In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices* (pp. 125-134). Reston, VA: NCTM.

Chapter 11

Describing Levels and Components of a Math-Talk Learning Community

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The transformation to reform mathematics teaching can be a daunting task. It is often unclear to teachers what such a classroom would look like, let alone how they might get there. In particular, the development of a discourse community is seen as a critical step in the implementation of reform, yet teachers often find this transition challenging. For example, research documents that when teachers open up their classroom to students' ideas, it can be difficult to manage the mathematical direction that instruction takes (Chazan, 2000; Sherin, 2002a; Stein, Engle, Smith & Hughes, 2008). This increased difficulty in predicting where a lesson will go may make it more difficult for teachers to anticipate and prepare for their role in instruction (Heaton, 2000; Sherin, 2002b; Smith, 2000).

This chapter addresses the question of how a teacher, along with her students, establishes the sort of classroom community that can enact reform mathematics practices. Specifically, the chapter presents a framework for the development of a *math-talk learning community.* By math-talk learning community, we refer to a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants. A primary goal of such a community is to understand and extend one's own thinking as well as the thinking of others in the classroom. This framework extends prior research on teacher change in the context of reform by describing key components of a math-talk learning community as well as the intermediary levels along which the community develops. The framework is particularly relevant today as the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO], 2010) require teachers to have students discuss their thinking and explain their reasoning.

The chapter is based primarily on an intensive yearlong case study of one teacher in an urban elementary classroom with Latino children (Hufferd-Ackles, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004). The teacher began the year by teaching in a fairly traditional manner. Over the course of the year, however, she had considerable success in implementing mathematics education reform, particularly in the area of whole-class discourse. Many educational reforms bypass classrooms with children from poor or non-English speaking backgrounds (Spillane, 2001) partly because such children are assumed not to be linguistically prepared to participate in reform-based practices. Thus, success in this case is particularly significant, for it supports the notion that urban classrooms with students who are below grade level in mathematics *can* function and learn as a math-talk learning community. All names used in the chapter (teacher and students) are pseudonyms.

This chapter is adapted from K. Hufferd-Ackles, K. C. Fuson, & M. G. Sherin (2004), Describing levels and components of a math-talk learning community, *Journal for Research in Mathematics Education, 35,* 81–116.

Method

Ms. Martinez taught grade 3 at a Catholic school in an urban U.S. city. She was observed and videotaped once or twice a week during the 1997–98 school year as she implemented the mathematics curriculum *Children's Math Worlds* (CMW) (Fuson et al., 1997). CMW is based on extended research into the manner in which children learn and understand mathematical concepts. It contains conceptual supports to help make mathematics personally meaningful to students and through which students can share their ideas with others. In this curriculum, students make mathematical drawings to solve problems and explain their thinking, and they label these math drawings and related equations to link them to the problem situation. CMW is currently published by Houghton Mifflin Harcourt as *Math Expressions*. Following each observation, Ms. Martinez was interviewed concerning her experiences teaching mathematics that day. In addition, three of Ms. Martinez's colleagues were observed and videotaped regularly as they used CMW.

Through a cycle of qualitative analyses of the data, we identified four key components along which a discourse community developed in Ms. Martinez's classroom. For each component, we also identified four levels of growth—Levels 0 through 3. Data collected in the the other three classrooms at the same school as well as additional observations in Ms. Martinez's classroom the following school year were used to confirm the robustness of the findings. Since its development, the math-talk framework that resulted from this study has been used in professional development situations with thousands of preservice and in-service teachers across many kinds of schools. Teachers expressed the belief that the framework is accessible to them and also doable; it provided them with a vision for change. Moreover, many teachers attributed changes in their practice to conversations about the framework held in such professional development sessions (e.g., Drake & Sherin, 2006).

The framework we show in this chapter (see table 11.1) is a modification of the earlier framework that reflects our subsequent use of that framework with teachers. In particular, we added a column for mathematical representations to highlight the central role such representations play in supporting math talk in the classroom; we collapsed the original "Source of mathematical ideas" and "Responsibility for learning" into the column "Building student responsibility within the community," and we added a first column, "Teacher role," to focus on major teacher actions at each level.

Results

The central result of this research is the articulation of the framework shown in table 11.1. This framework depicts growth in a math-talk learning community in two ways. First, it is made up of four developmental levels through which the case-study class moved. By developmental levels, we refer to changes in the teacher's and students' actions that occurred over time and built successively on one another. Level 0 in the framework represents a traditional, teacher-directed classroom. In the Level 1 classroom, the teacher in the study began to pursue student mathematical thinking, but still played the leading role in the math-talk learning community. In Level 2 the teacher began to stimulate and assist students to take on important roles in the learning community and receded from the central role in the math talk. In Level 3 the teacher coached and assisted her students as they took on leading roles in the math-talk learning community.

Table 11.1
Levels of math-talk learning community components

	Teacher role	**Questioning**	**Explaining mathematical thinking**	**Mathematical representations**	**Building student responsibility within the community**
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may tell answers.	Representations are missing or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students feel their ideas are accepted by the classroom community. They begin to listen to each other supportively and to restate in their own words what another student said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so others are able to follow their mathematical thinking.	Students believe they are math learners and that their ideas and the ideas of classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.	Students believe they are math leaders and can help shape the thinking of others. They help shape others’ math thinking in supportive, collegial ways and accept the same.

Second, the framework examines growth that occurred within each of five components from Level 0 to Level 3. The components that make up the framework are—

(*a*) teacher role: how the teacher participates in and organizes students to engage in mathematical learning;

(*b*) questioning: who serves as the questioner in the classroom, and what kinds of questions are posed;

(*c*) explaining mathematical thinking: who provides explanations in the classroom, and what kinds of explanations are offered;

(*d*) mathematical representations: how language, visual, and concrete supports are used; and

(*e*) building student responsibility within the community: the extent to which students' ideas are seen as central to the discourse and students share responsibility for the learning of their peers and of themselves.

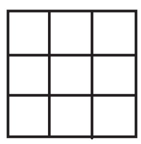
For the most part, growth occurred concurrently in each of the components of the math-talk learning community in Ms. Martinez's classroom. Here we illustrate this growth by focusing on the shifts that occurred in classroom questioning. For a detailed description of the growth along all components in the original framework, see Hufferd-Ackles, Fuson, and Sherin (2004).

Growth in the questioning component of the math-talk learning community

To further children's thinking about mathematics, it is important to find out what students know and how they think about mathematical concepts. Questioning students allows their responses to enter the classroom's discourse space to be assessed and built upon by others. Questioning challenges the thinking of the person being questioned by asking for further thinking about his or her work. As questioning built from Level 0 to Level 3 in Ms. Martinez's classroom, there was a shift from the teacher as the exclusive questioner to students as questioners along with the teacher. Another shift took place concurrently in the questioning component of the math-talk learning community—from a focus on *questioning to find answers* to a focus on *questioning to uncover the mathematical thinking producing the answers*.

Because the Level 0 math-talk learning community resembles the traditional, teacher-centered classroom, it is the teacher who assumes the role of question asking, and the goal of the teacher's questions is primarily to ask students to give answers to problems (but also often to control behavior by increasing listening). Early in the year, Ms. Martinez asked Level 0 questions that required only a brief answer, and she rarely followed up the students' responses with additional, more probing questions. Because the CWM curriculum prompted her to begin asking "Why?" and "How?" of students, Ms. Martinez quickly made the transition to Level 1 questioning. The excerpt that follows shows Ms. Martinez introducing the class to arrays (or squares making an area) for the purpose of scaffolding multiplicative understanding. Level 1 questioning is apparent in the types of questions Ms. Martinez asked and modeled. In the excerpt below and in all excerpts that follow, the actions of the teacher and students and our commentary on what was said appears in italics within parentheses.

Level 1 questioning: Teacher pursues student thinking



Ms. Martinez:	Now, who can tell me how many boxes of cereal I have in this container? *(She points to the 3-by-3 array she has drawn on the board.)* How many boxes of cereal do I have in this container, Carl?
Carl:	Nine.
Ms. Martinez:	Nine. How did you figure that out, Carl?
Carl:	Because I counted them. I counted them by 3s.
Ms. Martinez:	You counted them. You counted by 3s. Can you come up and show us?
	(Carl goes to the board to illustrate by pointing to the drawing.)
Carl:	I counted by 3s. There is 3 right here *(row 1 of boxes)*. Right there *(row 2)*. And there's 3 right here *(row 3)*.
Ms. Martinez:	So, it is like you are saying 3 + 3 + 3. What is another way we can count?

Level 2 questioning differs from Level 1 because of the shift made from the teacher as the sole questioner to the students as questioners as well. This new shift in Ms. Martinez's classroom began one day when several students were working at the board. In her efforts to engage the students who had finished the problem and were waiting in their seats, Ms. Martinez told them to think of one question to ask the explainers when they were finished. Liz explained her work at the board for the following problem: "Ana has 3 dolls. Maria has double the amount. How many are there all together?" To Ms. Martinez's surprise, the following dialog then took place.

Level 2 questioning: Students begin to question

Liz has written this labeled equation:

A *d* *J*
3 × 2 = 5
d

Ms. Martinez:	Okay, Santos?
Santos:	I wonder why she put the 5 in there.
Ms. Martinez:	Can you ask your question to Liz?
Santos:	*(To Liz)* Why did you put the 5 in there?
Liz:	Because it says, "How many are there all together?"
Saul:	How come there is a "d" under the 3?
Ms. Martinez:	Can you repeat the question to Liz?
Saul:	*(To Liz)* How come there is a "d" under the 3?
Liz:	Because it is for the dolls.

In this particular situation, Ms. Martinez could have involved herself in the discourse right away to discuss the error in Liz's solution. Instead, she waited to see if the problem in Liz's work would be clarified through the students' questioning. Students began to ask questions related to the issue of adding (3 + 2) rather than multiplying (3 × 2). Later, it took some further guidance from Ms. Martinez to resolve the issues embedded in this complex, two-step problem. However, Ms. Martinez was encouraged to see the beginnings of student-to-student math talk.

At the beginning of episodes of student-to-student questioning in later classes, the teacher often prompted the questioning process with statements like "Questions for people at the board?" Initially, many of the questions the students asked each other were modeled after questions that they had heard their teacher ask in class: for example, “What did you add?” “How did you come up with your answer?” and “Can you show us on your drawing?” A positive result of this new practice was that the students in the class who were not directly involved in the discourse were actively listening to the speakers so that they did not repeat the question another student had already asked. Sometimes students who were not outwardly participating in the questioning process gave evidence of their active listening by making comments. For example, one lower-achieving student often demonstrated active listening as he announced, “Someone already asked that.”

In the following Level 3 example, students contemplate whether one would get the same answer when subtracting two 3-digit numbers from left to right or from right to left. This excerpt demonstrates the highest level of student-initiated questioning in the framework, where students employ questioning in order to understand one another’s thinking and to understand the mathematics content.

Level 3 questioning: Student-initiated questioning

(Ms. Martinez is in the rear of the classroom, Jamie is stationed at the blackboard. He has been called on by Ms. Martinez to share his comments with the class about whether or not it is the same to add columns of numbers left to right or right to left.)

Jamie: No, because if you’re taking away any numbers you gotta take away from the other ones. Are you gonna start from the right?

Santos: What do you mean?

Jamie: Right when you’re taking away, yeah, subtraction, sometimes you gotta take away from the other numbers.

Maria: Sometimes you can start from the right *or* the left.

Jamie: How? Are you going to take one from the left, I mean from the right?

Maria: Sometimes it helps to write, like, when it’s subtraction, from the right or sometimes from the left.

Roberto: Either way, none of the numbers are going to change. Just do the same thing you’re gonna do from left to right, subtract the same thing you’re gonna do from right to left.

Jamie: Yeah, but that’s not gonna be the same answer.

(At Ms. Martinez’s urging, still from the rear of the classroom, the class moves on by choosing a problem to test. Following this, the conversation continued.)

This excerpt depicts a typical instance of initiative and persistence on the part of students that was common in Level 3 situations. Several students attempted to follow, challenge, and clarify Jamie’s thoughts about adding (or subtracting) from left to right, and interactions did not occur between only two people. Further, students were no longer dependent on the teacher to initiate the process of questioning *and* to keep it going. At the same time, the excerpt also illustrates how important the teacher’s role continues to be at even the highest level—Level 3. Ms. Martinez needed to intervene to clarify, to be sure students were fully satisfied, to suggest strategies for resolving differences, and to manage time by overseeing turn taking—though much of the conversation was managed by the students.

Structures and practices that support the math-talk learning community

We have found that two kinds of *Solve, Explain, Question, and Justify* classroom activity structures are effective in engaging all students in math-talk (see table 11.2). In both structures, all students solve problems simultaneously. In the first structure, as many students as possible go to the board to solve a problem while the rest of the students work at their seats. Then the teacher selects two or three students from the board who have interesting solutions, or need the chance to explain their work, to talk about their solution. Only two or three students need to explain their work because students usually cannot maintain concentration for more than two or three discussions of the same problem. Next, a different group of students goes to the board to solve the next problem. This process is very motivating. Most students enjoy solving problems at the board even if they do not get the chance to explain their work. While the students are working at the board, the teacher has a chance to see how solutions evolve. The teacher also gets a good sense for how individual students are doing. In one class period, many or even all of the students can get a turn at the board.

Table 11.2
The Solve, Explain, Question, and Justify classroom structure

Step 1 Solve:	All students solve.
Step 2 Explain:	One student explains and then asks, "Are there any questions?"
Step 3 Question:	Other students ask questions to clarify or extend.
Step 4 Justify:	The original explainer responds to the questions by explaining more (justifying the original explanation).

Notes: Any student at any time can ask for help from anyone. Typically another student explains, so the class loops through 2, 3, and 4 again. The discussion can now also contrast and compare the first and second solutions as well as others in the past. Explaining can be done in the whole class or in small groups so that more students have a chance to explain. Both versions have advantages. This can be called "Solve and Discuss" for short.

The second effective classroom structure allows every student in the classroom to explain his or her solution. Every student solves a problem at his or her seat. Then two or three students are selected by the teacher to go to the board to draw their solutions. The students left at their desks then pair up and explain their solutions to each other. Then the class discusses the solutions of those students at the board. Students at their seats can write their solutions on paper, which can be picked up and skimmed later by the teacher to see how students are progressing.

Building a nurturing meaning-making math-talk community requires leadership by the teacher and the help of all students in the class. Table 11.3 shows ways in which everyone in the class can assist the learning of others. These categories were based on categories used in reading by Tharp and Gallimore (1988) and are reported and discussed in Murata and Fuson (2006) and in Fuson and Murata (2007). *Engage and Involve* includes inviting all students to share ideas and questions, promoting analysis and discussion, and expecting that all students participate in developing understanding together in the community. *Managing* includes helping students monitor, be responsible for, and take ownership of their own learning. Initially the teacher engages and involves and also manages, but with encouragement and support students can also assist the math-talk community in these ways. *Coaching* involves two major categories. In *Show/Explain*, classroom members may model or instruct/explain. In *Focus and Extend*, the teacher or student coach may clarify, question, or give feedback. Teachers and students use gestures to connect the mathematical respresentations and the discourse in the classroom. Other specifics about building a math-talk learning community are summarized in Fuson, Atler, Roedel, and Zaccariello (2009).

Table 11.3
Responsive means of assistance that facilitate learning and teaching by all

A. Engage and Involve		
B. Manage		
C. Coach:	a. Show/explain	b. Focus and extend
	a1. model	b1. clarify
	a2. instruct/explain	b2. question
		b3. give feedback

Note: Students and teachers engage in all of these responsive means of assistance.

These specific means of assisting learning occur as students engage in the eight Standards for Mathematical Practices in the Common Core State Standards for Mathematics (NGA Center and CCSSO, 2010, pp. 6–8). They describe mathematical activities (practices) that teachers help students do in classrooms. These practices can be summarized as: Teachers continually assist students to do *math sense-making* about *math structure* using *math drawings* to support *math explaining*. A version of this summary, which we suggest is an easier way to remember the eight practices, appears in table 11.4. This summmary of the practices describes in more detail what we want to be happening mathematically in a math-talk learning community. Mathematics programs can provide supports for the teacher to build this kind of community (such as examples of rich mathematical practices within the teaching activities) and supports for teacher discussion and reflection (such as conceptual/mathematical unit overviews).

Table 11.4
Our summary of the eight mathematical practices of the Common Core State Standards for Mathematics (CCSSM)

Teachers continually assist students to do Math Sense-Making about Math Structure using Math Drawings to support Math Explaining.	
Math Sense-Making: Making sense and using appropriate precision	
1	Make sense of problems and persevere in solving them.
6	Attend to precision.
Math Structure: Seeing structure and generalizing	
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.
Math Drawings: Modeling and using tools	
4	Model with mathematics.
5	Use appropriate tools strategically.
Math Explaining: Reasoning and explaining	
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.

Note: There are eight CCSSM Mathematical Practices, but that is likely too many to keep in mind. So we suggest collapsing them into the four categories named above. The sentence at the top can assist teachers to use the Mathematical Practices every day and to improve in using and in helping students to use these practices.

Language tools for students to use in a math-talk setting can arise from the specific classroom community and fit its unique culture. One first-grade classroom community in Chicago developed the following language tools and used them regularly in discussing work with other students. Because they were developed collaboratively, they became tools that were helpful and nonthreatening. When a student used these phrases, the receiving student knew and understood what was being communicated.

"I can't really figure out what you did."

"I see that you ___."

"I get what you did, you ___."

"I have a question about what you did, ___?"

"You might want to ___."

Conclusion

Opening up one's clasroom to students' ideas is a critical first step in achieving a discourse community. Yet to be truly effective requires more. We have found that the levels and components we introduce here, along with classroom structures and means of assistance, describe the development of a classroom discourse community in ways that are accessible and useable by teachers and students. All teachers will need to continually adapt the frameworks described here to the changing needs of their own classrooms, but the frameworks give them a place to start and ways to evolve and grow with their students.

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