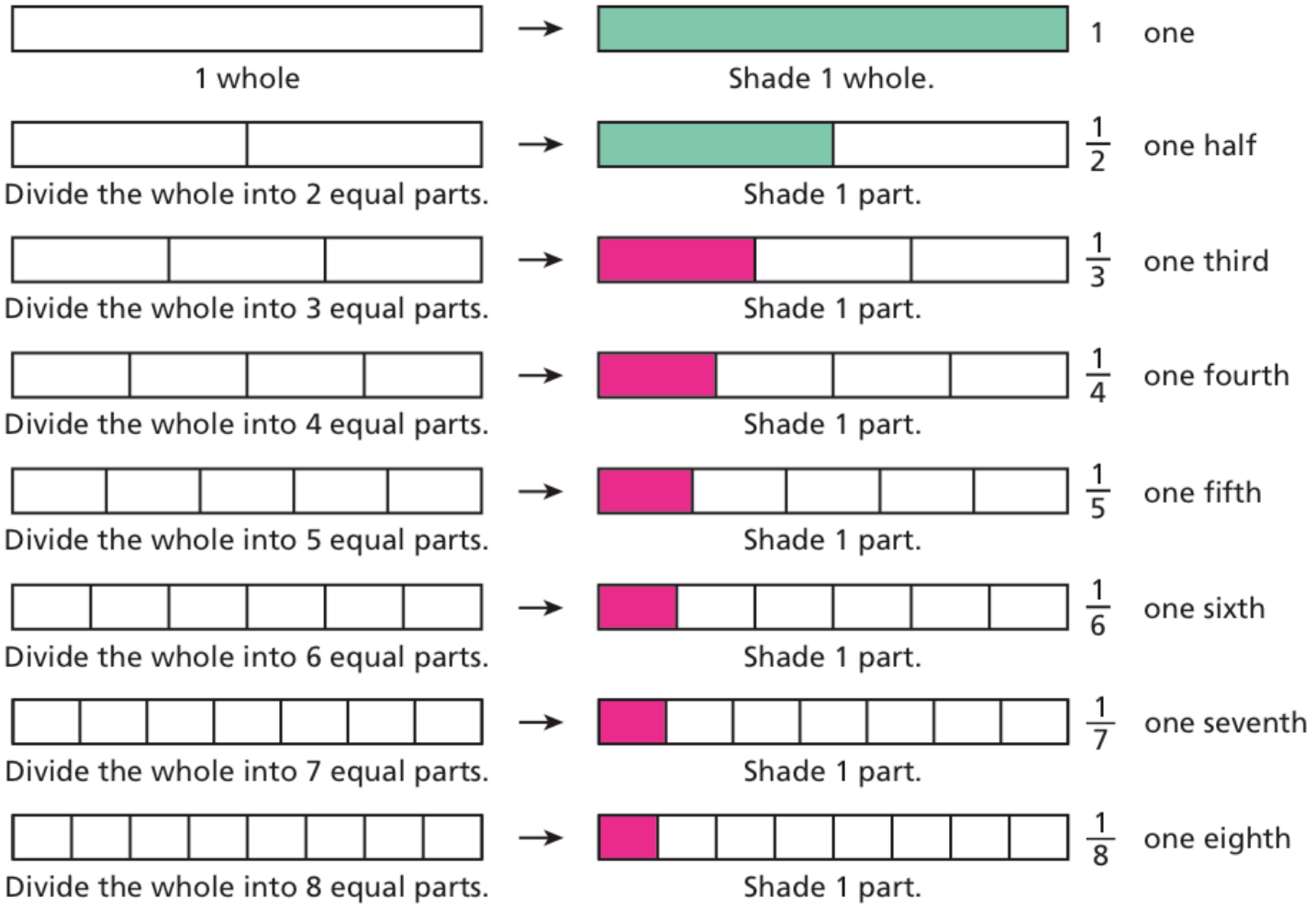
Grade 3 and Grade 4:

Recognize that comparisons are valid only when the two fractions refer to the same whole.

Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

G3 Compare Unit Fractions With Same Numerators

A **unit fraction** has a numerator of 1. Shade the rest of the fraction bars at the right below to represent unit fractions. What patterns do you see?



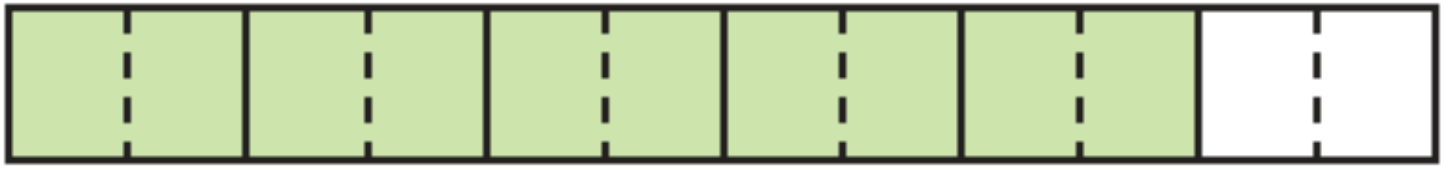
G4 Make Related Opposite Changes for Equivalent Fractions

Equivalent fractions are made by **dividing physically**
but **multiplying numerically**:

Physically divide (equal-fracture) each unit fraction **in the visual model** to get **more** but smaller unit fractions in the numerator and the denominator.
Numerically multiply the top and bottom of the written fraction to get **more but smaller** unit fractions.

You see the **numbers** in the **written fraction getting bigger,**
but you **do not see** the unit fractions getting smaller except in visual models.
You have to remember that a larger denominator is a smaller unit fraction.

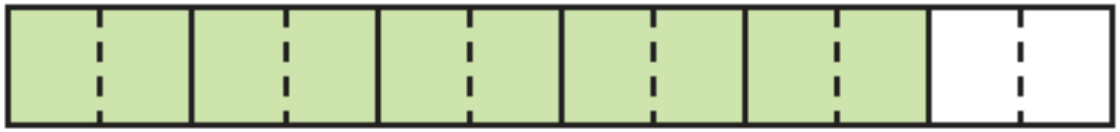
$$\frac{5}{6} = \frac{5 \bullet 2}{6 \bullet 2} = \frac{10}{12}$$



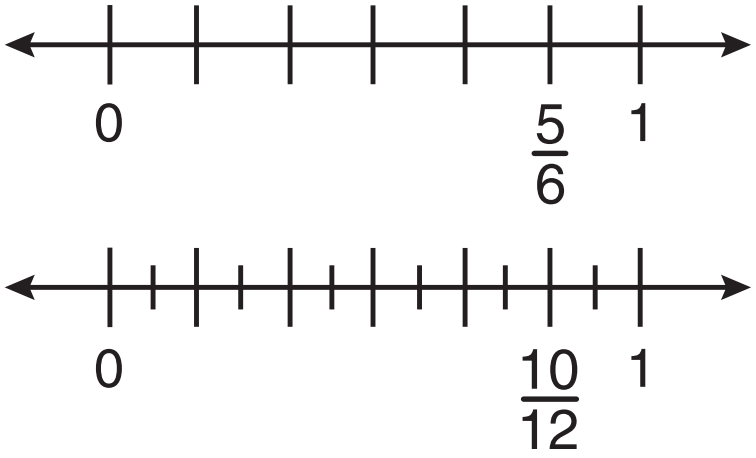
G4 More Visual Models for Equivalent Fractions

a. more but smaller parts

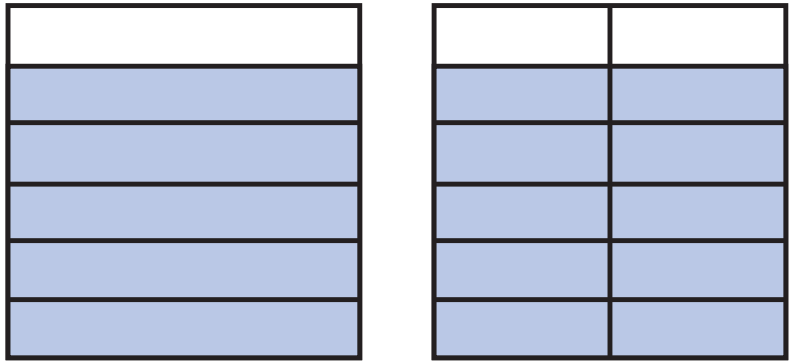
$$\frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$$



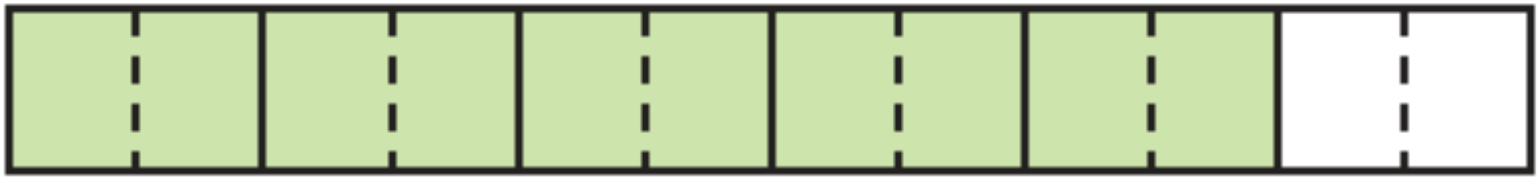
Number Line Model



Area Model



G4 Many Equivalent Fractions



$$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \frac{35}{42} = \frac{40}{48} = \frac{45}{56}$$

$\frac{5}{6}$: $\times 2$ $\times 3$ $\times 4$ $\times 5$ $\times 6$ $\times 7$ $\times 8$ $\times 9$

$\frac{10}{12}$: $\times 2$

$\frac{15}{18}$: $\times 3$

$\frac{20}{24}$: $\times 4$

$\frac{25}{30}$: $\times 5$

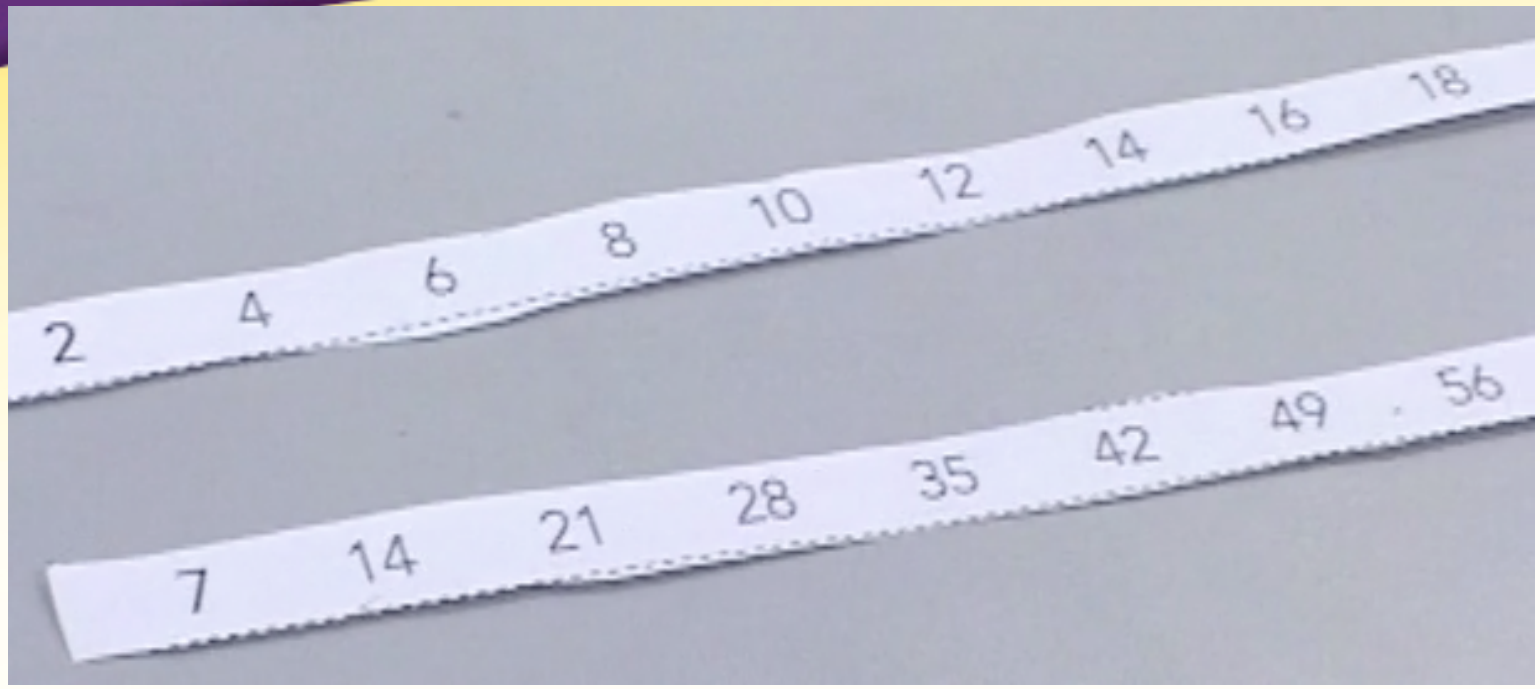
$\frac{30}{36}$: $\times 6$

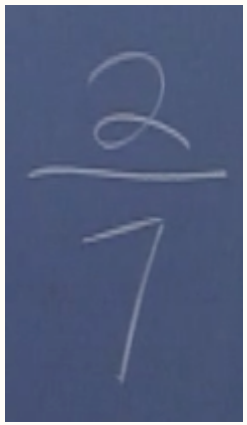
$\frac{35}{42}$: $\times 7$

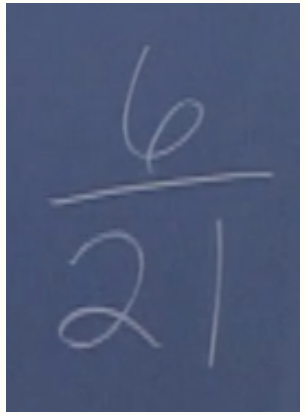
$\frac{40}{48}$: $\times 8$

$\frac{45}{56}$: $\times 9$

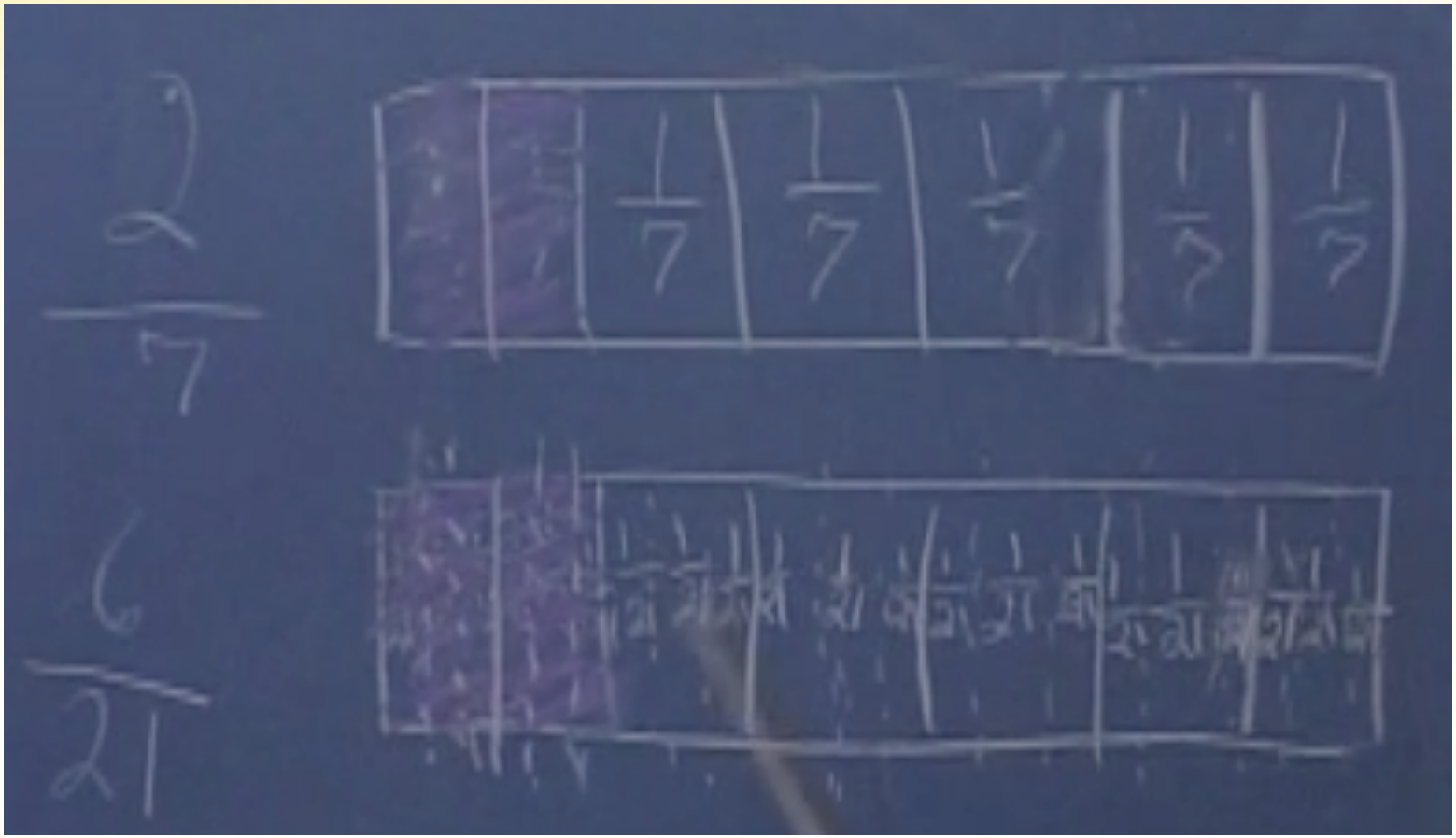
Fraction Strips for 2/7: Multiplication Table Rows



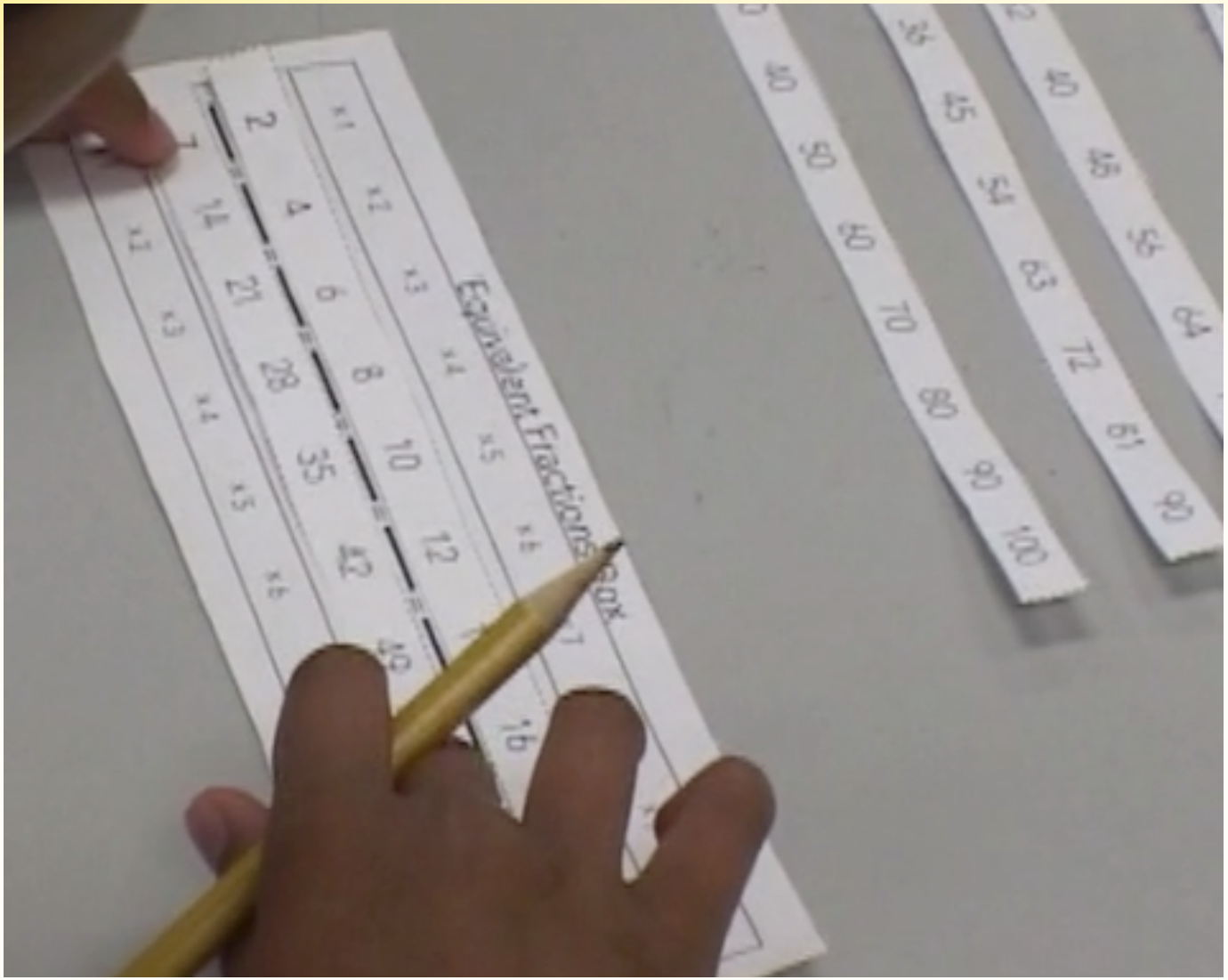




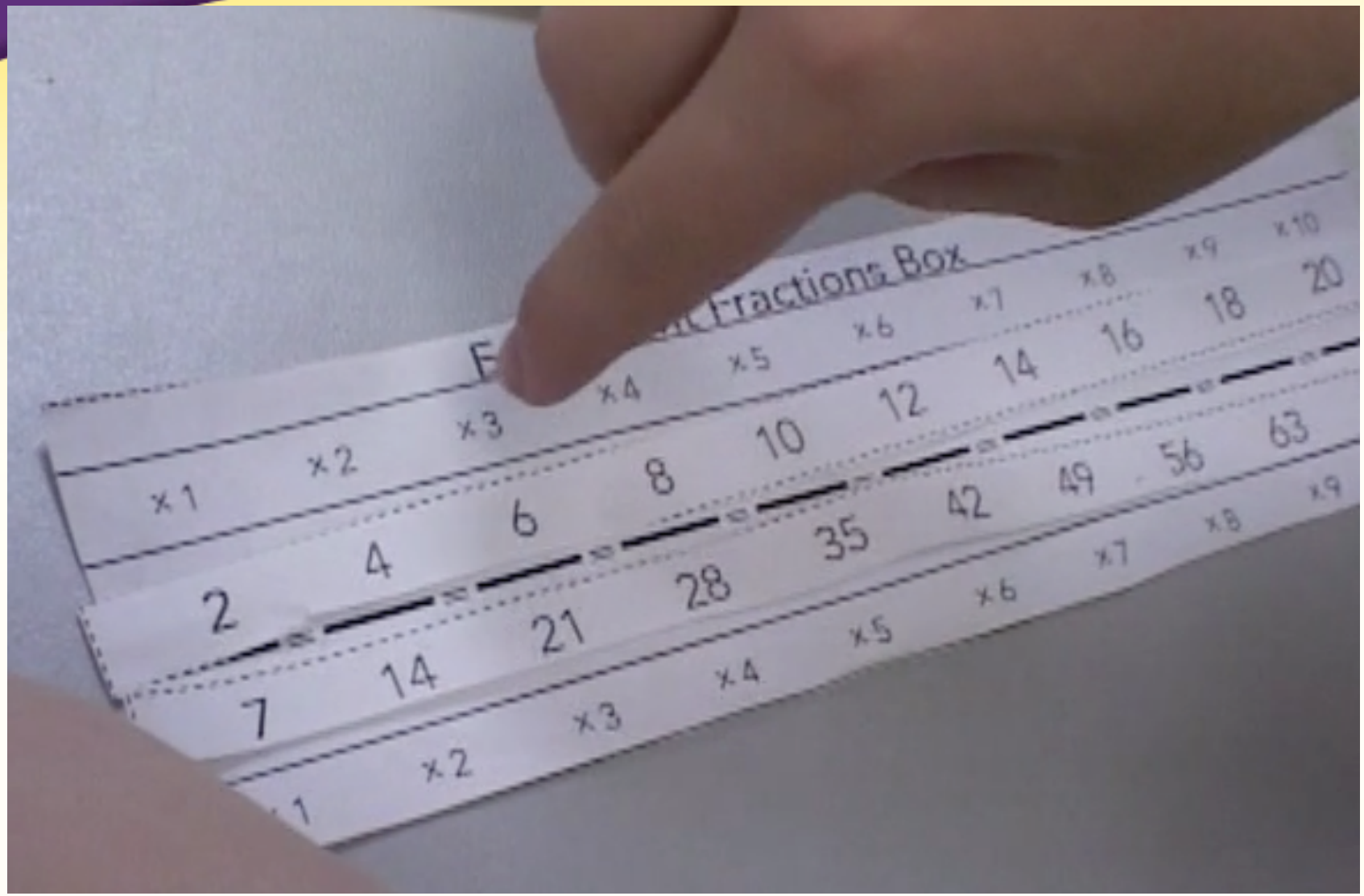
Drawing to show why the fractions are equivalent



Equivalent fractions box



Equivalent fractions box



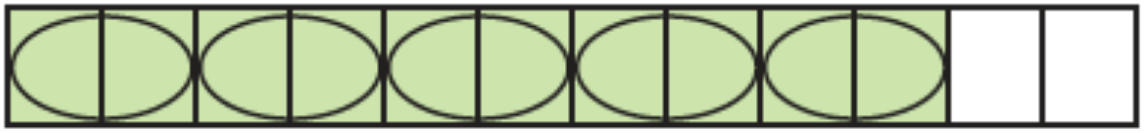
G4 Make Related Opposite Changes for Simplifying to an Equivalent Fraction

Equivalent fractions are made by **grouping physically**
but **dividing numerically**:

You see the **numbers** in the **written fraction getting smaller,**
but you **do not see** the unit fractions getting bigger except in visual models.
You have to remember that a smaller denominator is a larger unit fraction.

b. fewer but larger parts

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$



Common error when adding fractions is?

$$\frac{2}{7}+\frac{3}{7}=$$

G4 Adding and Subtracting Common Error

Common error:
Add fractions by adding the tops and adding the bottoms

$$\frac{2}{7}+\frac{3}{7}=\frac{5}{14}$$

***Children's Math Worlds* Solutions**

1. Offer many experiences writing sums of unit fractions where the denominator stays the same number and include drawings.

2. Find partners of a whole to practice this concept and use drawings.

1. Show addition and subtraction models. Write the sum or difference above the unit fraction as a middle step.

G4 Write Sums of Unit Fractions

▶ Sums of Unit Fractions

Shade the fraction bar to show each fraction. Then write the fraction as a sum of unit fractions and as a product of a whole number and a unit fraction. The first one is done for you.

9. $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$

10. $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 3 \times \frac{1}{8}$

11. $\frac{5}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 5 \times \frac{1}{5}$

12. $\frac{2}{12} = \frac{1}{12} + \frac{1}{12} = 2 \times \frac{1}{12}$

13. $\frac{4}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 4 \times \frac{1}{7}$

14. $\frac{7}{9} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = 7 \times \frac{1}{9}$

G4 Decompose Fractions

▶ Fifths that Add to One

Every afternoon, student volunteers help the school librarian put returned books back on the shelves. The librarian puts the books in equal piles on a cart.

One day, Jean and Maria found 5 equal piles on the return cart. They knew there were different ways they could share the job of reshelving the books. They drew fraction bars to help them find all the possibilities.

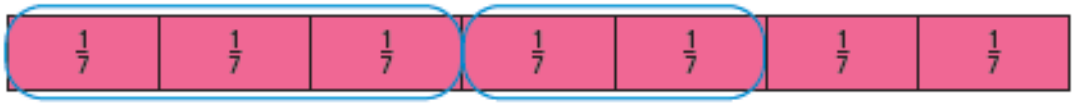
1. On each fifths bar, circle two groups of fifths to show one way Jean and Maria could share the work. (Each bar should show a different possibility.) Then complete the equation next to each bar to show their shares.
   Possible answers are shown.

1 whole = all of the books	1 whole	Jean's share	Maria's share
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$	$\frac{5}{5}$ =	$\frac{1}{5}$ +	$\frac{4}{5}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$	$\frac{5}{5}$ =	$\frac{2}{5}$ +	$\frac{3}{5}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$	$\frac{5}{5}$ =	$\frac{3}{5}$ +	$\frac{2}{5}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$	$\frac{5}{5}$ =	$\frac{4}{5}$ +	$\frac{1}{5}$

G4 Add and Subtract Fractions

▶ Add Fractions

The circled parts of this fraction bar show an addition problem.



1. Write the numerators that will complete the addition equation.

$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$ 3 sevenths + 2 sevenths = 5 sevenths

Solve each problem. Write the correct numerator to complete each equation.

2. $\frac{3}{9}+\frac{4}{9}=\frac{3+4}{9}=\frac{7}{9}$

3. $\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{4}{5}$

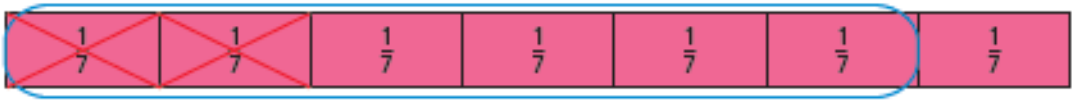
4. $\frac{2}{8}+\frac{5}{8}=\frac{2+5}{8}=\frac{7}{8}$

5. What happens to the numerators in each problem?
   The numerators are added together.

6. What happens to the denominators in each problem?
   The denominators stay the same.

▶ Subtract Fractions

The circled and crossed-out parts of this fraction bar show a subtraction problem.



7. Write the numerators that will complete the subtraction equation.

$\frac{6}{7}-\frac{2}{7}=\frac{6-2}{7}=\frac{4}{7}$ 6 sevenths - 2 sevenths = 5 sevenths

G4 and G5 Cases for Finding a Common Denominator

Strategies for finding the common denominator.

Analyze pairs of fractions into three classes:

A. One denominator divides the other denominator:

$$\frac{3}{5} ? \frac{7}{10}$$

Use the larger denominator as the common denominator.

I'll use 10, multiply by 2 to make 5 be 10:

$$\frac{3\times2}{5\times2} = \frac{6}{10}$$

$$\frac{6}{10} < \frac{7}{10}$$

B. No number except 1 divides both denominators (they are relatively prime).
Use the product of the denominators as the common denominator. Multiply each fraction top and bottom by the other denominator:

$$\frac{2}{3} ? \frac{4}{5} \qquad \frac{2\times5}{3\times5} ? \frac{4\times3}{5\times3}$$

$$\frac{10}{15} < \frac{12}{15}$$

C. Some number divides both denominators.
I can use the product of the denominators as the common denominator, but first I'll think of a smaller number that is a multiple of both.

$$\frac{2}{4} ? \frac{5}{6}$$

I'll use 12: $\frac{2\times3}{4\times3} ? \frac{5\times2}{6\times2}$

so

$$\frac{6}{12} < \frac{10}{12}$$

G5 5.NF.4

Any Fraction Times Any Fraction

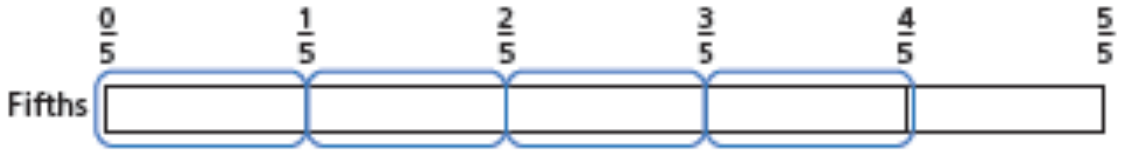
Length Model

Using Unit Fractions

Use Bar Models to Multiply Fractions

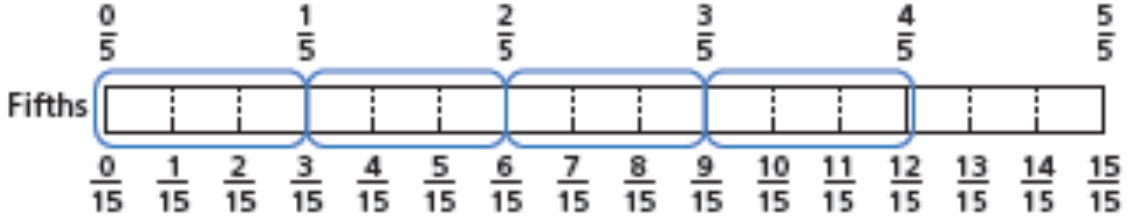
Miguel explains how to use fraction bars to find $\frac{2}{3} \cdot \frac{4}{5}$.

First, I circle 4 fifths on the fifths fraction bar.



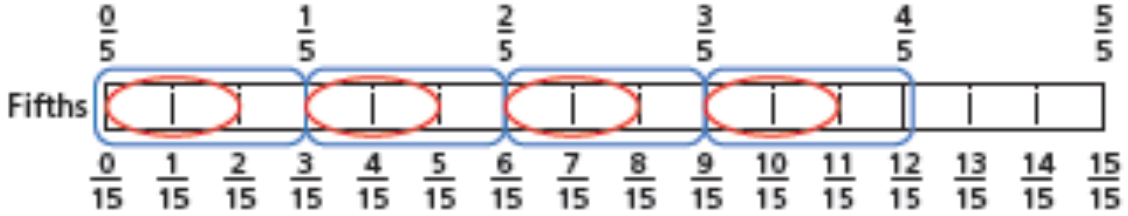
$\cdot \frac{4}{5}$

To find $\frac{2}{3}$ of $\frac{4}{5}$, I can circle $\frac{2}{3}$ of each fifth. But, first I have to split each fifth into three parts. After I do this, the bar is divided into fifteenths.



$\frac{\ }{3} \cdot \frac{4}{5} = \frac{\ }{15}$

Now, it is easy to circle 2 thirds of each of the 4 fifths.



$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$

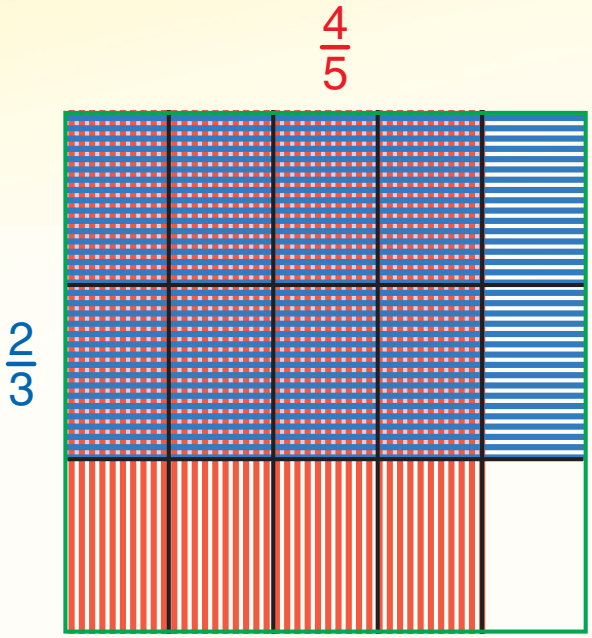
Each group I circled has 2 fifteenths, so I circled 4 groups of 2 fifteenths. That's 8 fifteenths in all. So, $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$.

G5 5.NF.4

Any Fraction Times Any Fraction

Area Model

Area Model



$\frac{2}{3}$ times $\frac{4}{5}$

$\frac{2}{3}$ of each of the $\frac{4}{5}$

$\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$

G5 Add, Subtract, Compare, and Multiply

▶ Add, Subtract, Compare, and Multiply

The fraction box to the right shows the same two fractions compared, added, subtracted, and multiplied.

	$\frac{1}{3}$ and $\frac{1}{6}$
>	$\frac{1}{3} > \frac{1}{6}$ or $\frac{2}{6} > \frac{1}{6}$
+	$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$
−	$\frac{1}{3} - \frac{1}{6} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
·	$\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

Complete the fraction box.

1.

	$\frac{2}{5}$ and $\frac{7}{10}$
>	$\frac{7}{10} > \frac{2}{5}$ or $\frac{7}{10} > \frac{4}{10}$
+	$\frac{7}{10} + \frac{2}{5} = \frac{7}{10} + \frac{4}{10} = \frac{11}{10} = 1\frac{1}{10}$
−	$\frac{7}{10} - \frac{2}{5} = \frac{7}{10} - \frac{4}{10} = \frac{3}{10}$
·	$\frac{7}{10} \cdot \frac{2}{5} = \frac{14}{50} = \frac{7}{25}$

2.

	$\frac{3}{5}$ and $\frac{4}{7}$
>	$\frac{3}{5} > \frac{4}{7}$ or $\frac{21}{35} > \frac{20}{35}$
+	$\frac{3}{5} + \frac{4}{7} = \frac{21}{35} + \frac{20}{35} = \frac{41}{35} = 1\frac{6}{35}$
−	$\frac{3}{5} - \frac{4}{7} = \frac{21}{35} - \frac{20}{35} = \frac{1}{35}$
·	$\frac{3}{5} \cdot \frac{4}{7} = \frac{12}{35}$

G5 Compare the Product to a Factor 5.NF.5

▶ Generalize

Complete the statement with *greater than*, *less than*, or *equal to*.

10. Multiplying any number, *n*, by a factor less than 1 gives a product ___less than___ *n*.

11. Multiplying any number, *n*, by a factor equal to 1 gives a product ___equal to___ *n*.

12. Multiplying any number, *n*, by a factor greater than 1 gives a product ___greater than___ *n*.

Multiplying a fraction by a fraction equal to 1 gives an equivalent fraction. It is the same as multiplying both the numerator and denominator by the same number.

$$\frac{4}{7} = \frac{4}{7} \cdot \frac{3}{3} = \frac{12}{21} \qquad \frac{4}{7} = \frac{4 \cdot 3}{7 \cdot 3} = \frac{12}{21}$$

Multiply the fraction by a factor equal to 1 to create an equivalent fraction. Answers will vary.

13. $\frac{4}{5}$ 14. $\frac{3}{11}$ 15. $\frac{5}{8}$

How would you want to divide fractions?

What would make sense from multiplying fractions?

G6 Seeing Division as Finding the Unknown Factor in an Area Situation

▶ Finding an Unknown Factor

1. On Gino's farm, there is a rectangular wooded area in a cornfield. This wooded section has area $\frac{8}{15}$ square mile and is $\frac{2}{3}$ mile long. How wide is the wooded section?

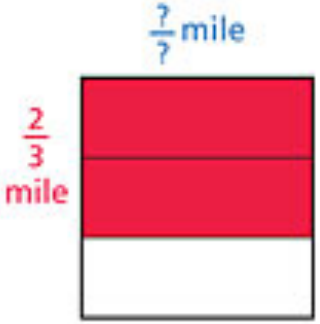
$\frac{4}{5}$ mile

We can write the division $\frac{8}{15} \div \frac{2}{3} = n$.

This is the same as the unknown-factor multiplication $\frac{2}{3} \cdot n = \frac{8}{15}$.

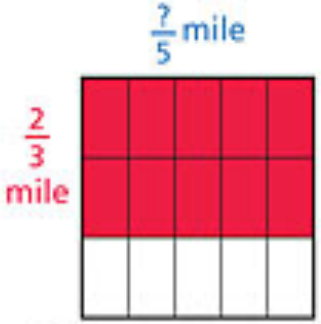
Step 1
Write an equation.

$$\frac{2}{3} \cdot \frac{?}{?} = \frac{8}{15}$$



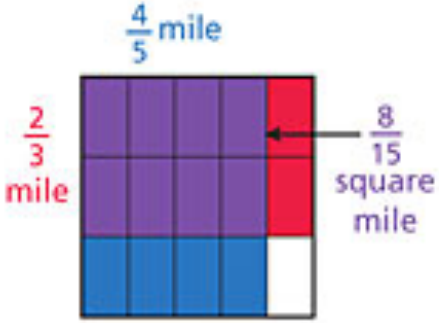
Step 2
Look at the denominators.

$$\frac{2}{3} \cdot \frac{?}{?} = \frac{8}{15}$$
$$3 \cdot 5 = 15$$
$$\frac{?}{?} = \frac{?}{5}$$



Step 3
Look at the numerators.

$$\frac{2}{3} \cdot \frac{?}{5} = \frac{8}{15}$$
$$2 \cdot 4 = 8$$
$$\frac{?}{5} = \frac{4}{5}$$



G6 Seeing Division as Finding the Unknown Factor in an Equal Groups Situation

2. The mugs at a restaurant hold $\frac{2}{3}$ cup of hot chocolate. The restaurant has $\frac{8}{15}$ cup hot chocolate left in its pot. How many servings of $\frac{2}{3}$ cup are in the pot?

$\frac{4}{5}$ serving

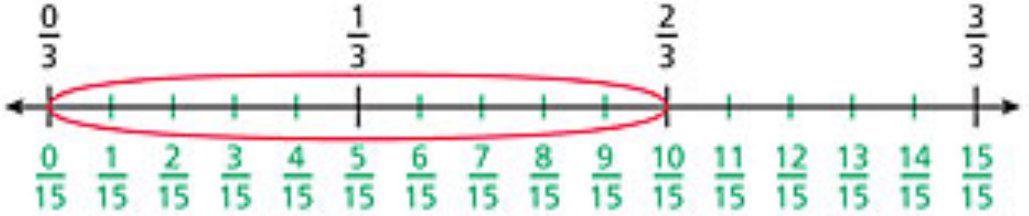
Step 1 Write an equation.

$\frac{?}{?} \cdot \frac{2}{3} = \frac{8}{15}$

Step 2 Look at the denominators.

Divide each $\frac{1}{3}$ into 5 equal parts to make fifteenths.

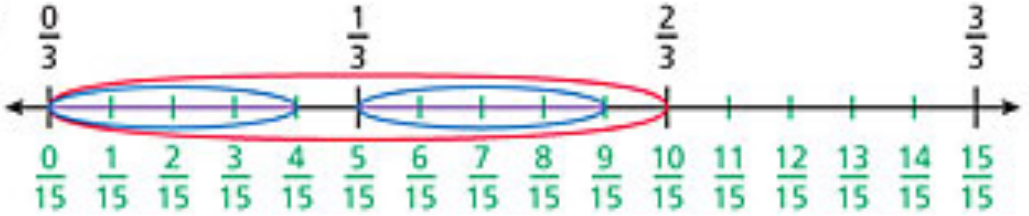
$\frac{?}{5} \cdot \frac{2}{3} = \frac{8}{15}$



Step 3 Look at the numerators.

Take 4 fifteenths from each of the 2 thirds to make $\frac{8}{15}$.

$\frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15}$



G6 Seeing Division as Finding the Unknown Factor Numerically

inverse operations

▶ Relating Multiplication and Division

Find the unknown factor in each equation. Then rewrite the multiplication as a division equation.

Multiplication Equation	Related Division Equation
3. $\frac{2}{3} \cdot \frac{\boxed{4}}{\boxed{5}} = \frac{8}{15}$	$\frac{8}{15} \div \frac{2}{3} = \frac{\boxed{4}}{\boxed{5}}$
4. $\frac{5}{7} \cdot \frac{\boxed{3}}{\boxed{8}} = \frac{15}{56}$	$\frac{15}{56} \div \frac{5}{7} = \frac{\boxed{3}}{\boxed{8}}$
5. $\frac{5}{8} \cdot \frac{\boxed{4}}{\boxed{9}} = \frac{20}{72}$	$\frac{20}{72} \div \frac{5}{8} = \frac{4}{9}$
6. $\frac{3}{4} \cdot \frac{\boxed{5}}{\boxed{9}} = \frac{15}{36}$	$\frac{15}{36} \div \frac{3}{4} = \frac{5}{9}$

Multiplication and division are **inverse operations** for all whole numbers, decimals, and fractions. One operation undoes the other.

$$\frac{2}{5} \cdot \frac{1}{5} \div \frac{1}{5} = \frac{2}{5}$$

G6 Unsimplify to Divide and See That You Have Multiplied by the Reciprocal

unsimplify

▶ Unsimplify to Divide

$\frac{2}{3} \div \frac{5}{7} = ?$

We cannot divide the numerator of $\frac{2}{3}$ by 5 or the denominator by 7.

To be able to divide, we need to unsimplify $\frac{2}{3}$. To **unsimplify** we rewrite it as an equivalent fraction so the numerators and denominators divide evenly.

Step 1 $\frac{2}{3} \div \frac{5}{7} = (\overbrace{\underbrace{\frac{2}{3} \bullet \frac{5}{5} \bullet \frac{7}{7}}_{\frac{2}{3} \text{ unsimplified}}}^{\frac{2}{3} \bullet 1 \bullet 1}) \div \frac{5}{7}$

Step 2 $= \frac{2 \bullet \overbrace{5 \bullet 7}^{5 \div 5 = 1}}{3 \bullet \underbrace{5 \bullet 7}_{7 \div 7 = 1}} \div \frac{5}{7}$

Step 3 $= \frac{2 \bullet 7}{3 \bullet 5}$

Step 4 $= \frac{2}{3} \bullet \frac{7}{5}$

1. How is the number you divide $\frac{2}{3}$ by in the original division problem related to the number you multiply $\frac{2}{3}$ by in the final multiplication problem?

You multiply by $\frac{7}{5}$, the reciprocal of original divisor, $\frac{5}{7}$.

G6 Two Ways to Divide Fractions

1. If you can, divide the top and bottom numbers (the numerator and denominator) of the product by the top and bottom numbers (the numerator and denominator) of the factor.

1. If you can't divide easily, flip the factor and multiply the product by it (multiply the product by the reciprocal of the factor).

Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings
that students can make and use in their own ways
in problem solving and explaining of thinking.

We want classrooms to be **using the mathematical practices**:
Students focus on math sense-making about math structure
using math drawings (visual models) to support math explaining.

Reason about unit fractions.

Supporting the CCSS NF Learning Progression and Avoiding Errors Known in Research by Using the Mathematical Practices

Professor Karen C. Fuson

Northwestern University

For more details about fraction errors and how to avoid them, see the Teaching Progression on Number Fraction at karenfuson.com. Teaching Progressions on other math topics are also at that website.