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Latino, Anglo, and Korean children's finger addition methods

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Understanding children's finger use developmentally

Educators in many countries view quite negatively children's use of fingers to add and subtract. They may attempt to eradicate such use by forbidding it, but conversations with teachers and researchers from many countries indicate that forbidding finger use only drives it 'under the desk'. Children still continue to use their fingers, but do so with their fingers hidden from the sight of the teacher who has forbidden such use. Children continue to use their fingers because they cannot present number situations to themselves without using concrete objects. Later they learn mental methods of adding and subtracting and do not need objects. But all children of all cultures need objects to show numbers for quite a long time. And most cultures have invented finger methods to show numbers and to add and subtract them.

Finger use can eventually be negative if children remain in their most primitive stage: direct modelling of the number situation by counting all the fingers or taking away fingers without using the ten structured nature of fingers. These primitive methods are fairly slow and eventually can slow children up. But all finger methods evolve into more rapid and effective methods, which are first done with fingers and later may evolve to being done mentally. Children can be supported to learn these more advanced methods within their own cultural pattern.

In one such series of instructional interventions, Anglo first graders in the United States were helped to move from counting all to counting on in addition. They also learned to count up to solve subtraction combinations. Counting on and counting up are more rapid and accurate finger methods (see later discussion and Table 1). Children in these studies learned a special one-handed method of showing numbers in which the index finger was pressed to a table to show one, the index finger and longest finger were pressed to show two, etcetera for three and four. The thumb alone was pressed to show five (and the other fingers were up in the air with the small finger at the top), and then the fingers for one, two, three, and four were pressed with the five to show six, seven, eight and nine. This method enabled children to use their non writing hand to keep track of how many they were counting on or counting up while they held their pencil in their writing hand. This method was therefore very rapid and children enjoyed doing it.

This approach was quite successful with first graders of all achievement levels. They learned to add and subtract combinations to 18 rapidly and accurately, and they were able to use their finger methods to solve multidigit addition and subtraction problems (Fuson, 1986; Fuson & Fuson, 1992: Fuson & Secada, 1986; Fuson & Willis, 1988). This developmental approach led to an acceleration of at least a year in children's single digit addition and subtraction over traditional methods of teaching in which children count objects or pictures of objects and then are supposed to memorize the number facts. Subtraction was as rapid and accurate as addition when it was done as counting up. Children ordinarily do subtraction as counting down, which is much more difficult than counting on (Baroody, 1984; Fuson, 1984). Most of the high achieving children went on to learn mental methods in second grade. More of the low achieving children continued to use their finger methods through the second grade year, but they were very proud to be able to be adding and subtracting such big numbers accurately.

Culturally different finger methods: Anglo, Latino, and Korean methods

Knowledge about children's informal methods of thinking can enable educational practice to begin with such ways of thinking and move on to more advanced ways of thinking. Understanding ways in which different cultures support children's thinking can illuminate aspects that may not be so clear from remaining embedded within the practices of a single culture. Such understanding can also directly influence educational practice in multicultural countries that educate children from many different cultures.

The objectives of this paper are to describe such cultural differences in the area of early addition thinking for children from three cultures: Korean children, English speaking children in the United States (Anglo children), and Spanish speaking children in the United States who have come from Mexico, Central America or South America (Latino children). Anglo children follow a well documented developmental sequence of addition solution procedures (see Fuson, 1992a; 1992b, for recent reviews of these methods). Korean children follow a different developmental sequence (Fuson & Kwon, 1991; 1992; 1992). These methods will be summarized and compared to a third developmental sequence of addition solution procedures exhibited by Latino urban first, second and third grade children.

These new Latino methods were initially identified in interviews of Latino children in a large urban school. Six second and third graders who used such solutions were then given monthly interviews throughout the school year; these results are summarized in Fuson and Perry (1993). Four first graders were added to the study in mid-year to examine the earliest levels in this new developmental sequence of addition methods. The teachers of these children were unaware of the finger methods used by the children, so they could not be the source of the methods. Some children explicitly said that they had learned their finger methods at home or at school in another country.

Children from these three cultures show numbers on their fingers differently. Most Anglo children begin with all fingers folded down and then raise (unfold) the index finger, raise the next three fingers in order (long, ring, small), and then raise the thumb. The Latino and Korean children begin by raising the thumb and move successively across to the little finger, or they begin with the little finger and move successively across to the thumb. Korean children may fold or unfold fingers to show numbers. Most of the Latino children raise fingers as they count, but a few begin with all fingers raised and fold fingers down as they count. This way to show fingers is more widespread among Latino children than are the particular methods of finger addition reported next. At least half of the Spanish speaking and English speaking children in our urban Latino school show numbers by beginning with the thumb or little finger.

The finger methods described here are based on two interviews with about 50 first and Latino second graders and some classroom observations of about 150 of their classmates, an interview with 36 Korean first graders (18 children from each of two schools), and multiple studies (by various researchers) of Anglo children in the United States in which the levels of solution strategies given in Table 1 were identified (see Fuson, 1992a; 1992b, for reviews of these studies).

Developmental sequences of addition, methods in the three cultures

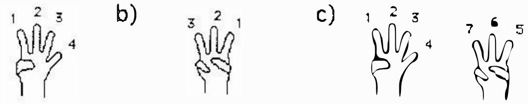
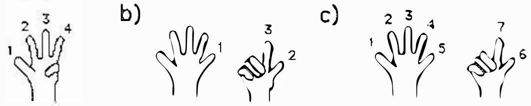
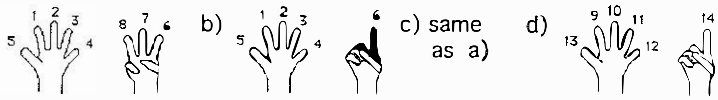
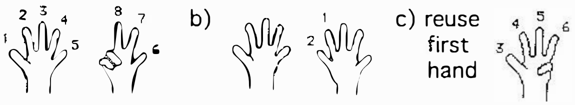
Level 1: Counting all

The solution sequences from all three cultures begin by counting all: The child makes fingers for one addend, then makes fingers for the other addend, and then counts all of these fingers. For addends of five and less, Anglo children show each addend on a separate hand. Latino and Korean children show the second addend by continuing with the adjacent fingers.

Each of these approaches has advantages and disadvantages. The Anglo method shows *each addend* more clearly, and addends of five and less can always be shown with a standard finger arrangement. But the sum will not be shown in the same way (4+1 looks quite different from 2+3, and these both look different from five fingers on one hand). The Latino and Korean continuous method does not show the addends very clearly (they become subsumed in the sum), and the second addend must be counted rather than using a known finger pattern. But this method shows *the sum* very clearly because a given sum always uses the same fingers. Children therefore can learn a pattern for a given number and not have to count all the fingers to find the sum.

If each addend is over five, children using the Anglo method must show fingers for the first addend and then reuse those fingers to show the second addend. Then fingers for each number must be put up again and counted all together to find the

Table 1: *Word sum and finger sum addition methods*

Method	Solution	Finger and word use
Word Sum	Count all under ten	a) b) c) 
Finger Sum	Add all under ten	a) b) c) 
Word Sum	Count all over ten	a) b) c) same as a) d) 
Finger Sum	Add all over ten	a) b) c) reuse first hand  d) recognize sum: 2 hands + 4 fingers = 14
Word Sum	Count on	The second hand of step c) in count all under ten; step d) in count all over ten; the last word tells the sum
Finger Sum	Add on	a) Put up fingers for first addend b) Step b) as in count all c) Recognize the number of fingers in the sum
Word Sum	Ten-structured	1) Use tens words in the counting on to separate the sum into ten and ones: "nine, ten, one ten and <u>one</u>, one ten and <u>two</u>, one ten and t<u>hree</u>, one ten and <u>four</u>" 14 2) Use two extra hands (of a friend or a drawing on paper) to make a) and b) in count all over ten (the extra hands are the fives in 6 and 8); the two fives make ten, and the 1 and 3 combine to make 4: ten four is 14 Eventually numbers over five could be abbreviated on each hand (e.g., as 1 and 3); these show the amount over ten and the two fives are invisible (the invisible fives method)
Finger	Ten-structured	The fingers structure the up-over-ten method in adding on over ten: they first show the amount needed to make ten, and then hold that amount while one reuses a hand to show the amount over ten (the rest of the second addend); children often move on to abbreviate the first addend by just showing it with the fingers over five, thus avoiding reusing fingers

<u>Note</u>. The word sum method is that previously identified in the research literature as used by English speaking children in the U.S. The finger sum method is the newly identified Latino method. Korean children show the finger sum method, but reuse fingers by folding down if they folded up originally. In all of these methods, children usually for a long time raise fingers successively while they count. Addends that can be learned as certain finger patterns (each addend in the word sum method, the first addend and the sum in the finger sum methods) eventually may be raised all at once or recognized without counting.

sum. If the sum is over ten, children using the Latino and Korean method must also reuse fingers over ten. It may take some considerable time for children to be able to generalize their finger method to problems over ten and reuse fingers to show such problems.

Level 2: Counting on or adding on

In the next more advanced level, children conceptually embed the addends within the sum and can consider a given finger or number word as being in an addend and a sum at the same time. In the earlier counting-all method, the addends and the sum had to be considered successively. Children at this embedded level can make conceptual advances with either embedded addend. Anglo children can make a count-cardinal transition (Fuson & Secada, 1986; Fuson, 1988) and abbreviate the counting of the first addend. In such counting on, they just count on as many words as are in the second addend (e.g., 4+3: 'four, five, six, seven'). When counting on with second addends that are very large, a child must make a conceptual advance for the second addend and invent some method of keeping track of the number words counted on. A common method is to use the fingers to keep track of the second addend. Fingers no longer show the first addend; they just are put up successively while the words count the sum.

For the Latino and Korean method, the roles of the fingers and number words are the opposite of those for Anglo method. In the Anglo counting on method, the words say the sum, and the fingers show the second addend. In the Latino and Korean methods, the fingers show the sum, and the words say the second addend. For example, to add 4+3, children use a known finger pattern to put up four fingers and then count aloud the second addend (say 'one, two, three') while putting up the next three fingers. The fingers then show the sum of seven fingers, which the child recognizes by the canonical pattern as a sort of 'Gestalt' or count.

When the sum is greater than ten, these Latino and Korean methods diverge. Korean children distinguish fingers above ten by doing the opposite of their ordinary way of using fingers. For example, for 8+6, if they unfold numbers to ten, they unfold eight all at once, unfold two more saying, 'one, two,' and then fold down four fingers saying, 'three, four, five, six.' The answer is the total number of fingers used: 'ten four' (ten fingers unfolded and four fingers folded). Korean children's number words for all two digit numbers state the ten in a regular way (12 is 'ten two' and 53 is 'five ten three').

The Latino children reuse their fingers over ten by raising them in the same way as they did for the numbers up to ten. A Latino child therefore has to remember that he had five whole fingers on a hand that now may be only partially raised. For example, after adding 8+6, a child will typically have nine fingers raised (five from the second hand raised and four from the four fingers over ten). Latino children might use someone else's fingers as the extra fingers before they can reuse their own fingers. Even then they may experience difficulty with sums over fifteen, which require reusing fingers on the second hand. Eventually Latino children anticipate when they will need to reuse fingers, and they abbreviate the first addend by not showing the first five fingers (e.g., they show eight by raising three fingers or six by raising one finger). The second hand can then be used for eleven through fifteen. At this level they can deal with sums over fifteen by reusing the first hand.

Some Latino children also did a combined Latino/Anglo method. They successively unfolded fingers while counting for the first addend and then continued on from that finger to show the second addend on the fingers. However, they counted on instead of counting the second addend from one (how they kept track of the second addend is not clear).

Anglo counting on is easily used for sums over ten, and the difficulty of finding sums over ten may serve as a pressure to invent counting on. The Latino method is more difficult than the Anglo method (because some method of reusing fingers must be found) and than the Korean method as well (because a reusing method is provided by the Korean culture). However, the Latino method is more concrete and related to counting all.

Level 3: Ten-structured potential of the methods

The Anglo solution methods are unitary; ten plays no special role in them. If English number words were ten-structured in a regular way, like Asian words, the number words would support children's splitting the second addend into the amount to make ten plus the amount over ten. For example, a Korean child using the Anglo counting-on method would say for 7+5: 'eight, nine, ten, ten one, ten two' while raising five fingers to count on the five more. The ten-structured words would help the child see that he is adding three to the seven to make ten and then adding on two more. This then leads to the mental ten-structured method of adding up over ten.

Unlike the unitary Anglo solutions, the Latino methods do have the potential to support children's solutions structured around ten. The fingers in the Latino method show the second addend being separated into the amount to make ten and the amount over ten. The fingers also show the sum as ten plus the amount over ten (8+6 is ten fingers plus four fingers). This ten-structured method can be seen on the fingers first and then later can be done mentally. This ten-structured method is powerful and general and is conceptually and procedurally helpful for multidigit computation.

This ten-structured method is taught to children in China, Japan, and Korea (Stigler, Fuson, Ham & Kim, 1986), but it is not generally taught in the United States. Both the Korean number words and the Korean finger methods support such ten-structured thinking. Many Korean first graders by the middle of the year could carry out such solutions mentally (Fuson & Kwon, 1992). This way of thinking of a sum between 10 and 18 is also very helpful for multidigit addition: the answer is already split into the ten that will get traded into the next column and the amount over ten that remains in the given column.

A child speaking a European language, all of which have many irregularities between ten and eighteen, does not even have to think of the actual sum (e.g., twelve). Thinking of the answer as 'ten and two' is actually much more helpful. In fact, it may also be helpful to introduce a regular tens words system into the classroom, adapted from the European language of the child. We have used such tens words with both Anglo and Latino children (83 is 'eight tens and three ones' or 'ocho dieces y tres unos') (Fuson & Fraivillig, 1993). They have supported children's construction of multiunit concepts of tens and ones, children's use of these multiunit concepts rather than their more primitive unitary conceptions that are suggested by number words in European languages.

When we began our research, many Latino second and third graders in our study classrooms were still using primitive count-all methods for finding single-digit sums in multidigit problems. They clearly need support to move through some developmental sequence, either the Latino or the Anglo sequence, to more conceptually advanced and efficient methods. It also seems worthwhile examining whether Anglo children can learn the Latino or Korean finger methods that could support their ten-structured thinking.

Teachers were generally unaware that their children were using the Anglo or Latino methods. Understanding these three finger methods can provide perspective to teachers with multicultural classrooms and enable them to examine other finger or mental methods children from other cultures may bring to their classrooms.

Finally, there is the important issue of whether using fingers is a 'crutch' that is helpful at first but later interferes with children's more mature mathematical problem solving. Many teachers in the United States feel strongly that any use of fingers is a crutch and will hurt children later, so they forbid children to use fingers in the classroom. Of course, what happens all over the country then is that children do not give up using their fingers. They simply 'go underground' and hide their fingers under their desks, thereby losing most of the time the visual aspects of finger solutions and retaining only the feel of the fingers. Unless the teacher provides some alternative way that children can present objects to themselves when adding and subtracting, this suppression of finger use actually can interfere with children's numerical thinking. Young children's early concepts of number and of addition and subtraction are as operations on collections of objects; children cannot think about numbers abstractly. When, instead, children are allowed to use their fingers in class *and* they are supported to use more advanced finger solution procedures, all children in the regular classroom do move on from the most primitive solution procedures to more rapid, accurate and sophisticated methods. Many children move on to rapid mental methods for most or all single digit sums and differences. This was true in our earlier one handed finger studies, and it is becoming true presently in our work with Latino children. All first graders of all achievement levels can move from counting all to counting on or adding on in a conceptual way, understanding that they do not have to count all of the items in the first addend but can begin the count with how many items there are in that addend (rather than doing a rote procedure of 'put the first number in your head' or some other meaningless count on procedure). Many second graders can move on to ten-structured or other thinking strategies (e.g., 6+7=13 because 6+6=12). Both counting/adding on and ten-structured and thinking strategies are rapid and accurate enough to be used in multidigit addition, subtraction, multiplication, and division. Some very low achieving children were still using counting on in grades 3 and 4, but this was not interfering with their more advanced mathematical problem solving. Thus, finger methods above the first level are sufficient for children to do more complex mathematical problem solving; these more advanced finger methods are not crutches that hold children back.

We are not suggesting that children do not learn any addition or subtraction combinations from memory or that fingers are the only solution methods allowed or supported in the classroom. Many children know certain combinations from memory even before they enter school (e.g., one plus one is two, two plus two is four). Almost all children seem to be able eventually to memorize most of the facts below ten so that fingers or other objects do not have to be used. But the combinations above ten seem to be more difficult to memorize, and these addition combinations interfere with and are interfered with by the later learning of multiplication facts. So we use fingers, objects, and dot drawings and patterns for sums and differences up to ten, and then move on with fingers and ten-structured objects and dot drawings for sums and differences between ten and eightteen, while children also practice learning the facts up to ten. It is also very important for teachers to help children move from primitive counting all to counting on or adding on and then move on to more advanced methods when individual children can handle such a move. Without such support, many children do stay at the first counting all level for many years of school. It is the developmental conceptual approach to children's use of fingers that is important and that keeps fingers from retarding children's mathematical progress.

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