El Mercado in Latino Primary Classrooms: A Fruitful Narrative Theme for the Development of Children's Conceptual Mathematics

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Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, April, 1995.

The research reported in this chapter was supported in part by the National Center for Research in Mathematical Sciences Education, which is funded by the Office of Educational Research and Improvement of the U.S. Department of Education under Grant No. R117G 10002, and in part by the National Science Foundation under Grant No. RED 935373. The opinions expressed in this chapter are those of the authors and do not necessarily reflect the views of OERI or of NSF.

We are very grateful for the extraordinary support and cooperation of the classroom teachers, Elba Cora and Myrna Núñez, and of the school principal, Karen Carlson.

Abstract

The "el mercado" situation, a narrative framework based on students' own buy/sell transactions, was used as an on-going strand of the mathematics curriculum in a first- and a second-grade Latino inner-city classroom. El mercado became a shared on-going story created by each class as a social group. Teacher and students turned to el mercado to provide meaningful contexts for a range of mathematically problematic situations involving single-digit and multidigit addition, subtraction, and multiplication including word problems. Students enacted via role-playing in pairs a variety of buy/sell situations. The use of money and the buy/sell transactions were very positively charged for the students and created sustained involvement. However, children's mathematical knowledge obtained by real world practices was quite limited, and activities to support children's construction of such knowledge needed to be designed for the classroom. We identified conflicts in goals between learning through cultural practices and learning in schools, and discussed differences in the consequent scaffolding of performance. We proposed and discussed a third kind of concepts and learning to mediate Vygotsky's (1934/86) spontaneous and formal (scientific) concepts and learning: "referenced concepts" and "learning referentially."

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The past two decades have brought an explosion of research in two overlapping areas that can support the current national and international efforts toward reform of school mathematics teaching (see National Council of Teachers of Mathematics, 1989, 1991, and Wirszup & Streit, 1987, 1990, 1992, for reports of these efforts). These two areas are research on ethnomathematics and research on children's methods of solving mathematics problems. The literature on ethnomathematics has contributed greatly to our understanding of how children's and adults' understanding of mathematical topics is supported by and arises within cultural practices (e.g., see D'Ambrosio, 1992, for an overview; Nunes, 1992, for a recent review; and Lave, 1988; Mukhopadhyay, 1987; Nunes, Schliemann, & Carraher, 1993; and Saxe, 1991, for detailed studies). This literature has reported large gaps between the skills and understandings built outside of school by participation in mathematical cultural practices and the skills and understandings built by participation in mathematics classrooms in schools. The research on children's methods of solving mathematics problems has documented many methods that differ from those taught in classrooms (see Grouws, 1992, for reviews of the literature in many areas of mathematics). The literature on addition and subtraction word problems and on single-digit and multidigit addition and subtraction methods has been especially large and detailed and has described developmental progressions in such methods that usually occur independently of, and sometimes in spite of, school classroom activities (see Fuson, 1992b, 1992c, for reviews of this literature). This literature has also indentified various kinds of errors that arise in children's attempts to use school-taught written algorithms.

These literatures identify tensions and differences between street (out-of-school) mathematics and school mathematics and between informal mathematics (child invented, often in school) and formal mathematics (school taught written symbolic algorithms) (Ginsburg, 1977). These contrasts might be phrased in terms of Vygotsky's (1934/1986) terms "spontaneous concepts" and "scientific concepts" or "learning spontaneously" and "learning scientifically." Spontaneous concepts are those learned implicitly through ordinary life in contextualized activities. Scientific concepts (not to be confused with concepts of or about the sciences) are hierarchical, learned intentionally and explicitly in school, and learned by relatively more mediated experience (e.g., verbal discourse, written and/or oral) rather than direct experience, and have comparatively more cultural-historic contribution to the concept. In mathematics, this cultural-historic contribution includes the written symbol systems for writing mathematical concepts and operations. These frequently are difficult for young children because they are so unrelated to their referents and may even be misleading rather than merely opaque (e.g., Fuson, 1992a; Fuson & Kwon, 1991, 1992/91). Vygotsky postulated that there is an interaction between spontaneous concepts and scientific concepts such that the specific real world attributes of spontaneous concepts become attached to the related scientific concepts, enriching them, and the scientific concepts bring order to the organization of spontaneous concepts. The scientific concepts can become symbolic tools for thought. However, how these effects occur or what facilitates them was not clear from his writings.

The ethnomathematics literature raises the obvious possibility that basing school mathematics classroom experiences on children's mathematical experiences outside of school might bridge the gap ordinarily existing between these two kinds of experiences and might help make school mathematics learning more meaningful. The literature on children's methods of solving problems suggests that school mathematics classroom experiences be based on children's methods of solving problems and provide opportunities for children to move through the developmental progressions of such methods.

However, ethnomathematical contexts are limited as classroom contexts for learning because most real world mathematical contexts have limited context-dependent solutions that are not necessarily readily generalizable. For example, some of the engineers interviewed by Nunes (1992) could not deal with any blueprint ratios other than the three simple ratios they knew and used in their work. For the child candy sellers discussed in Saxe (1991), only particular ratios could be used to fill a box of candy, so the sizes of boxes constrain the ratios these children consider. Furthermore, in many of the contexts studied, social supports are available for missing solution steps so that there may be an extended period of functioning (years as an apprentice or as a candy seller) during which mathematical learning is quite limited. This combination of contextual constraints and the availability of social supports for missing solution steps can severely limit the mathematics that is learned. For example, only 60% of the sellers aged 6 to 15 could accurately give change from Cr$5000 for a purchase of Cr$3800 (Saxe, 1991). They were much better than were nonsellers of similar ages, but this still seems quite low for a task that would at first glance seem to be well within their experience. However, the uneven number Cr$3800 is not likely to be experienced by the sellers very often. To simplify the purchasing, they intentionally price goods with simple round prices using multiples (i.e., selling multiple amounts of candy for a given round amount of money). This limits what they need to know and speeds up each buying/selling interchange (consistent with their economic goals), but it also limits the mathematical tasks they consider.

In contrast, the goals of the mathematics classroom in modern first-world countries are to help children construct generalizable knowledge and solution methods that will work for a wide range of situations. In the past this goal has ordinarily been met by teaching mathematics in a decontextualized manner so that it will be maximally generalizable. The difficulty with this approach has been that the mathematics learned has been so decontextualized that it has been almost devoid of meaning for many children. This has rendered it inapplicable. Many children cannot apply the mathematical algorithms and symbolic moves to real situations or even to word problems.

Many people in mathematics education, and in other subject matter domains, are now searching for some balance between contextualized and decontextualized approaches so that children can learn concepts and procedures with meaning but also learn powerful generalized abstractions that they can use in various settings. In some settings, such as small villages or rural areas, ethnomathematics that does not go much beyond the contextualized needs of the culture is quite sensible, probably much more sensible than decontextualized methods that do not work as well in those circumstances. In the United States it is much more difficult to predict the mathematical needs of any individual, or indeed of anyone given the rate of technological change, so the need for at least partially generalizable concepts and strategies is important. This search for balance is not a recent one. It has been a tension throughout this century as various Deweyian programs have attempted to bring aspects of the real world into classrooms. But NAEP data (National Association of Educational Progress) indicate that many children still have problems with applications and even with algorithms (e.g., Brown et al., 1989; Kouba et la., 1988).

We report here a project that is attempting to bring such a balance into the primary school classrooms of poor urban children from Spanish-speaking backgrounds. To accomplish this, we engage in coordinated analyses of the target mathematical domains and of target real world situations. However, we also concentrate very heavily on trying to understand and help children construct concepts in the mathematical domains under consideration. We call this approach children's conceptual mathematics. We focus on children, and therefore on situations that are familiar to them, and on mathematics, and therefore on standard vocabulary and symbols and eventually on generalizable mathematical practices. The word "conceptual" ties these extremes together, indicating that any mathematical practices must be the children's own practices and be comprehensible and meaningful to them. We try to invent and/or use objects and symbols and words that will help provide meaning to the standard vocabulary and symbols. We explore how Vygotsky's hypothesis about relationships between spontaneous and nonspontaneous concepts and learning can be enacted and understood in the primary school classroom.

The major mathematical foci of our project are single-digit addition and subtraction, the whole range of addition and subtraction word problems, and multidigit addition and subtraction. Children ordinarily do not have support in the classroom for moving through the developmental progression of single-digit addition and subtraction methods nor do they experience the whole range of addition and subtraction word problems, unlike children in the Soviet Union, who do solve this whole range, Stigler, Fuson, Ham, & Kim, 1986. We sought to accomplish both of these goals. Construction of adequate conceptions of multidigit numbers requires that children experience quantities that are grouped into tens and the remaining loose ones (and, later, grouped into hundreds and thousands as well as tens and ones). Children in the United States have relatively few such experiences in their real lives. We do not have the metric system in which units are ten-structured, few objects are packaged and sold in tens, and irregularities in the number words for 2-digit numbers obfuscate the groups of ten (for details see Fuson, 1990, and Fuson & Kwon, 1992/91), unlike number words based on Chinese in which the tens are explicitly described (52 is said "five ten two" and 12 is "ten two").

We began by attempting to identify potential activity domains in children's everyday lives that could support a sufficiently broad range of mathematical topics to warrant extensive curricular building in that area. We felt that such a coherent approach would support children's learning efforts better than a scattered introduction of "culturally related topics" and would also be more consistent with the strong narrative aspect of some Latino cultures (Benjamin, 1993; Escamilla, 1993; Vasquez, 1992). Consistently following one major theme for a major time period also seemed important in inner-city schools, where bureaucratic school systems and constant pressures from a scarcity of resources create many crises, changes, and discontinuities in teacher's and children's lives. We hypothesized that such a theme could provide a familiar and coherent grounding for mathematical thinking. Our goal was to develop mathematical learning activities in the classroom that would be culturally affirming rather than discrepant for a range of Spanish-speaking children while also comfortable for the children not from Spanish-speaking backgrounds. This identification process of topic areas co-occurred with the pedagogical work.

Money is one area in which children can experience groups of ten, if quarters (25¢ coin) and $20 bills are ignored. Money is a real-world object that can be used to carry out addition and subtraction in ten-structured ways (e.g., adding dimes to dimes and pennies to pennies for 2-digit addition). Buying and selling situations also cover a wide range of addition and subtraction word problem situations and can therefore potentially be used to help these be meaningful to children. During the 1992-93 school year, we used buying and selling experiences acted out in the front of the first-grade classes as contexts for children's learning about story problems. These involved food preparation needs in the home linked to needed purchases in the store. These activities seemed to be helpful and enjoyable for the children, who began to participate in the acting out of the stories. We also used buying and selling situations involving packaging in tens (e.g., boxes of ten donuts and loose donuts) for adding and subtracting multidigit numbers (see Fuson, Smith, & Lo Cicero, 1997, for a report).

This paper is a report of our 1993-94 attempt to extend money and buying/selling situations to as many mathematical domains as possible, while functioning within our focus on children's conceptual mathematics. We undertook interactive analyses of these real-world situations and of our target mathematical topics to identify possible classroom activities. We then designed particular classroom activities and homework activities. Trying these out in classrooms led to revisions and new or adapted classroom activities. Often just articulating problems with an activity or assumption enabled progress to be made. Our report here is necessarily linear and cannot reflect all of the recursive steps in this development of activities. The project is on-going as we continue to modify, improve, and identify new problems to be overcome. We did make considerable progress during this year, but also had substantial limits to our success. Our experience has led us to a preliminary analysis of this domain and its potential for mathematical learning, to some general principles concerning this kind of endeavor, and to an increased understanding of some inherent conflicts between learning through cultural practices and learning in schools. In this paper we present these results of our analysis of our experience and enough background concerning what we actually did to contextualize our more general results.

Attributes that seemed to us central to the mathematical functioning reported outside of school and that we tried to retain in our classroom situations are (a) the activities are situated in real-world activities that are meaningful and important to the child, (b) the mathematical operations can be done concretely and orally with some sort of objects (at least initially) rather than carried out with written mathematical symbols, (c) these methods eventually can be done orally without objects, and (d) participation in the activities is scaffolded by more experienced participants so that functioning is possible.

Our Intervention

The setting

The school is located in a predominantly working-class Latino neighborhood. Over 90% of the students qualify for free or reduced lunch. The majority of students are first-generation children (born in the United States or in Mexico) whose parents have immigrated from Mexico. Some students are from Puerto Rico, a few students come from a range of Central and South American countries, and a few students are African-American or European-American or children whose parents have recently immigrated from Europe. At each grade level from one to five, there is a Spanish-speaking and an English-speaking class. Spanish-speaking children are bused in from six other elementary schools to complete the Spanish-speaking classes. Math classes and most other classes are carried out almost entirely in the specified language (English or Spanish). Many children in the English-speaking class also speak Spanish, and some children in the Spanish-speaking classes also speak at least some English. Parents can choose which class their child attends, but the child must demonstrate sufficient competency in English to be placed in the English-speaking class. There is one kindergarten class in which both English and Spanish are used. The number of students in the first-grade Spanish-speaking class varied through the year from 12 (in September before children came from another school and entered late) to 24; this range for the second-grade Spanish-speaking class was 22 to 25.

The teaching/learning activities were primarily developed by project staff. The teaching was done in Spanish three days a week by a fluent bilingual project teacher. The first-grade teacher participated in the project during both years. Over this period she increasingly became involved in teaching within the project framework and contributed many helpful ideas. The second-grade teacher was new to the project in the second year and also increasingly became involved in teaching within the project approach; at the end of the year buying/selling activities were done rarely because of an interdisciplinary unit focusing on whales. The project teacher had worked with the 1993-94 second-grade class when they were in first grade; excellent rapport existed between this class and the project teacher. Both classroom teachers were from Puerto Rico. The project teacher grew up in Argentina, her parents are from Mexico, and she lived for six years in Puerto Rico. Bilingual project researchers were from Argentina, Brazil, and Spain.

<u>The mathematical topics and buying/selling situations</u>

Our project explored ways to support children's construction and use of ten-structured conceptions of single-digit addition and subtraction, multidigit numbers, and multidigit addition and subtraction and children's solving and posing a wide range of addition and subtraction word problems. Buying and selling situations and money could be used in many ways within each of these areas. Tables 1 and 2 outline the results of our interactive analyses of these mathematical domains and real-world situations. These tables cover the whole range of first and second-grade topics. First graders did not get to the most advanced topics in the tables (beyond 2-digit addition), and second graders did little work on the easiest topics during the 1993/94 school year.

Money in the United States is not entirely ten-structured (the 25¢ coin and the $20 bill are exceptions). However, use of the 1¢, 5¢, and 10¢ coins and $1, $5, and $10 bills allows 2-digit numbers within either coins or dollars to be constructed from tens and ones so that children could learn and use the place value meanings of these numbers (e.g., 52 as 5 dimes and 2 pennies). Use of the 5¢ and 1¢ coin and the $5 and $1 bill helps children construct and use a conception of the numbers 6, 7, 8, and 9 as 5 + 1, 5 + 2, 5 + 3, 5 + 4. These conceptions then can help children add and subtract in ten-structured ways (e.g., 7 + 8 = 5 + 2 + 5 + 3 = 5 + 5 + 2 + 3 = ten and five = 15). Thus, money can be used as pedagogical objects that children can use to solve problems by counting out quantities and then adding them, taking from one quantity, or comparing the quantities made with money. Adding with carrying can be shown by trading ten pennies for a dime or ten $1 bills for one $10 bill. Subtracting with borrowing requires one to "go to the bank" or to a friend to get ten pennies for a dime or ten $1 bills for one $10 bill.

Multidigit numbers can arise in buying and selling contexts either as multidigit costs for a single object (thus using money) or as a large quantity of purchased objects. In the latter case, ten-structured concepts can be supported by packaging such objects in groups of tens and then hundreds and thousands. Pedagogical objects that can be repeatedly packaged and unpackaged for multidigit addition and subtraction would be helpful for this aspect of buying and selling. We tried various such objects, but did not solve this practical issue satisfactorily in this year. We did find some products that were packaged in tens (pens, markers, ball gum, Kit-Kat candy bars, cookies). These were used in class to develop packaging language, especially for addition and subtraction, and to practice counting by ten. In the first-grade class buying and selling milk for the family was a major focus. We used milk cartons from the school lunchroom (rinsed and closed) for many early enactments. Somewhat later in the year milk cartons fastened by velcro in packages of ten were used to provide ten-structured packaging language. These worked well for whole class demonstrations, but were too large and too time-consuming to make for each child to have his/her own 2-digit quantities.

What the children primarily used instead as their own packagable materials was a drawn quantity system the second graders had learned in first grade (Fuson, Smith, and Lo Cicero, 1997) and the first graders learned again during this year. Children drew 2-digit quantities as columns of ten dots (or circles) and extra dots horizontally to the right. These columns were eventually connected by a vertical stick drawn through them, and finally only the ten-sticks were drawn. Thus, 52 was made with 5 ten sticks and 2 dots. Children added with these ten-sticks and dots by making another ten when possible (by circling ten of the dots), and they subtracted by opening (unpackaging) a stick to make ten dots when necessary to subtract ones (see examples in Figure 1). These sticks and dots could represent packages of various goods bought and sold. Packaging language was used frequently in buying/selling situations in the classroom when ten-sticks and dots were used for solutions and at other times. Children became quite good at and used to describing opening a package of ten and making a new package of ten and used this language adaptively in new situations.

Many of our activities were based on children's stories about their own buying/selling activities. Early in the year the second-grade children began describing their own experiences selling with family members in Mexico. Over half of the children had had such experiences. Examples are as follows:

1) "My mother and I used to go to the bakery and the baker gave us bread that we sold around our neighborhood. We had to get up very early to get the bread."

2) "My mother used to make tamales, and we sold them at 1,000 pesos in the streets."

3) "We sold used clothes on the beach. Many times my brothers and I were left alone to watch the clothes, and sometime we sold some."

4) "My aunt made candy to sell. I remember the recipe: water, sugar, and flavor (agua, azúcar y gusto)."

From these experiences we developed a major classroom activity structure for buying/selling situations called "playing mercado" (jugar al mercado). Pairs of children played mercado, with one child taking the role of the buyer (comprador/a) and the other child the role of the seller (vendedor/a). Because so many children had such experience, it seemed likely that children could participate in such a pair buying/selling situation rather than watching a single enactment in front of the class, as we had done earlier. Importantly, the second-grade teacher was open to allowing children to work in pairs frequently. A deeper analysis of children's functioning within this activity structure is given in Lima (1995b, 1995c). The mercado playing could involve different mathematical tasks; these tasks are given in parentheses in Table 1. Thus, this single classroom activity structure could be adapted to a range of mathematical learning goals. Initially children played freely, and later we used various balance sheets to structure their recording and reflections (see Figure 2).

Our thinking with respect to playing mercado was that it met the four attributes identified earlier as important in ethnomathematical experiences outside of school. Children had such buying/selling experiences either directly through participation or by watching their parents in stores, money could serve as concrete objects for calculation, counting and adding methods with money could later be done orally without money, and more expert children could scaffold less expert ones in the pair situations. Scaffolding of learning also occurred on the whole class level because pairs of children were asked to describe their buying and selling interactions after playing mercado. This permitted the teacher to highlight and discuss with the class a range of solution methods.

The first graders also played mercado, but to a much lesser extent. Most of the uses were before Christmas. This was partly because the first-grade teacher was not so enthusiastic about using pairs in the class and partly because the class had an unusual number of low-performing children. This created a large gap between this bottom half and the more advanced top half of the class, and resulted in separate activites much of the time from January on. This mercado was more structured than that in the second grade. Each child got a fixed amount of money (usually $10), and the seller had milk cartons to sell. Each child filled out a balance sheet that specified how much they had at the beginning, how much they spent or received, and how much money they had at the end. At the end one or two pairs of children would take turns explaining to the rest of the class what they had done. This explanation was sometimes used by the teacher to pose word problems to the rest of the class.

Children were asked to play mercado at home with their parents several times. In October second graders played at home and brought in written records of their buying and selling. Parents had many different ways of organizing these records. From this work we elaborated many word problems and worksheets. Later in the year more structured assignments were made, and play money was sent home so that children could focus at home on the mathematical topics being done in class.

Children also did various other activities with the money. They made different amounts with coins and made one amount in different ways. These ways were then discussed in the whole class. They made change from $1.00, again discussing various methods of doing so. Both subtraction and counting up methods were discussed. We made our own dollars by Xeroxing on green paper fairly realistic "fake money" about 2/3 the size of real money. These could be cut apart with a paper cutter, so large amounts of money could be made fairly quickly and inexpensively. We primarily used $1, $5, and $10; $100 bills were used for some later activities. We mostly used real pennies and dimes, but sometimes used paper coins the teachers had from the Open Court Real Math textbook series (Willoughby et al., 1991).

A mini-theme in the second-grade class followed one of the student's actual selling experiences (the fourth example given above). The second-grade class cooked candy and packaged it in sandwich bags and then sold it to the first-grade class for pennies. Children used real sales receipts a community business person had given the class. Before the candy was made, children were asked to find out the prices of the ingredients and to name the stores where they found these ingredients. This information was then used to compare prices and to make graphs and word problems.

Word problems were introduced in the first-grade class through a series of enactments of family cooking needs and resulting purchases as a store, focusing especially on purchases of cartons of milk for individuals. Children participated in these enactments. Gradually these situations were described in words rather than acted out. Children later solved a range of word problems about buying and selling that were not limited to this particular context. The second graders had solved most main types of word problems when they were first graders. They solved and posed a wide range of word problems concerning buying/selling situations. In some cases a given situation was described, and children posed various questions about it (see Table 2). Second graders also solved problems with irrelevant information and 2-step problems (see Table 2 for examples).

The fit of costs and sensible quantities of goods varies greatly with the value of money in given countries. Countries with huge rates of inflation support the construction and use of concepts of very large monetary quantities (in the hundreds of thousands in Brazil). Items commonly purchased by children (e.g., ice cream, small toys) use 4-digit numbers in Italy and

Japan. These items used to cost single-digit or teen numbers in this country; now they cost 2-digit numbers. Most grocery items in this country cost 2-digit or 3-digit numbers. These facts constrain the types of realistic problems that can be given to children at early, middle, and late stages within the first and second grades. We tried to be as realistic as possible in all problems, but occasionally (especially at the very beginning of first grade) had to violate realistic prices in order to function in the numerical learning zone of the children.

What Worked?

We found that both first and second graders possessed robust "mercado scripts" that enabled them to engage in buying and selling pair activities. Playing mercado worked well as a classroom activity structure. Children enthusiastically and creatively role-played buying and selling and embellished with talk, objects, and physical actions the basic situations given to them in various ways to make them socially detailed and personal. In the lunchroom and elsewhere outside of class, the second graders began talking in excited and involved ways about mathematics and about playing mercado. Many people in the school became interested in what was happening in the classroom to create such enthusiasm about mathematics in the children. The success of playing mercado as a classroom activity does not mean that all aspects of the mathematics carried out within each pair was correct; limitations in mathematical understanding are described below. But children understood enough and cared enough about such interactions that they participated in enthusiastic and sustained ways.

The use of money was positively charged for childen and also created sustained involvement. Working with money allowed children to learn more about this mathematical domain and to use money as a problem-solving tool in buy/sell situations involving addition, subtraction, and multiplication.

As indicated in Tables 1 and 2, buying and selling situations provide familiar contexts for a wide range of first- and second-grade mathematical topics. We were able to develop and use classroom activities that fit all topics listed in the tables. Because of this range of topics, el mercado became in each class a shared on-going story created by each class and the teachers as a social group. Teachers and students turned to buying/selling situations to contextualize and provide meaning for a range of mathematically problematic situations. They would sometimes turn to a buying or selling situation as a source of clarification when stuck in some mathematical problem or issue. For example, during December interviews, one of the first graders was incorrectly solving an addition problem as a subtraction. The problem was written from the perspective of a seller and asked about the amount of money accrued (Miguel had $4. Jolger paid him $3 for a milk carton. How many dollars does Miguel have now?). When the interviewer repeated the problem making clear that Miguel was the seller, the child immediately solved it correctly. Children frequently used packaging language to explain their

2-digit addition or subtraction methods, first setting their problem as buying some specified product and then (in addition) describing trading as making a new package of ten to go with the other packages of ten or (in subtraction) opening a package to get enough to take away some.

The second graders frequently used buying/selling situations as contexts for word problems they made up. The second graders became quite fluent in their mercado vocabulary and concepts, so that they could easily and correctly go from the mathematical operation to the real world situation and from the situation to the mathematical operation. Even low-achieving children, who had trouble with some 2-digit aspects of the mercado, could produce consistent word problems that reflected both the perspective of the buyer and the seller.

Students were able to experience over time each of the complementary interdependent buy/sell roles. They thus experienced the progressive effects of a single buying role (money decreases and products increase) or selling role (money increases and products decrease), and some students began to understand the inverse relation of these roles.

There was a considerable focus on students explaining their activities both orally and in written form. The classroom teacher and project investigators would listen to some of these explanations individually. Frequently after playing mercado, pairs of students came to the front and described their buying and selling. Because the teacher later asked other students to answer questions about these reports, the class was attentive and students were aware of different strategies used to solve problems. Children later in the year wrote such descriptions for homework. These activities facilitated reflection on and sharing of solution strategies. Children became better at describing and explaining their transactions as well as better at carrying them out. By the end, even the lower-achieving children could describe their buying/selling activities.

Children became quite good at identifying various mathematical questions within a given buy/sell situation. An example from the second-grade class on April 18 illustrates this. The problem given by the teacher was:

> En tu caja tienes \$.65. Viene el comprador a comprar 3 productos. Cada uno cuesta \$.23. El cliente paga con un billete de \$1.00. (In the cash register you have \$.65. The buyer comes to buy 3 products. Each one costs \$.23. The client pays with a \$1 bill.)
>
> <u>Teacher</u>: ¿Qué pregunta voy a escribir aquí? (What question am I going to write here?)
>
> <u>Student 1</u>: ¿Cuánto cambio va a recibir? (How much change is he going to get?)
>
> <u>Student 2</u>: ¿Cuánto tenía al principio el vendedor? (How much did the seller have initially?)
>
> <u>Student 3</u>: ¿Cuánto va a pagar el cliente por tres productos? (How much is the client going to pay for the 3 products?)

Student 4: ¿Cuánto dinero le va a quedar al vendedor? (How much money is the seller going to have left?)

Thus, in one problem situation children can pose and solve addition, subtraction, and multiplication questions.

What Was Problematic?

There were substantial limitations in our buying/selling situations as sources of learning. Many first-grade and some second-grade children did not know the values of coins, and few knew the names of all the coins. Most children did use the words "penny" or "un centavo" (and sometimes "un peso", the Mexican basic monetary unit) and "cora" (presumably a Spanglish version of "quarter"). However, they did not necessarily know the values of these coins. Both "penny" and "cora" meant for some children "coin" ("I have pennies/quarters/coins in my pocket today.") as well as the specific penny or quarter coins. Very few children knew the word "nickle" or "dime." They called nickles "un cinco" or "cinquito" and the dime "un diez" or "diecito." These terms use the Spanish words for five and ten (five is cinco and ten is diez) or use diminutives of these words, but many children did not access or use these meanings. These terms were just standard names they heard around them, and they did not process them quantitatively. The second-grade classroom teacher asked us to use the terms "moneda de veinticinco (coin of twentyfive), moneda de diez, moneda de cinco, and un centavo" because she thought these would be easier for all children to understand. "Moneda de veinticinco" was also called "veinticinco centavos," "moneda de diez" was also called "dieci centavos," and "moneda de cinco" was also called "cinco centavos." For some children coin names were associated with speaking English, and they shifted partly into English when discussing them (e.g., they might count in English instead of in Spanish or use some English phrases).

A considerable number of children had trouble making quantities with the coins, and more had difficulties adding and subtracting with them. Many first graders found it difficult to count by tens (dimes) and by fives (nickles); coordinating the counting by all three values (1, 5, and 10 for pennies, nickles, and dimes) when making quantities from all three kinds of coins was even more difficult. Some of the second graders also had these difficulties in switching values when counting. Some children also had trouble differentiating the bills for $1 and $10 and intermixed these. Bills in the United States are not nearly as distinguishable as those in some other countries, which use size (bills for larger quantities get larger) and/or color to differentiate bills. In the United States all bills are the same size and the same color (green on one side and greyish black on the other). They are differentiated only by the dollar amount, which is written in small numbers in each corner and in words on some places, and by marginally different pictures, most of which very busy and complex and not readily differentiable at a glance (e.g., the pictures of men have similar backgrounds and one must focus on the features of the person to differentiate them). The common use of nonnumeral features to differentiate bills is exemplified by Saxe's results (1991) with Brazilian street candy sellers who could readily differentiate bills when the numbers on them were masked but who could not identify them when only the numbers on them were used (i.e., they did not even notice the numbers in their use of the bills).

Many of our second graders could function well within either of the 2-digit base hundred structures we used -- coins or $1 and $10 bills -- but had difficulty integrating these values. They did not know or could not use the fact that there are ten dimes in one dollar. These two settings for 2-digit addition and subtraction instead stayed for many children independent 2-digit domains rather than an intergrated 4-digit domain. In fact, the way in which we read money is as two adjacent 2-digit fields (thirty six dollars and thirty six cents) and not as 4-digit numbers (three thousand six hundred thirty six cents). Therefore solution methods for money that use sequence counting in English or Spanish will function within these two 2-digit quantity domains (coins and dollars) rather than use 4-digit terminology.

Many first graders and some second graders had inadequate understanding of the concept of getting/giving change (recibir/dar cambio). For children speaking English, this may be exacerbated by the different meanings of this word. "Change" can refer to the coins one has in one's pocket or purse as well as the the coins one receives back when one overpays for a purchase (e.g., uses a $1 bill to pay for something costing 78¢). The first meaning interfered with understanding the second meaning for some children when first dealing with getting/giving change in the classroom.

Children's difficulties with this concept also stemmed from the fact that in the real world the nonmathematical aspects of buying/selling are more salient and obvious than are the mathematical aspects. Children see their family member give money to the store keeper and get money back. They are ordinarily not privy to the amounts involved, and so make several different hypotheses about them. Early in the year, some second graders playing an exact purchase mercado (where they have to give the exact amount to buy something) with $10 bills and $1 bills dug into their pockets to find coins to "give back change." Their view was that one always got change, and change always involved coins. In the real world this is often true; people may rarely have or pay with the exact amount. Some first graders in the middle of the year described getting/giving change as "paying back," as in this interview done in the classroom during buying/selling pair activities involving change:

<u>Child</u>: In the store I go to, they ... they ... they pay my sister back.

<u>Interviewer</u>: Why are you giving him this (money)?

<u>Child</u>: To pay him back.

Some children thought that they should pay what an object cost and then get that same amount of money back as change. Others thought they should give back one less (a budding notion that what you are buying costs something). Some just expressed this as "you get some money back." Other children knew that they should give back a specified amount but could not reliably find that amount. Most of these children thought that you should always get back change, probably because this was the situation they saw the most. With experience, many children knew or learned how to calculate the amount of the change using ten-sticks and dots or numbers, but some did not always do so accurately. Others could figure out the amount of change using ten-sticks and dots or numbers but could not necessarily give back change with money because this required counting by tens, fives, and ones. Finally, some children could give back change using money as well as calculate the change using numbers or ten-sticks and dots.

The scaffolding of learning by the children in a pair was quite variable (see Lima, 1995b, 1995c, for more analysis). Some children were excellent learning supports, adapting their help to the needs of their partner. Many other children ignored errors or just did a problem for their partner, not necessarily even being sure that their partner could or did watch them as they solved the problem. We tried to get children to describe and explain what they were doing to each other so that they could get practice at such reflective descriptions of their actions. This met with mixed success, with some children doing this frequently and very well, and others rarely and/or cursorily.

Our failure to find actual objects that could be bought and sold in packages of ten meant that children for the most part could not actually package or unpackage quantities except with ten-sticks and dots. Children and the teachers did use packaging language with the ten-sticks and dots, and this was quite helpful. Packaging language was also used in story problems the children and teacher made up. Many children seemed to understand 2-digit addition and subtraction well without needing actually to package and unpackage, but such experiences might have been quite valuable for children having difficulties.

Our focus on children learning money, and our conclusion that monitoring 2-digit quantities of both money and goods would be too complex initially, resulted in an asymmetry in the salience of the results of the buying and selling roles. In actuality, as the seller accumulates money, the quantity of goods decreases. As the buyer accumulates goods, the quantity of money decreases. The children frequently kept track of their money using a balance sheet. Therefore, it was clear that the buyer steadily lost money; the compensating acquisition of goods was not as clear because we did not have such a focus on it. Also, much of the time children gathered goods to sell from their desks, pockets, etc. They enjoyed creating their own sales items, but the money was more desirable than these objects. For this reason, both children in a pair sometimes wanted to be the seller (because the seller accumulated money). However, in general, turn-taking and who got to be what role was not an issue.

Reflections on General Issues

Our two years of work advanced our thinking concerning several theoretical issues. We will discuss these issues in terms of our buying/selling situations, although our reflections apply to a range of mathematical situations and probably to other subject matter domains as well. Results from related literature that became available to us during and after the project efforts reported here are woven into this discussion.

Conflicts in goals and implications for scaffolding mathematical learning

The first issue concerns intertwined aspects of (a) the goals of buying/selling situations inside the classroom and outside of it and (b) the quantity and nature of scaffolding available in these two contexts. The goals for buying/selling in the real world are economic (Saxe, 1991). Exact answers are not necessarily important (e.g., in Brazil, the phrase "the skin of a mouse" used to refer to a Cr $100 bill because it was so worthless, Saxe, 1991, p. 136). Non-arithmetical issues like location of selling and time for each transaction are quite important, and these may be inversely related to arithmetical accuracy. For example, taking time to calculate very accurately takes away from selling time (see also Beach, 1990, for a discussion of this issue in Nepalese stores). Furthermore, the social environment within which the candy sellers (and other child sellers) function is a very benign one: There is ample adult or child expert support available to carry out any steps that the child does not know. Therefore, mathematical learning is incidental rather than the goal, or even a goal, of the situation. This results in a very prolonged learning curve in which many of the oldest children still carry out direct modelling solutions with the physical objects. For example, 53% of the 12- to 15-year-olds found out how much a proposed selling ratio would bring them for the whole box of candy by repeatedly putting the specified groups of candies in the box while counting up the money with each placement (e.g., for selling 3 candies for Cr$1000, they would put 3 candies, say $1000, put 3 more and say $2000, etc.).

In contrast, our goal for our classroom buy/sell situations was not economic gain, but learning of the mathematical concepts in those situations. However, the children's goals were the same as in the real-world buy/sell situations: to buy and sell and to accumulate money. Their goal then led them most often to scaffold each other in the same way the scaffolding is done in the real world: They told what to do on a behavioral level or did it for the other child. This was appropriate scaffolding for their view of the situation: It allowed them to get on most quickly with the task of buying and selling. But it did not contribute maximally to the other child's learning, which was our goal. The teachers worked to try to have the children adopt the goal of helping each other learn and understand. This did become a functional social norm in the classrooms. Children were extraordinarily attentive during whole-class discussions, presentations by pairs, and enactments of mercado activities, and there were general whole class feelings to the classroom activities. But within the pair mercado activities, scaffolding was still frequently less than maximal and too often (for our goal) more focused on scaffolding in the classical sense of doing particular steps for another so that the children could get on with the activity of buying and selling, without regard for the learning taking place.

Furthermore, especially at the beginning in both classes, there were not enough children with enough knowledge to scaffold everyone who needed help doing parts of the buying/selling. Most first graders needed to learn (a) to count to 100 by ones, fives, and tens, (b) to recognize the coins, and (c) to coordinate counting by ones and tens (and later by ones, fives, and tens) when counting coins of different values. Many second graders had to learn at least several of these competences. Targeted focusing on each of these aspects of money knowledge would seem to be helpful rather than always embedding each of them within a whole buying/selling scenario.

Our proposed solution to this need for more expertise in the classroom is to have a first knowledge-building phase in which we spend more time on the prerequisite knowledge identified above and on individual rather than pair versions of buying/selling activities. The goal is to enable more children to construct more of the required mathematical knowledge before moving so heavily into the pair buying/selling situations. During this prerequisite knowledge-building phase, the expert children can be used to help children who need it, with the explicit goal to help them learn how to do something by themselves (and be able to explain it). The explicit goal during this phase will be for everyone in the class to learn this prerequisite knowledge (e.g., make quantities with money) and to help others learn. This group norm of learning and helping others learn can then explicitly be carried over into the pair buying/selling activities that would begin more heavily in the second phase. This first phase would need to create in the classroom a balance between the number of available helpers and the number of children who need helping and raise almost all children to a minimal level of functioning (e.g., making quantities with money needing only occasional help). When such a balance is achieved, the class could move into the second phase.

For some of this first phase, using explicit teacher-student pairs where children take turns playing the roles of teacher and student could support this group norm of explicit learning and explaining by everyone. The task for the teacher would be to monitor his/her student's solution methods, give feedback if there is an error, and help by questioning and other indirect methods, using direct showing or doing for the student only as a last resort. Children would change roles with each problem. Some initial work indicates that at least the top five first graders in a classroom can with help learn to function quite well in this situation, providing very competent learning support for the lowest-achieving children in the class (Fuson &

Smith, 1995; Bechorner & Taniguchi, 1995). How well this can extend to others in the class, and what state of mathematical knowledge is necessary, are still to be ascertained.

Analysis of children's concepts of the domain and of the learning supports in the real-word situation

In addition to a task analysis of situational and mathematical features, it is important to ascertain children's conceptions and misconceptions in the domain. Only by such a conceptual analysis can activities be adapted to decrease misconceptions and errors and to support children's construction of adequate understandings. We began such an analysis concerning money and change by identifying above several problem areas in which children had limited or erroneous knowledge.

This unfolding conceptual analysis was accompanied by an increasing realization that the outside world was not as supportive of the mathematical features of buying/selling situations as we had assumed. As discussed above, social aspects and the physical exchange of money were salient features to all children. The amounts involved were not so readily available, so children constructed various inadequate notions of "change," for example. We think that this is a general problem with many real-world mathematical and scientific situations. Many such situations vary considerably in the transparency of the concepts involved and in the social support available for seeing or learning the concepts involved. Therefore, use of real-world situations in the classroom requires a careful task analysis of the features that are crucial and of the availability of these features to the onlooker. Special pedagogical materials may be required to help children see and understand the relevant features of the domain. Technology in the form of similations that make apparent important constructs (e.g., rate) can be very helpful in such learning.

The value of money was a problematic aspect of buying/selling situations. The features of both bills and coins in the United States obfuscate and even mislead (a dime is smaller than a penny and nickel) rather than clarify these values. Pedagogical supports to help children ignore the irrelevant features and construct adequate quantitative meanings would be very useful. As an indication of how difficult these meanings are for children, Griffin, Case, and Sandieson (1992) reported that only 60% of 6-year-olds can answer which is worth more (or is the bigger amount) when shown two $1 bills versus one $5 bill. Knowing and combining the different quantities in bills and coins is an especially complex and difficult problem. They found that only 40% of 8-year-olds could say how much money the following amounts are all together: $5, $2, $1, 25¢, 10¢, 2¢ (Canadian money is similar to that in the United States except that they have 2¢ coins and still use $2 bills). Only 3% of 6-year-olds and 87% of 8-year-olds could tell which was worth more: $1 and 20 pennies or $5 and one penny.

A recent report by Brenner (1994) elucidates these issues of the transparency of out-of-school money experiences of young children and difficulties in children's money knowledge as well as gaps between children's in-school and out-of-school money learning. Brenner interviewed and observed Hawaiian children ranging from preschool age through second grade. The observations were made both in school, where the mathemetics classrooms used traditional textbooks, and in children's neighborhoods, where children's buying and selling in small stores were the focus. There was a "child culture" of shopping in Hawaii in which prices are simple, so that they involve little use of pennies or of making change, and buying is scaffolded by the adult seller as necessary. The result is that the buying experiences have limited opportunities for calculation, learning the whole range of money meanings, or making change.

Brenner found a substantial gap between the money knowledge children constructed from their out-of-school experiences and that expected or used by the school. Preschoolers knew the names of dollars and quarters because they saw these in use for real purchases, but they had no sense of quantities involved in any particular coin or bill. Kindergarteners began to have some knowledge of pennies and dollars, but not quantity knowledge of any money. Second graders knew that a dollar was four quarters, but not other values for a dollar (e.g., not ten dimes = one dollar). They could also break down a quarter into dimes and nickles.

However, in contrast to this real-world knowledge of larger money quantities, school knowledge began with pennies and concentrated heavily on pennies and nickles, presumably because the primary school curriculum focuses initially so heavily on numbers below ten. Children never made money quantities as large as a dollar, so they never had an opportunity to learn the quantity value of a dollar or how it related to dimes, nickles, and pennies. Nor did children make quantities from dimes and pennies, thereby potentially elucidating the tens and ones meanings of written symbols.

Thus, Brenner found a large gap between in-school and out-of school money foci. Furthermore, neither the classroom nor the out-of-school learning opportunities used or helped children construct base-ten knowledge that could be related to place value. Real world purchases were most often made with quarters, and school money activities did not use dimes and pennies, or even just dimes, nickles, and pennies, to make quantities. Neither setting helped children construct quantitative understanding of a dollar in terms of coins except for the quarter/dollar relationship in real life. The types of operations on money supported in the two contexts were also different. Real life shopping encouraged qualitative comparisons of two quantities ("Which is more, the price or my money?" in order to ascertain whether one can buy certain things) and subtraction ("How much will I have left after I buy that?" so as to know if I can buy anything else). School emphasized making various quantities from money using pennies to get exact quantities like 42¢, a practice nonexistent in real life. Therefore, traditional textbooks need to be changed considerably to use money as a support for children's understanding of place value and multidigit addition and subtraction. They will also need to provide a range of learning opportunities for children to learn to count by fives and tens and to combine the different counts by ones, fives, and tens in order for all children to use money in this way effectively.

Some children do not even have much experience with money. We found a wide range of knowledge in our children, none of which came from school experience other than from our project. Guberman (1994a, 1994b) also reported considerable variability in the experiences Latino children in Los Angeles had with money outside of school, and Brenner (1994) found this with Hawaiian children. Brenner did find that 97% of her children went grocery shopping with their family. There may be cultural as well as individual differences in the amount of such experiences. Guberman (1994a, 1994b) found that Latino children in Los Angeles had considerably more exposure to and responsibilities that involved buying and selling than did Korean children in Los Angeles. Latino children frequently made small purchases for their family or for themselves, while Korean culture viewed children's contact with or use of money as quite negative and therefore limited it. A preliminary discussion with Guberman concerning his results with Latino children (personal communication, April, 1992) encouraged us in our el mercado efforts. Lima (1995a) described initial results from our project concerning our children's purchasing in Chicago.

<u>Achieving the balance between contextualized and decontextualized learning</u>

Our 2-year effort to explore how spontaneous and nonspontaneous concepts can relate and enrich each other in the classroom has led to an extension of Vygotsky's (1934/86) dyadic distinction between spontaneous and scientific concepts. First, to avoid confusions from Vygotsky's use of the term "scientific" concepts as his contrast with spontaneous concepts, we will substitute the term "formal" concepts. This term captures most of the features Vygotsky discussed for this term (see the beginning of this paper for a brief summary of these). Second, we find it useful to discuss both a noun form (concepts) and a verb form (learning) for each of these terms, thus, spontaneous concepts and learning spontaneously as well as formal concepts and learning formally. Third, we have found it necessary to intersperse a third kind of concepts/learning between spontaneous and formal (see Figure 3) because these two differ so much in so many ways that it is difficult to bridge them directly. We conceptualize our mathematics classrooms as intermediaries between (a) the real world and (b) the formal world of mathematics and of traditional school mathematics learning. In these classrooms our focus is on helping children construct "referenced concepts" by "learning referentially." In the classroom real-world mathematical practices and experiences are lifted from their full embedded complexity, and the mathematical features of such practices and features become the focus. Activities range from those close to the real world (e.g., classroom simulations of real-world practices) to those close to traditional decontextualized formal school mathematics practices (e.g., doing numeral problems). There are special activities focused on helping children relate meaningful referents to formal mathematical words and symbols. We emphasize children using solution methods they understand, but also help children learn at least one correct method and also try to help children move on to more advanced methods. "Referenced concepts" necessarily means a concept with a referent that is meaningful for each particular child; therefore learning referentially carries a much heavier responsibility for the teacher to monitor each child's understanding and to adapt the learning activities to needs of various children in the class. We chose to use the term "referenced concepts" rather than "meaningful concepts" to capture the responsibility of the teacher and curriculum designer to provide referents that will be meaningful to children. Children will search for meanings in any setting including the classroom and will use any meanings they have from outside the classroom. But they cannot bring new or clear or accurate referents into the situation; that is the responsibility of the adults in the educational sphere.

Our classroom buying/selling situations, activities with money, and extensive use of buying/selling word problems provided a strong core of referenced concepts that enabled many children to learn referentially for many concepts. Children's spontaneous concepts of buying and selling were sufficient to provide at least an initial functional level of mathematical meaning for a range of mathematical topics. However, as discussed in the previous section, the classroom must then also provide support for many children to construct more adequate concepts of this domain.

In designing classroom versions of real-world practices that can highlight and support the mathematical features, enough of the mathematically irrelevant real-world features must be retained to make them real and familiar and comfortable to children without being too distracting or costly in money or time. Our children, especially in the beginning, spent a considerable amount of time on the physical settings of their buying/selling contexts (e.g., setting up shop) or on the social embroidery involved. This left less time for the mathematical aspects of the situation. But these aspects were important to them and probably contributed to their enthusiasm. Later on, it seemed easier to focus on the mathematical aspects, as the children constructed a familiar classroom version of real-world practices. Thus, simulations and enactments may be able to be successively stripped down to their mathematical features.

Our analyses and curricular design work was simultaneously both bottom-up from the real-world situation and top-down from the mathematical topics. This bidirectional approach seemed to help us to find and use so many relationships. Two recent projects have reported more bottom-up efforts at designing school mathematical learning experiences. Both were successful in terms of student interest and involvement and mathematics learning, but both authors reported limitations in the mathematics that was able to be approached. One project is the application of the "funds of knowledge" project to mathematics (Civil, 1992, 1993, 1994), and the other focuses on the integration of mathematics with other subject matter areas (Henderson & Landesman, in press). The "funds of knowledge" project seeks first to uncover by teacher visits to children's homes "the essential bodies of knowledge and information that households use to survive, to get ahead, or to thrive" (Moll et al., 1992, p. 2). Then teachers, with university researchers, attempt to design mathematics learning experiences based on these funds of knowledge from children's homes. Civil described such efforts in the areas of games (1994), construction (1993), and money (1992). Both Civil (1994) and Henderson and Landesman (in press) raised as issues the limited scope of the mathematics learned in their approaches. This issue is related to our earlier discussion of the limited nature of contextualized mathematical practices. Such practices can provide a meaningful start for mathematics learning, but they may well not be sufficient. Using students' funds of cultural knowledge or building interdisciplinary units usually will need to be supplemented within a classroom in which mathematics is learning referentially by some more schoolish activities that provide practice on needed competencies and some problem situations that provide more generality to children's learning in the contextualized situation.

Practical problems in implementing classrooms in which mathematics is learned referentially

There are practical as well as theoretical problems in attempting to bring the real world into the classroom. Considerable social, organizational, and time constraints have combined to create the formal learning of formal concepts in traditional classrooms. It takes considerably less energy for a teacher to pass out worksheets and walk around helping individuals (or work at her desk) than to organize and monitor pair interactions or whole-class discussions. On some days in some classrooms a considerable number of children were not able to focus on the mathematical aspects of pair situations and degenerated into social play (this was more true in the English-speaking classrooms where we tried some buying/selling activities than in the classrooms discussed here). Keeping track of which concepts which child understands and does not understand and adjusting classroom activities to the learning zones of individual children is much more difficult and complex than giving written tests and assuming that the performance captures competence or understanding. It takes considerable time and energy of the teacher to gather, organize, and use the entities from the real world, and it involves some (and sometimes considerable) monetary cost. To do this many times throughout the year for many different topics is just not possible for teachers in inner-city schools. This is a major reason we looked for one single coherent real-world frame that could be carried through many different mathematical topics. Using this frame would minimize the gathering and organizing of materials and the establishment of routines by the teacher.

There are also practical problems involved in the design of such classrooms, a major one being the amount of time such design takes for both researchers and teachers. Designing our activities took a huge amount of time, and Henderson and Landesman (in press) report the same for their integrated curriculum approach. Civil in her implementation expressed concern that the "implementation did not always capture the mathematical potential (1994, p. 5)" and identified lack of time as a major reason. She also suggested that the early stage of the collaboration between teachers and researchers was a limiting factor. It takes a great deal of trust to communicate about, design, and modify classroom activities that function well. Also of concern is the necessity to plan as one goes along if one is truly basing the classroom activities on children's experiences and thinking as it evolves in the classroom. This almost inevitably means that one will miss opportunities for seeing and building mathematics arising out of unfolding units because such seeing takes time, experience, and opportunity for reflection, which is not necessarily available under the daily pressure of curriculum building and teaching. Simple limits of planning time, classroom teaching time, and curriculum-making time did not allow us to try nearly as many aspects of bying/selling situations as we considered worth trying. It is impossible to overestimate the amount of time it takes to build coherent systematic engaging learning experiences that adapt to children's unfolding mathematical thinking. Simply tracking this learning, and the learning zones of all children in a class, is a formidable task. This time limitation is a major reason to try to expend such energies on situations that many children experience and to try to build into the curricular materials easy ways for teachers to make local adaptations to fit children's experiences.

Conclusions

We conclude that buying/selling situations are an excellent sustained and sustainable setting within which children can construct a range of mathematical understandings. These situations were familiar to all children in our school, albeit with a wide range of understanding of the mathematical specificities involved in them. These buy/sell transactions and the use of money generate considerable enthusiasm in children. For children from cultures in which people engage in considerable amounts of informal buying, selling, and exchanging of goods, the classroom experiences were also culturally affirming. They allowed children to talk about their own experiences outside of school, thereby linking school and family life, and enabled children to use in school their knowledge constructed in these real-life experiences. Children who only had lived in Chicago also could relate to these buying/selling situations in school because we also used examples from local stores, and they could discuss their experiences in these stores. Using these buying/selling situations also conveyed the norm that we expected mathematics learning to be useful in and connected to daily life, not just something that took place during mathematics class.

We identified many different mathematical topics in grades 1 and 2 that could use buying/selling situations and money, and designed and implemented teaching/learning activities for these topics. These activities worked well as classroom activity structures. However, children's experience with buying/selling situations yielded for most children only general schemata about buying/selling and money that were not sufficiently detailed in their mathematical knowledge to enable children to function independently in buying/selling situations. Many children did not adequately understand change, and most could not initially make change. Most children did not have quantitative understandings of money and could not make quantities from different coins. They needed many experiences that would allow them to construct such understandings. Coordinating three different counts (by ones, by fives, and by tens) in order to make amounts with pennies, nickles, and dimes is a complex cognitive task requiring considerable support and practice for many children. If money is to be used to help children build quantitative notions of place value and of multidigit addition and subtraction, activities need to focus on making amounts with dimes and pennies and only later also use nickels and quarters.

We identified a difference between the goals of buying/selling situations in the real world and in our classroom and a consequent conflict in the nature of scaffolding of such missing knowledge. The economic goals in the real world lead to scaffolding by doing for, which does not contribute to our classroom goal of learning the missing knowledge. This classroom goal is accomplished more readily by scaffolding by doing with or helping to do. Such conflicts may be common when real-world practices are brought into the classroom as potential learning situations.

We proposed a third kind of concepts and learning to mediate Vygotsky's (1934/86) spontaneous and formal (scientific) concepts and learning: "referenced concepts" and "learning referentially." The focus of our classrooms was on children's conceptual mathematics. This required us continually to consider the references for concepts we were using in the classroom and to ascertain whether those references were meaningful for children. This focus on "referenced concepts" and on children "learning referentially" enabled us to design classroom versions of real-world practices that would focus on and support the mathematical features of those practices and to design versions of traditional classroom symbolic written work in which the symbols had meaningful referents for the children. This middle ground provided many specific links between spontaneous (but not well formulated mathematically) concepts from the real world and formal (but decontextualized and subject to erroneous interpretation) concepts from mathematics.

Finally, we briefly acknowledged that building a classroom in which children learn referentially is difficult and complex especially if one takes seriously the differences in the meanings children construct and the rates at which they do so. Because the costs of developing any real-world frame are considerable, it seems preferable for any such frame developed by one person or group to be one that is experienced widely and able to be adapted readily to local variations. This is the case with buying/selling experiences.

Overcoming the lack of knowledge of many children of the value of coins led to the invention and use of penny/dime strips that had ten pennies vertically on one side with a break between the top and bottom five pennies so that the ten could be seen at a glance. These pictures of pennies were the size of real pennies. On the back centered was a picture of a dime that was the actual size of a dime. Children used these strips with the penny sides showing to count by tens and to solve addition and subtraction problems using real pennies as the loose ones that did not make a ten. Later children used the dime sides with loose pennies. We also used penny/nickel strips with five pennies on one side and one nickel (actual size) on the other side. Two nickel strips matched visually one dime strip. These strips were very powerful for helping children build concepts of tens and fives and ones (e.g., see Fuson, Grandau, & Sugiyama, 2001; Fuson, Lo Cicero, Hudson, & Smith, 1997). But we also wanted a way for children to record their thinking with visual tens and ones as well as with place-value notation as they added and subtracted. So we then moved to having children draw through the ten pennies (and just a column of ten dots) to make one ten-stick. Children then used such drawings (e.g., 42 was 4 ten-sticks and 2 circles to show the ones) to make 2-digit numbers and add and subtract them (see Fuson, Smith, & Lo Cicero, 1997, 2002).

For more details about the penny/dime and penny/nickel strips in K, G1, G2 and the drawings of tens and ones (later extended to hundreds, and thousands), see:

Fuson, K. C., Lo Cicero, A., Hudson, K., & Smith, S. T. (1997). Snapshots across two years in the life of an urban Latino classroom. In Hiebert, J., Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P., Making sense: Teaching and learning mathematics with understanding (pp. 129-159). Portsmouth, NH: Hienemann.

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Fuson, K. C., Smith, S. T., & Lo Cicero, A. (2002). Supporting Latino first graders' ten-structured thinking in urban classrooms. In J. Sowder & B. Schapelle (Eds.), Lessons Learned from Research (pp. 155-162). Reston, VA: NCTM.

Also in this website menu click on Teaching Progressions and choose NBT1 and NBT2 to see the Teaching Progression of building and using place value concepts in single-digit and multidigit addition and subtraction.

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Table 1
Mathematical Concepts Set Within Discussion and Enactments of Buying/Selling Situations

Word problems (about buying/selling situations) [see Table 2]

  Money (buying /selling)

  names of coins[a], values of coins, values of bills,

  coin equivalencies, bill and coin equivalencies

Making quantities (buying with exact amount)

  counting by 1's to make amounts

  numbers 6, 7, 8, 9 as 5+1, 5+2, 5+3, 5+4 (nickel + pennies)

  counting by 1's, 10's, and 5's to make 2-digit numbers (coins or bills)

Single-digit addition (buying 2 or more entities)

  adding 2 numbers using pennies (or $1 bills) as counters

  ten-structured addition for 2 numbers ≥ 5 made with nickels (2 nickels = 10)

  adding 3 or more numbers using pennies or nickels and pennies

Single-digit subtraction (finding out how much money/how many goods you have left)

  numbers between 10 and 20 as one ten (dime) and some ones (pennies)

  subtracting using coins

Multidigit addition (buying 2 or more entities at multidigit prices or buying 2 or more lots of large quantities of packaged items)

  adding 2 2-digit numbers using coins or bills (1, 5, 10)

  adding 3 or more 2-digit numbers using coins or bills (1, 5, 10)

  adding 2 3-digit numbers using $1 and $5 bills and coins (1, 5, 10)

Multidigit subtraction (finding out how much money/how many goods you have left)

  subtracting 2 2-digit numbers using coins or bills (1, 5, 10)

  subtracting 2 3-digit numbers using $1 and $5 bills and coins (1, 5, 10)

Unknown addend problems (making change, how much more money do I need?)

  single-digit numbers with sums to 10, with sums to 18

  2-digit

  3-digit

Multiplication (buying 2 or more of the same item: total cost or total quantity)

  1-digit x 1-digit

  1-digit x 2-digit[b]

Inverse relationship between amount of money and quantity of goods (complementary interdependent buy/sell roles)

Commercial terms (used in some or all buy/sell activities)

  change, balance sheet, check, bank, buyer, seller, gain, loss

Measurement (constructing tiendita in classroom)

  metric measures

[a]Coin names for pennies, nickles, and dimes are not usually used by Spanish-speakers in our Chicago locale. The value is used as the name, e.g., un cinco (one five).

[b]The second graders also posed for themselves the problem 32x32.

Table 2

Mercado-Related Word Problems Used In First And Second Grade

Change-Add-To unknown result	Change-Add-To unknown change	Change-Add-To unknown start
Jessica tenía 7 caramelos y compró 2 caramelos más. ¿Cuántos caramelos tiene Jessica ahora?	Sara tenía 25 dólares en su caja. Edwin vino a la tienda y le compró algunos caramelos. Ahora Sara tiene 32 dólares en la caja. ¿Cuánto costaron los caramelos?	Isabel tiene una tienda de caramelos y le vendió algunos caramelos a Eliany por $7. Ahora Isabel tiene $18. ¿Cuántos dólares tenía Isabel antes de venderle los caramelos a Eliany?
Miguel had 4 dollars. Jolger paid 3 dollars for a milk carton. How many dollars does Miguel have now?	Eliany had 5 packets of ten candy and 7 loose ones and went to the store and bought some more candy. Now she has 2 packets of ten candy and 6 loose ones. How much candy does Eliany have now?	Pablo had some pencils and bought 9 more. Now Pablo has 16 pencils. How many pencils did Pablo have to start with?

Change-Take-From unknown result	Change-Take-From unknown change	Change-Take-From unknown start
César tenía 24 centavos y se compró un caramelo que costó 11 centavos. ¿Cuántos centavos tiene César ahora?	Sara compró dos docenas de huevos pero en el camino se le rompieron algunos huevos. Al llegar a la casa Sara sólo tenía 18 huevos. ¿Cuántos huevos se le rompieron a Sara en el camino?	Carlos tiene una tienda de pasteles. Lucía le compró algunos pasteles que costaron $29 y le pagó con un billete. Carlos le dió a Lucía $21 de cambio. ¿De cuántos dólares era el billete con el que pagó Lucía?
Doridalia had 32 dollars y went to the store and paid 13 dollars for some crayons. How many dollars does Doridalia have now?	Edwin has 45 pieces of candy in his store, and he sold some to Mario. Now Edwin has 27 pieces of candy in his store. How many pieces of candy did Mario buy?	Mitzi went to Roberto's store and bought 2 packets of ten peanuts and 8 loose ones from him. Now Roberto has 4 packets of ten peanuts and 7 loose ones. How many peanuts were there in Roberto's store before Mitzi bought her peanuts?

Put Together unknown Total	Put Together unknown part	Put Together unknown part
Miguel tiene 2 monedas de veinticinco centavos, 1 moneda de cinco centavos y 4 monedas de diez centavos. ¿Cuántos centavos tiene Miguel en total?	Quiero comprar un juguete que me cuesta doce dólares. Solamente tengo siete dólares. ¿Cuántos dólares más necesito?	Rodrigo fue a la tienda y compró una libreta y unas crayolas. La libreta le costó $16, pero no se acuerda de cuánto le costaron las crayolas. En total las libretas y las crayolas costaron $33. ¿Cuánto costaron las crayolas?
Mario bought 3 packets of ten colored pencils and 3 loose ones. Edwin bought 2 packets of ten colored pencils and 9 loose ones. How many pencils did they buy altogether?	If I want to buy a cake that costs $.82 and only have $.49, how many more cents do I need?	Anayeli has a flower shop. In her shop there are 13 roses and some carnations. Altogether there are 37 flowers in Anayeli's shop. How many carnations does Anayeli have in her shop?

Compare unknown difference	Compare consistent	Compare inconsistent
Sergio vende sus caramelos a 27 centavos la bolsa y Jennifer vende sus caramelos a 35 centavos la bolsa. ¿Cuántos centavos más cuesta una bolsa de caramelos en la tienda de Jennifer?	En la tienda de Yalitzia una bolsa de papas cuesta $1.75. En la tienda de Laura una bolsa de papitas cuesta 20 centavos más barata que en la tienda de Yalitzia. ¿Cuánto cuesta una bolsa de papas en la tiendá de Laura?	Yohana y Emiliano fueron a la tienda a comprar libretas. Yohana pagó $6 más que Emiliano por sus libretas. Si Yohana pagó $16, ¿cuánto pagó Emiliano por sus libretas?
Carlos went to the store to buy a can of beans. On the shelf there are two cans of the same size. The brown can costs $1.25, and the yellow can costs $1.42. How much money will Carlos save if he buys the brown can?	My friend and I went to the store to buy notebooks. My friend paid $.64 more than I did. If I paid $1.68, how much did my friend pay?	Rodrigo has 16 pieces of candy in his store. Rodrigo has 7 pieces of candy more than Sergio.How many pieces of candy does Sergio have?

Multiple numbers	Multiple steps
Isabel tiene una tienda de dulces. Primero vino Sara y le compró 28 dulces. Luego vino Mario y le compró 44 dulces. Más tarde llegó Lucía y le compró 27 dulces. ¿Cuántos dulces vendió Isabel en total?	Ayer Roberto tenía $43 en su banco. Hoy Roberto se compró un libro que costó $17 y unas crayolas que costaron $12. ¿Cuántos dólares tiene Roberto en su banco ahora?
Nancy sold 12 crayons to Pablo, 25 crayons to Jennifer, and 9 crayons to Sara. How many crayons did Nancy sell altogether?	Yalitzia had $50 in her bank, and she bought a doll that cost $23. Then Yalitzia sold the doll to her friend for $29. How many dollars does Yalitzia have in her bank now?

Extra information	Story with multiple questions
En la tienda de Eliany hay 27 caramelos de chocolate y 16 caramelos de limón. Su mamá le trajo 15 caramelos de chocolate más para su tiendita. ¿Cuántos caramelos de chocolate tiene Eliany en su tienda ahora?	Mitzi tenía $53 en su banco y fue a la tienda y compró una muñeca que costó $25. Ayer Mitzi le vendió la muñeca a Isabel por $32.
Alexis has $25 in his bank, and he bought some pencils for $6 and a notebook for $7. How many dollars did Alexis spend today?	¿Cuánto dinero tenía Mitzi en su banco después de que comprara la muñeca? (How much money did Mitzi have in her bank after buying the doll?)
	¿Cuánto dinero tenía Mitzi en su banco después de que le vendiera la muñeca a Isabel? (How much money did Mitzi have in her bank after selling the doll to Isabel?)
	¿Cuánto dinero ganó Mitzi? (How much money did Mitzi earn?)

Multiple steps (multiplication)	Story with multiple questions (ten-structured packaging language)
Mario compró 6 chocolates a 9 centavos cada chocolate y pagó con un dólar. ¿Cuántos centavos va a recibir Mario de cambio?	La tienda de dulces de Willmynette vendió tres paquetitos con diez dulces cada uno y siete dulces sueltos. La tienda de Gabriel vendió cinco paquetitos de diez [five packets of ten] y dos dulces sueltos [and two loose cakes].
Sergio bought 7 cakes at 30 cents each, and he gave the seller $5.00. How much money will Sergio get as change?	¿Quién vendió más dulces? (Who sold more cakes?)
	¿Cuántos más vendió? (How many more?)
	¿Cuántos dulces vendieron los dos? (How many cakes did they buy between the two of them?)

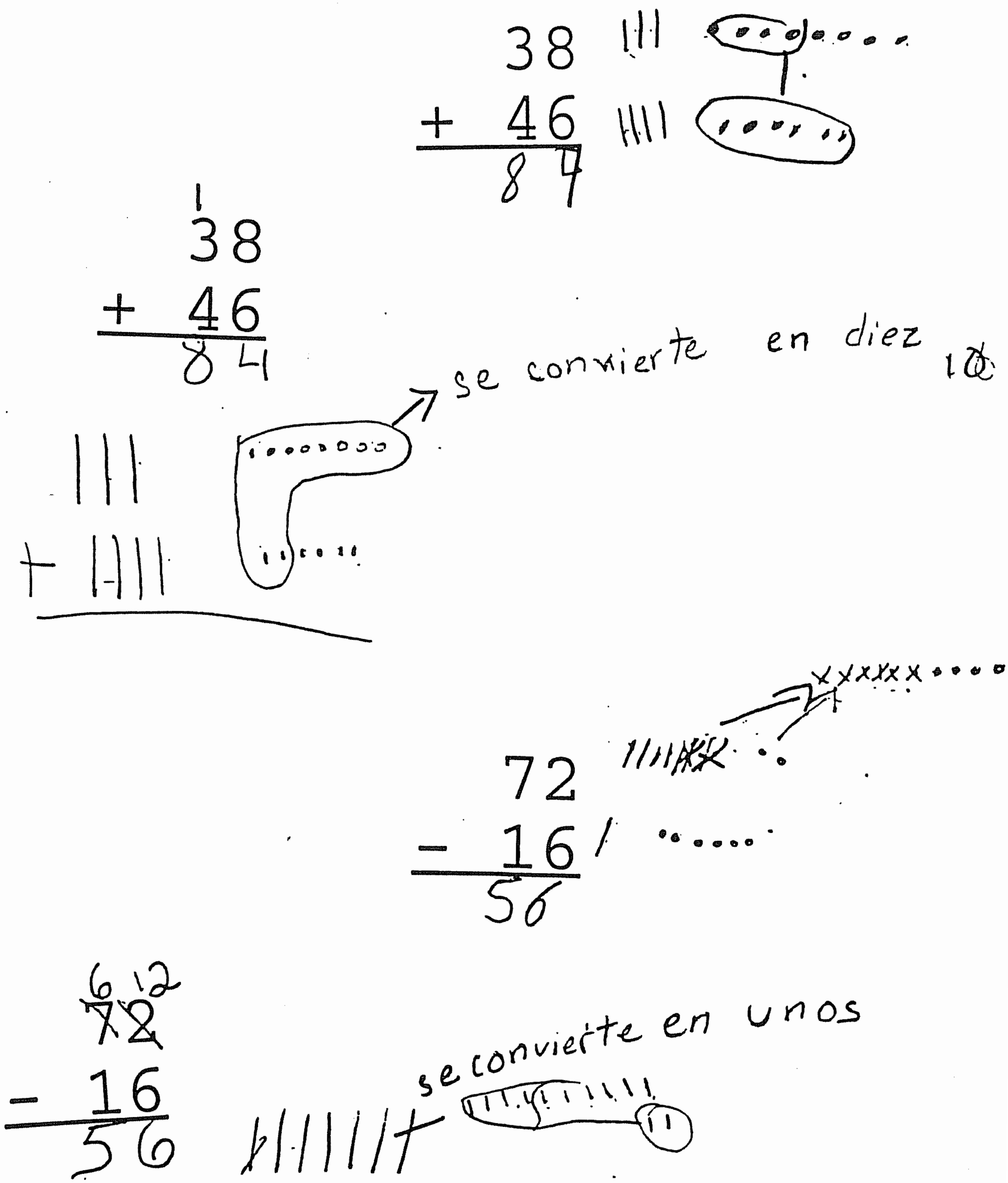

Figure 1: Children's ten-stick and dots solution methods for 2-digit addition and subtraction

Nombre ________________ Fecha __________

Producto: ________________

Cantidad	Precio cada uno	Precio Total	Balance

Nombre Sergio ________________

Fecha 11-3-93 ________________

2do grado

vendedor-a [Se] comprador-a [Se]

Descripción	Cant.	Precio		balance	
				dieces	unos
				5	5
		dieces	unos		
Javer	3	0	4	5	1
Marcadores	22		6	4	5
Plumas	2		2	4	3
miton	1		5	3	8
Foto	1	1	2	2	6
livreta	1		5	2	1

Figure 2: Mercado balance sheets

Formal Concepts	Learning Formally
Standard decontextualized mathematical symbols, words, procedures	Traditional decontextualized school mathematics practices Worksheets, symbols, rote procedures Meanings attended to briefly if at all
Referenced Concepts	**Learning Referentially**
Mathematics lifted from the real world but still connected to it; mathematical features of a real-world situation made salient Pedagogical supports (objects, symbols, words) may provide references Formal concepts linked to meaningful real-world or pedagogical referents	Classroom versions of real-world practices highlight and support mathematical features Goal is to learn meaningfully and be able to explain own understandings and methods Activities may focus on formal concepts but provide meaningful referents for these
Spontaneous Concepts	**Learning Spontaneously**
Mathematics embedded in the natural and cultural world	Experiencing mathematical concepts through individual and cultural practices

Figure 3: Spontaneous, Referenced, and Formal Concepts and Learning: An Extension of Vygotsky's Dyad