Korean Children's Single-Digit Addition and Subtraction: Numbers Structured by Ten
Author(s): Karen C. Fuson and Youngshim Kwon
Source: *Journal for Research in Mathematics Education*, Vol. 23, No. 2 (Mar., 1992), pp. 148-165
Published by: National Council of Teachers of Mathematics
Stable URL: http://www.jstor.org/stable/749498
Accessed: 24-01-2017 21:23 UTC

Journal for Research in Mathematics Education
1992, Vol. 23, No. 2, 148–165

KOREAN CHILDREN'S SINGLE-DIGIT ADDITION AND SUBTRACTION: NUMBERS STRUCTURED BY TEN

KAREN C. FUSON, *Northwestern University*
YOUNGSHIM KWON, *Yonsei University*

Korean children's ability to solve addition problems with sums of 10, single-digit addition problems with sums between 10 and 18, and single-digit subtraction problems with minuends between 10 and 18 was assessed in interviews given at the end of the first semester of first grade, before children had studied problems involving numbers larger than 10 in school. These children showed considerable competence with all three kinds of problems, solving correctly 95%, 85%, and 75% of these problems, respectively. Almost two-thirds of the solutions of the problems above 10 were addition or subtraction recomposition methods structured around ten or known facts. Korean children demonstrated two finger methods that allowed fingers to be reused to show sums over 10. These methods and the regular named-ten Korean number words for numbers between 10 and 18 ("ten, ten one, ten two, ten three,..., ten eight") support Korean children's learning of three efficient recomposition methods structured around ten.

Korean children show considerably more competence in adding and subtracting single-digit and multidigit numbers than children in the United States (Song & Ginsburg, 1987), but we have little knowledge about the solution procedures used by Korean children. In particular, it is not clear whether Korean children follow the same developmental sequence of solution procedures constructed by children in the United States (Baroody & Ginsburg, 1986; Carpenter & Moser, 1984; Fuson, 1988a, in press-a, in press-b; Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983) and are just accelerated along this sequence or whether Korean children show a different sequence of solution procedures that is supported in their culture or schools.

One reason to think that Korean children might demonstrate different solution procedures is the structure of the Korean number words and the evidence that these words enable Korean children to think differently than children in the United States about numbers between 10 and 100. The Korean number words have a regular structure that tells how many tens and ones are in each number; in contrast, the English number words are irregular between 10 and 100 and do not clearly present these numbers as composed of tens and ones. Korean children actually must learn two counting sequences, an informal Korean system used for counting objects in the world and a formal system based on Chinese that is used in school and for all calculation. Both systems are structured similarly between ten and twenty: the numbers 11, 12, 13,..., 18, 19 are said as "ten one" "ten two" "ten three"..."ten eight" "ten nine" (see Table 1). The formal system also explicitly names the tens in the decades in a completely regular fashion (two ten, three ten, four ten,..., nine ten), whereas the informal system is more like English in that the decade words share varied amounts of phonetic similarity to the basic words for two, three, and so on, but do not clearly make these links or name ten. Song and Ginsburg (1988) reported that 3- and 4-year-old Korean children could count in both Korean systems about as high as their age-mates in the United States counted in English. Both groups counted less high than Chinese children using the Chinese regular named-ten decade words (Miller & Stigler, 1987). By kindergarten, Korean children counted more accurately in their formal system than English-speaking children in the United States.

The differences between the formal Korean system and English in the explicitness of one or more tens in a number word leads to a difference in how kindergarten and first-grade children speaking these two languages think about the numbers

Table 1
Korean Formal and Informal Systems of Number Words

	Formal Korean System		Informal Korean System	
Number in written marks	Korean	English translation	Korean	English translation
1	Il	One	Hana	One
2	Ee	Two	Dool	Two
3	Sahm	Three	Set	Three
4	Sah	Four	Net	Four
5	Oh	Five	Tasut	Five
6	Youk	Six	Yasut	Six
7	Chil	Seven	Ilgop	Seven
8	Pal	Eight	Yadul	Eight
9	Coo	Nine	Ahop	Nine
10	Ship	Ten	Yul	Ten
11	Ship il	Ten one	Yul hana	Ten one
12	Ship ee	Ten two	Yul dool	Ten two
13	Ship sahm	Ten three	Yul set	Ten three
14	Ship sah	Ten four	Yul net	Ten four
15	Ship oh	Ten five	Yul tasut	Ten five
16	Ship youk	Ten six	Yul yasut	Ten six
17	Ship chil	Ten seven	Yul ilgop	Ten seven
18	Ship pal	Ten eight	Yul yadul	Ten eight
19	Ship coo	Ten nine	Yul ahop	Ten nine
20	Ee ship	Two ten	Sumool	New(a)[two]
21	Ee ship il	Two ten one	Sumul hana	New(a)[two] one
29	Ee ship coo	Two ten nine	Sumul ahop	New(a)[two] nine
30	Sahm ship	Three ten	Sulheun	[Three ten]new(b)
31	Sahm ship il	Three ten one	Sulheun hana	[Three ten]new(b) one
40	Sah ship	Four ten	Maheun	New(c)new(b)
50	Oh ship	Five ten	Sheheun	New(d)new(b)
60	Youk ship	Six ten	Yesun	[Six]new(b)
70	Chil ship	Seven ten	Itheun	[Seven]new(b)
80	Pal ship	Eight ten	Yadeun	[Eight]new(b)
90	Coo ship	Nine ten	Aheun	[Nine]new(b)
100	Bak	Hundred	Bak	Hundred

Note. English translation words in [ ] mean that the Korean word is phonetically related to the Korean word translated by the English word within the [ ]. The new(a), new (b), new(c), and new(d) mean new phonetic elements not obviously related to previous Korean number words. The Korean spellings are the authors' efforts to communicate the sound of the Korean words; we departed from the Song and Ginsburg (1988) spellings in several places.

between 10 and 100. Miura, Kim, Chang, and Okamoto (1988) asked Korean and American first graders and kindergarteners to show numbers between 11 and 42 with base-ten blocks (the tens blocks were ten unit blocks long and were marked off to show the individual units). Korean children were much more likely than the United States children to show such numbers with the tens blocks and ones blocks, whereas United States children predominantly just counted out single blocks, even for the large numbers. Korean kindergarteners showed numbers in tens and ones more frequently than first graders in the United States did. Although the Korean kindergarteners initially preferred single block collections, when asked to show the numbers another way, every Korean kindergartener across trials showed all five numbers in both ways (as a unitary collection of single blocks and as tens and ones). Only 13% of the United States first graders showed all five numbers in both ways, and half of them never showed tens and ones for any of the numbers.

A second reason to think that Korean children might demonstrate different solution procedures is that they might have instructional support for addition and subtraction recomposition methods structured around ten. Children in mainland China, Japan, and Taiwan are taught three such recomposition methods in first grade (Fuson, Stigler, & Bartsch, 1988). Chinese and Japanese number words also show the regular named-ten structure displayed in Korean words; several Asian languages show this influence of regular named-ten Chinese words (Fuson & Kwon, 1991a). These Chinese and Japanese number words make these recomposition procedures easier because they explicitly name the ten and the ones. In the addition recomposition method *up-over-ten* one addend is recomposed into (a) the number that will make ten with the other addend and (b) the leftover number. For example, 8 + 6 is recomposed as 8 + 2 + 4 = 14 ("ten four" in Chinese and Japanese). In subtraction the *down-over-ten method* is the reverse of the up-over-ten method. The subtrahend (the number being subtracted) is decomposed into (a) one part that matches the part of the minuend that exceeds ten and (b) the rest, which is then subtracted from ten to give the answer. So for 14 – 6, the 6 is decomposed into 4 (in Chinese and Japanese "ten four – four" to get down to ten) and 2, which is subtracted from 10: 10 – 2 = 8. In the *take-from-ten method,* the whole subtrahend is taken from the ten, and this difference is added to the number that exceeded ten: 13 – 6 (in Chinese and Japanese "ten three – six") is calculated by subtracting 6 from 10 and adding this difference to 3. In English, one extra step is necessary for all of these methods: changing an English "teen" word into a ten and some ones (e.g., thinking of fourteen as ten and four ones).

To begin to explore these issues, the Korean textbook series was examined for the existence of the ten-structured recomposition methods, and middle-class Korean first graders were interviewed in the middle of the year. The Korean children were given sums to 10 to ascertain the extent to which they had learned one of the major prerequisites for the ten-structured methods, and they were given addition and subtraction single-digit problems with sums between 10 and 18.

METHOD

Middle-class first graders from two schools in Seoul, Korea, participated in the study. Six children were drawn at random from each of three classrooms in each school, for totals of 18 children from each school. Classrooms in these schools were large, with about 50 children in each class. Children were interviewed near the end of the first semester. Because Korean children must be 6 years old when they begin first grade, these children ranged in age from 6 years 4.5 months to 7 years 4.5 months at the time of the interview.

An examination of the textbook series used throughout Korea indicated that a different textbook is used each semester of the first grade. The textbook for the first semester contained only addition and subtraction combinations up through 10, and all three ten-structured methods (up-over-ten for addition, down-over-ten and take-from-ten for subtraction) were in the first half of the textbook for the second semester of first grade. Discussion with teachers in the two schools from which children were interviewed indicated that they did teach these methods (most Korean teachers do follow the textbooks). At the time of the interview, halfway through the year, they had completed the book for the first semester and had not begun the book for the second semester. Therefore, they had not yet discussed any of the ten-structured methods.

Children were interviewed in the school but outside the classroom by the second author, a native Korean speaker experienced in interviewing children. Each child was told that the interviewer was interested in how children thought about addition and subtraction. Children were first asked two addition combinations whose sum was ten (6 + 4 and 7 + 3) to determine the extent to which children immediately knew the sums to 10. They were then asked three addition combinations (8 + 5, 9 + 7, and 7 + 6). Finally, children were asked three subtraction combinations that were the reverse of the three addition combinations (13 – 5, 16 – 9, and 13 – 6). Each combination was presented in numerals on a card and asked orally as the card was presented. Sheets of paper and a pencil were visible and available to the child. If a long pause (20 seconds or more) occurred and a child did not exhibit any attempt at solving a combination, the interviewer suggested that the child draw a picture. If the child gave an answer without any indication of the solution method, the interviewer asked the child how he or she had found the answer. Children's verbatim responses and any visible solution method (uses of fingers, drawings on the paper) were recorded in Korean by the interviewer.

The recording sheets were translated into English by a Korean teacher living in the United States. Each child's response to each combination was classified by the first author using these translations and by the second author using the original Korean protocols. The anticipated solutions were the solutions that children in the United States use and the Korean methods structured around ten. New solutions identified in the classification process were the Korean folding and unfolding finger methods and other solutions using Korean finger patterns; these are described in the results. Each solution was given a descriptive code agreed on by the two coders. The categories for addition were as follows: *counting all* required that the child count objects presenting all of the sum (i.e., say the sequence of number words from one to the sum), *finger patterns* required that the child produce finger patterns for the addends without counting out each finger to form the pattern, *counting on* involved a count that began with one addend and continued on to the sum, *recomposition* involved breaking up one or more of the addends in order to form a sum related to the final sum, and a *known fact* required that the answer be given immediately (immediate responses had been coded as such in the interview) and that the child claimed that he or she knew or had learned the answer. Categories for subtraction procedures were *separate,* which involved making the sum objects and then marking out the known addend and then counting the remaining objects to find the unknown addend; *recomposition,* which involved down-over-ten or subtract-from-ten; and *known fact,* which used the same criterion as addition known facts. Coder agreement was 98% on the sums to ten, 94% on the sums over ten, and 98% on the differences. Coding disagreements were discussed, and the classification was mutually agreed upon.

RESULTS

Fingers Used to Present Numbers

We observed three ways in which Korean children used their fingers to show sums; in two of these methods fingers were reused in order to show a sum larger than 10. The three methods are described in this section, and their use on various problems is given in the next two sections. These finger methods were not used by many children because these children were so advanced, but we think they are important for understanding possible developmental sequences of solution procedures for Korean children. The protocols were sufficient to classify uses as falling within one of these three methods, but because we did not anticipate all the variations in the first method, the protocols were not detailed enough to distinguish among these variations on a trial-by-trial basis.

The first method, which we will call the *contiguous folding/unfolding method,* had several variations that share the same basic structure. The hands were held up facing the counter and the thumbs faced out. Counting began with the thumb or with the little finger. If counting began with the thumb, it moved linearly across all of the fingers to the little finger, continued on the other thumb, and moved across to the little finger on that hand (see Figure 1). Beginning with the little finger was just the reverse of beginning with the thumb: counting moved from the little finger across the fingers, in order, to the thumb and then moved from the little finger on the other hand to the thumb on that hand. Some children began on the left hand and moved to the right, and some began on the right hand. Children counted all by folding (folding down fingers as each count is made) or by unfolding (beginning with folded fingers like a fist and unfolding the fingers successively while counting). In the folding method of counting all, the first addend was counted as fingers were folded, the second addend was counted as the next fingers were folded (the second addend did not begin separately on the second hand but continued across the contiguous fingers), and then to find the sum all of the fingers were counted again (they

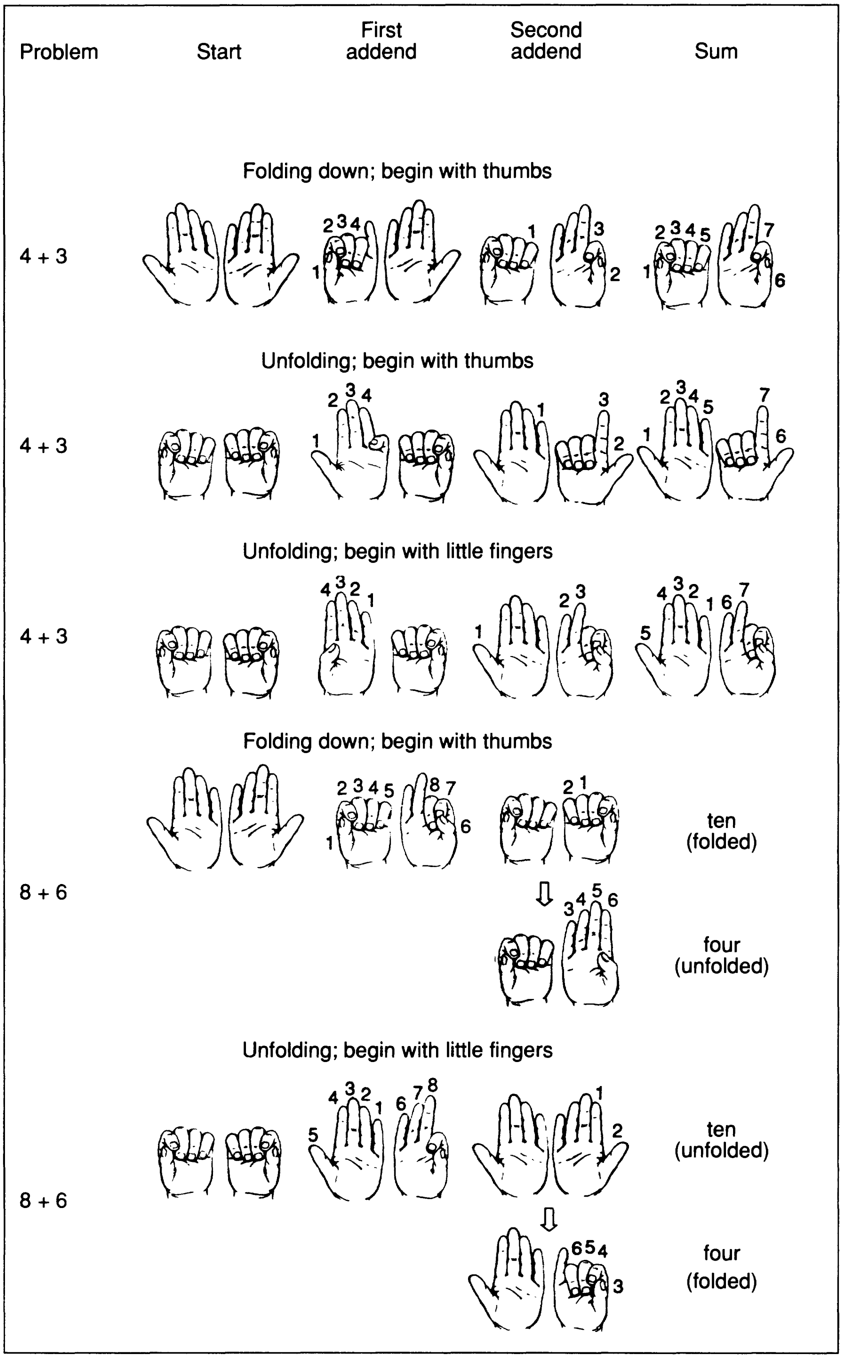

Figure 1. Korean children's addition with the contiguous folding/unfolding methods.

remained folded but were wiggled as they were counted in the sum count). In the unfolding method, fingers for the first addend were unfolded as they were counted, fingers for the second addend were unfolded as that addend was counted (again the unfolding continued across the ten fingers rather than starting on a separate hand), and the unfolded fingers were all counted again to find the sum.

The contiguous folding/unfolding methods of showing numbers have simple ways to reuse fingers to show numbers over 10: folded fingers are unfolded over ten, and unfolded fingers are folded again over 10. In the folding method, fingers are folded to make numbers up to 10 and then are unfolded again to show numbers over 10. In the unfolding method, fingers are unfolded to make numbers up to 10 and then are folded again to show numbers over 10. In both cases the undoing of fingers is usually done in the reverse order: in the folding method the last fingers folded are the first fingers to be unfolded, and in the unfolding method the last fingers unfolded are the first to be folded. In Korean number words the sum then is very easily said as "ten the-number-of-unfolded-fingers" in the folding method (ten fingers were folded and then some were unfolded) and as "ten the-number-of-folded-fingers" in the unfolding method (see Figure 1). These methods also support both of the prerequisites for the recomposition up-over-ten method (see Figure 1). After one addend has been made, the fingers clearly show the number needed to make ten: it is just the rest of the fingers. The fingers also can help in the splitting of the second addend into its two numbers: the fingers needed to make ten and the leftover amount of the second addend (the remaining second addend fingers). Using the fingers in the Korean way to add numbers whose sum exceeds 10 clearly structures that addition procedure in the up-over-ten method, visually asking for the number to make ten and then also producing the leftover rest of the second addend (and hence the answer) as the second addend fingers are counted out. These finger methods also can support both subtraction recomposition methods.

A different method of reusing fingers was shown by some children who used an *add fives* variant of the contiguous folding/unfolding methods. Here the child put each addend on a separate hand. The switch from folding to unfolding (and vice versa) occurred when one hand became filled (completely folded or unfolded to five). Thus, to make 7, a child who unfolded to five unfolded fingers would then fold down two fingers, and a child who folded to five folded fingers would unfold two fingers. Using this method allowed children to add the two fives (the completely folded or unfolded hands) to make ten and to add the visible unfolded (or folded) fingers to find the part over ten. For example, for 8 + 7, the two fives made ten, and the three (in 8) on one hand and the two (in 7) on the other hand make five, for an answer of ten five (15).

The third method of showing sums was to use the contiguous folding/unfolding method of showing numbers (beginning with the thumb or little finger) to show the second addend when counting on. For example, for 8 + 6, the child would say "eight," and then count "nine, ten, ten one, ten two, ten three, ten four" while folding (or unfolding) the thumb through little finger on one hand and the thumb on the second hand.

Addition solution procedures

Children solved 95% of the sums to ten correctly. The solution procedures used are listed in Table 2 in developmental order from primitive to advanced. Almost three-fourths of the sums to ten were known facts. Of the remaining solutions about half involved finger patterns or counting on, and about half involved a procedure that was not clear to the observer and was not explained adequately by the child. Most children used the same solution procedure to find both sums to ten.

Table 2
*Percent of Use of Addition Solution Procedures by Korean First Graders (*N = *36) at Midyear*

Solution procedures	Sums to ten	Sums exceeding ten
"I don't know" or no attempt	—	2
Count all		
Drawing on paper	—	2
Fingers	—	2
Finger patterns	5	—
Count on		
No visible keeping track of second addend	8	10
Using Korean folding/unfolding fingers to keep track of second addend	1	5
Recomposition		
Up-over-ten		
Mental	—	39
Using contiguous folded/unfolded fingers	—	6
Add-fives and then add amounts over five		
Mental	—	2
Using folded/unfolded fingers on each hand	—	2
Related addition fact	1	2
Known fact	71	11
Procedure unclear	13	16

Children gave correct answers to 87% of the attempted combinations with sums over 10. Most of the errors were made with the more primitive solution methods or the procedures that were unclear. About half of the children used the same solution procedure across all three combinations. Three of the 36 children used procedures that ranged from counting all with fingers to a recomposition method or a known fact, a wider range than typically shown by children in the United States. The rest of the children displayed solutions that were at most two apart in the order shown in Table 2 (the procedure-unclear solutions are not within the developmental sequence).

The count-all solutions were used rarely, and even counting on from one addend was only used on 15% of the combinations. The most common solutions were recomposition solutions, and most of these solutions were mental recompositions. The descriptions of the up-over-ten method were quite clear, though they varied in their completeness. Representatives across this range for solving 7 + 6 are the following:

Ten three. Six is from three plus three. Add three to seven. Make ten. Add the remainder three to ten, and it makes ten three.

Ten three. If I take four from seven, six will be ten, and seven will be three.

Ten three. In order to make seven to be ten, three is needed. In six, there are two threes. So add the remainder three to ten.

Ten three. Six is from three plus three. So the answer is ten three.

Ten three. Seven plus three plus three is ten three.

Ten three. After adding three to seven, I added three more.

Ten three. Seven plus three equals ten. Three remains. So ten three.

Although only a few of the children used folded/unfolded fingers to show the recomposition methods, two kinds of evidence hint that children using mental recomposition might have used fingers to recompose at one time. First, the descriptions of the procedure seemed similar for children using fingers (the last two examples above) and those not using fingers (the rest of the descriptions), except that some of the former were more complete explanations, as might be expected by more advanced children. Second, three children volunteered that they used fingers in their head to find an up-over-ten answer.

Subtraction solution procedures

The solution procedures used for the three subtraction combinations with minuends between 10 and 18 are given in Table 3. On attempted combinations, children gave about the same proportion of wrong answers as they had in addition (85% in subtraction and 87% in addition). But because children refused to attempt more subtraction than addition combinations (10% versus 2%), fewer correct answers were obtained in subtraction than in addition (75% versus 85%). About half the children used only a single solution procedure across all three subtraction problems, a sixth used both recomposition methods, and all but two of the remaining children used methods within one step in Table 3.

Table 3
Percent of Use of Subtraction Solution Procedures by Korean First Graders (N = 36) at Midyear For Minuends Exceeding Ten

Solution procedures	Percent
"I don't know" or no attempt	10
Separate	
Drawing on paper	12
Recomposition	
Down-over-ten	
Mental	38
Using folded/unfolded fingers	4
Subtract-from-ten	
Mental	14
Using folded/unfolded fingers	4
Known fact	4
Procedure unclear	15

Sixty percent of the solutions involved some form of the subtraction recomposition procedures that were to be taught in the second semester. Twice as many of these solutions were down-over-ten as were subtract-from-ten procedures. A few children carried out these solutions using Korean folding/unfolding fingers, but most of the solutions were done mentally and then described to the interviewer. As with the up-over-ten procedure for addition, the fullness of these descriptions varied across children. Representative examples for 13 – 6 are the following:

Down over ten

Seven. The six has three more than the three. Ten three minus three is equal to ten. Then take away three again. That leaves seven.

Seven. After taking three, take three from ten again. It will be seven.

Seven. Six is from three plus three. Take three, ten remains. Ten take away three again. It becomes seven.

Seven. Three from ten three and three from ten is six. So seven is left after six was taken away.

Subtract from ten

Seven. Ten take away six leaves four. And there is three more. Add together, three and four make seven.

Seven. I take six from ten. And I add four to three.

Seven. If I take six from ten, four remains. Then adding three makes it seven.

DISCUSSION

Korean children's solution procedures

The rapid and accurate responses on the sums to ten demonstrated that most of the Korean first graders at midyear had the major prerequisite for the ten-structured methods: they knew the sums to 10 or could find them by some efficient method. Furthermore, these Korean first graders in midyear showed remarkable competence at solving the more difficult single-digit addition and subtraction combinations with sums between 10 and 18. Almost two-thirds of their solutions were advanced solutions that involved known facts or recomposition methods structured around ten, and addition and subtraction had similar combined percentages for these two solution categories. The fact that addition and subtraction for sums between 10 and 18 had not yet been discussed in school raises the question of how these children learned these advanced solutions. Several children spontaneously mentioned working at home on workbooks or with parents, and some children mentioned learning at a special school they attended after regular school. We did not question all the children about where they learned the recomposition methods or whether they figured (or perhaps fingered) them out themselves because we did not expect so many to know them already.

Ascertaining the developmental sequences actually followed by Korean children, and how these sequences are supported by the culture, would seem to be a worthwhile endeavor, given the precocity demonstrated by the Korean first graders in this sample. Three possible sequences were suggested by our data. One possible sequence (see Fuson & Kwon, 1991b, for a more complete discussion) uses the Korean folding/unfolding finger methods. In this sequence Korean children begin adding by counting all using the Korean folding/unfolding methods. Because the second addend is made on the fingers immediately next to and following the fingers used to make the first addend, particular finger patterns can with experience become associated with particular numbers for the first addend and for the sum. These associations enable children to carry out a more advanced solution procedure in which they fold (or unfold) the fingers for the first addend all at one time, fold (or unfold) while counting each of the fingers for the second addend, and then recognize the pattern of fingers that represent the sum. Several children used such pattern methods to find the combinations that made ten. Ten is a particularly easy sum to recognize, so this finger path is very supportive of this prerequisite of the ten-structured methods. When the sum is over ten, use of this finger-pattern method supports the up-over-ten method because the counting of the second addend is visually broken into two parts by the using up of the ten fingers (for 8 + 6, count "one, two") and the making of the fingers over ten (count "three, four, five, six"). Repeated use may enable children to use a method in which the finger pattern for the first addend is made, the second addend is made by two patterns of fingers that complete the ten fingers and then show the rest of the second addend, and the sum is then recognized as "ten four." Either of these methods might be able to be done with mental visual images instead of real fingers, as suggested by the children who said they did the ten-structured methods using fingers in their heads.

A second developmental sequence that may be followed by some Korean children is the developmental sequence invented by many children in the United States in which counting on follows counting all (Baroody & Ginsburg, 1986; Carpenter & Moser, 1984; Fuson, 1988a, in press-a, in press-b; Steffe & Cobb, 1988; Steffe et al., 1983). Counting on is opposite to the contiguous folding/unfolding methods in the first developmental sequence, because in those methods the words say each addend and the fingers show the sum but in counting on the words say the sum (e.g., "eight, nine,..., ten four") and the fingers show the second addend (they keep track of how many second-addend words are said so that the words can stop when the second addend has been made). Some children in this study did use Korean folded/unfolded fingers to keep track of the second addend as they counted on. However, other Korean children did not use a visible method of keeping track while they were counting on. Because the Korean number words count how many fingers there are over ten (the four remaining of the six) while simultaneously counting the sum as "ten four," it is not clear whether these children were counting on or whether they might have been using fingers as in the first developmental sequence and just seeing or feeling the two fingers to make ten as they said "nine, ten" and then counted the four more fingers to make six ("ten one, ten two, ten three, ten four").

Korean children used procedures that suggest a third possible sequence for addition. They structured each addend between 5 and 10 as five and the leftover; the fives were then combined to make ten and the two leftovers were combined into the part of the sum over ten. Again, this method is easier with Korean number words than English number words. For example, for 6 + 7, the leftovers one and two just equal the part over ten in the sum—the three in the sum "ten three." This method is like the common United States method in that each addend is put on a separate hand, but this method of reusing fingers by folding and then unfolding (or vice versa) on each hand enables any single-digit addend to be put on one hand and then the fives can be combined into a ten.

The possible developmental sequences of subtraction solution procedures are not as clear as the addition sequences in our data. Children used either a primitive object-separating solution that paralleled counting all for addition or an advanced recomposition or known-fact solution. Intermediate subtraction methods might have been uncovered if we had asked children who used primitive methods to count or to use their fingers and if we had suggested these methods instead of suggesting drawing on paper. At the very least, such intermediate subtraction methods are not as readily available as they are for addition because some children did spontaneously show intermediate addition methods for all three developmental sequences: they used their fingers to show the up-over-ten method, counted on, and used their fingers to show the add-fives method. Subtraction counterparts for these addition intermediate methods would be to use folded/unfolded fingers to show the down-over-ten or the subtract-from-ten method, count down and/or count up, and use fingers to make the add-fives number that will make the known sum with the known addend (e.g., for 13 – 7, make 7 on one hand and then make on the other hand the fingers that will make 3 with the 2 fingers from the 7). For the first and third paths, it may be that the addition recomposition methods precede the subtraction methods, and that by the time children do the subtraction methods they do not need to use fingers but can think through the recomposition mentally.

Solution procedures and cultural supports in the United States

Children in the United States lack the cultural supports for ten-structured methods identified above for Korean children. First, textbooks do not support the ten-structured methods or the prerequisites for them (Fuson, et al., 1988; Fuson, in press-a). Textbooks in the United States also present sums and differences to 18 later than Korean textbooks. Many United States textbooks published in the mid-1980's did not even include all sums and differences to 18 in the first-grade textbook (Fuson, et al., 1988), and the next wave of textbooks often included these combinations but only in the last chapter of the text (Fuson, in press-a). In the first half of the Korean first-grade, second-semester text, combinations to ten are presented and practiced in several different ways, and then all three ten-structured methods are introduced. Second, the English words do not name the ten and the ones in numbers between 10 and 20. Therefore this ten is not available in a sum to suggest a ten-structured method or to provide the actual recomposition that is used in the ten-structured method. Instead, English words require that a child learn the decomposition into ten and some ones for each number word between ten and eighteen (e.g., twelve is ten plus two) and make this decomposition as an extra step when using the ten-structured methods. Third, there is not a culturally supported way to reuse fingers to show numbers larger than 10 when finding sums and differences to 18.

Although some children in the United States do invent interesting ways to use fingers to show sums over ten, a considerable number of second graders do not solve this problem successfully (e.g., Steinberg, 1984, 1985).

There also seem to be some cultural supports in the United States that propel children along a developmental sequence of unitary conceptions of numbers in which ten plays no special role and numbers over ten are not composed of a ten and some ones. The current research literature about children in the United States describes only the second developmental sequence discussed above for Korean children: children in the United States move from (a) counting all to (b) counting on for addition and counting down and counting up for subtraction to (c) derived facts (relating a given fact to a known fact by decomposition of one or both addends) and finally to (d) known facts (Baroody & Ginsburg, 1986; Carpenter & Moser, 1984; Fuson, 1988a, in press-a, in press-b; Steinberg, 1984, 1985; Steffe & Cobb, 1988; Steffe et al., 1983). All of these solution procedures, except for the derived-fact solution up-over-ten that some United States children do use, involve only unitary conceptions of numbers in which numbers over ten are just a collection of that many units. The English number words support this unitary developmental sequence by their failure to name explicitly the ten and the ones in numbers between 10 and 20.

This developmental sequence may also be supported by the common way in which children in the United States show addition with their fingers. Few researchers have focused explicitly on the ways in which children present addends on their fingers, but an examination of the drawings researchers present and their descriptions of finger solution procedures do lead to a coherent picture of finger use (e.g., Baroody, 1987; Siegler & Shrager, 1984). Children in the United States commonly show numbers by raising in succession contiguous fingers from the index (pointing) finger to the little finger and then raising the thumb. With small addends of five or less, each addend is shown on a separate hand. For example, to show 4 + 3, four fingers are raised on one hand and three fingers are raised on the other hand, and then all of the fingers are counted. This method of showing addends is very clear for small numbers, but it makes addition and subtraction of sums over ten difficult because it takes two hands to show one addend over five and there are not two more hands to show the other large addend. This difficulty is solved by counting on. The child just begins the final sum count with the first-addend number, and the fingers then may easily be used to show the second addend in order to keep track of the second-addend words counted on, up, or down. Thus, for 8 + 6, the child will just say, "eight, nine, ten, eleven, twelve, thirteen, fourteen" while extending six fingers in correspondence with the second-addend words nine through fourteen. When counting on, neither the fingers nor the irregular English number words for the teens signal that something special is happening at ten, so the child has no special support for inventing ten-structured methods. In contrast, both the Korean words and the Korean folded/unfolded fingers signal that a repetition is occurring at ten, and both of these support the decomposition of the second addend into the part to make ten and the part over ten that is the sum word.

The way in which addends are presented on fingers can affect the developmental sequence of solution procedures invented by children. The identification of different developmental sequences through single-digit addition and subtraction for Korean children raises the possibility that some children in a multicultural country such as the United States experience conflict between the method of using fingers at home and the method they may learn at school (from other children, even if not from the teacher) or between their native number-word sequence and the English number words they use at school. Research on ways in which children from different cultural subgroups within the United States use fingers would be helpful in assessing this potential interference problem.

Implications for instruction in the United States

It is clear that using fingers or other materials in particular ways can support and direct children's understanding of addition and subtraction along different paths. We have at present few data that suggest which are the best methods to use with English-speaking children in the United States. One alternative is to support rather than suppress and force underground the typical United States counting-on developmental sequence. This approach leads to considerably accelerated learning of addition (Fuson & Secada, 1986; Fuson, 1987; Fuson & Fuson, in press). Because children find it so much more difficult to count backward than forward (Baroody, 1984; Fuson, Richards, & Briars, 1982), supporting children's learning of subtraction as counting up (rather than as counting down, which results from having only a take-away meaning for subtraction) also leads to considerable acceleration in subtraction (Fuson, 1986b; Fuson, 1988b; Fuson & Willis, 1988; Fuson & Fuson, in press). Children following this alternative were able to solve all sums and differences to 18 by the end of first grade, and their solutions were rapid and efficient enough to be used in four-digit addition and subtraction in second grade (Fuson & Briars, 1990).

A second alternative is to support ten-structured methods in the classroom. The only research that has done this to date has included the up-over-ten method as one of several thinking or derived fact strategies. This research has provided evidence concerning when children in the United States learn the prerequisites for the up-over-ten method; it indicates that under usual school instruction children in the United States are considerably delayed compared to Korean children in learning these prerequisites. In the present study we found that most first graders by the middle of the year immediately recalled combinations that summed to 10. Steinberg (1985) reported that half of her middle-class second-grade sample initially did not recall such combinations; instead, for example, given seven, they had to count to find out that three more made ten. For the other prerequisite that is necessary in English but not with Korean number words—knowing the teens words as composed of a ten and some ones—more than a third of these second graders had to count to find the English word for such sums (e.g., they said "ten, eleven, twelve" to find that twelve is "ten plus two" or counted four past ten to find that "ten plus four" is fourteen). Few of these second graders used any of the ten-structured methods at the beginning of the study.

In the instructional studies supporting ten-structured methods, Thornton (1978; 1990; Thornton & Smith, 1988) reported that teaching a range of thinking strategy (including counting on small numbers) and derived fact procedures resulted in better addition and subtraction performance than ordinary school textbook instruction did. These studies were with first graders and used a sequencing of number combinations by solution strategy in which most combinations benefiting from ten-structured methods were last. Most children did not reach these combinations by the end of first grade, so it is not clear how easily they could have learned them. Steinberg's (1984; 1985) study used Thornton's materials and concentrated on derived-fact procedures with middle-class second graders. Most of them did learn the up-over-ten method, but they used it on only 22% to 41% of their derived-fact solutions over various interviews rather than using it for all problems as our Korean first graders did (the U.S. second graders used doubles and other related fact solutions instead of just using the up-over-ten method). Some children in Steinberg's study preferred to count on rather than switch to up-over-ten; they had counted on rapidly and accurately at the beginning of the study and could do so within the time limit of the 2-second "recall" test on the final interview. A pilot study had indicated that children had considerable difficulty learning the down-over-ten method but they instead spontaneously used on subtraction problems an unknown-addend up-over-ten method: for 14 – 8, "eight plus how many to make ten (two) plus how many more (four) to fourteen is six." Therefore this method was taught instead; it was learned and used by some children. A couple of children did invent the down-over-ten and the subtract-from-ten methods and used one of these fairly consistently. Most of the down-over-ten solutions involved some counting down, so they were really a combination of the Korean method and unitary counting down. These studies are not the best test of teaching the ten-structured methods because they emphasized a range of solution procedures, leaving relatively little time for the ten-structured methods. Studies that focus directly on these methods, and perhaps explore differences among the three subtraction ten-structured methods (unknown-addend-up-over-ten, subtract-from-ten, down-over-ten), would provide a better test of how ten-structured methods may be acquired by children in the United States.

The use of ten-structured words for two-digit numbers and of the Korean finger methods to support children's learning of ten-structured methods might also be evaluated. However, the contiguous folding/unfolding methods may be problematic for some children in the United States who are already into the common counting-on developmental sequence, because these two approaches use fingers and words in opposite ways. The ten-structured words could be helpful in any of the developmental sequences. In the common counting-on path, these words enable counting on to be abbreviated into the up-over-ten method, because the ten separates the second addend into the part to make ten and the part over ten and counts this second part as it counts the sum. As discussed above, the tens words also support the other two Korean developmental sequences. The "tens words" ("ten, one ten and one, one ten and two, ..., one ten and nine") could be introduced and used as words that tell the meaning of the English words. Having two explicit linguistic structures, one the usual unitary English words already known by most children entering kindergarten and the other the ten-structured words that support thinking of the written two-digit marks as tens and ones, might facilitate children's invention and use of ten-structured methods and also provide them with words that would clarify their discussions of their adding and subtracting methods.

Making the decision about which methods to support is complicated by the fact that children need to use single-digit sums and differences within other more complicated computations involving multidigit numbers. Single-digit addition and subtraction methods structured around ten are easy to use and efficient in working with multidigit numbers, and they eliminate the need for children to translate between their unitary answer for a given column and a ten-structured conception in order to understand regrouping (Fuson, 1990a). Children in the United States who do not have tens words must make a translation between these two conceptions. For example, in an earlier study (Fuson, 1986a) children found a unitary single-digit sum for a column, wrote it out at the side, and then looked at it to see the ten and how many ones (e.g., wrote 12 and then "saw" the 1 ten and 2 ones). Eventually, this translation becomes very rapid for many children, but initially it may interfere with conceptual understanding of the trading required in multidigit problems. Use of the tens words might facilitate this understanding and make multidigit addition and subtraction as easy as it is for Korean children, who do not have to make such a translation (Fuson & Kwon, in press). Having the tens words as supports may be particularly important for children who are inventing their own methods of multidigit addition and subtraction (cf. Fuson, Fraivillig, & Burghardt, in press). However, the ten-structured methods are not necessary for children in the United States to understand multidigit addition and subtraction. If they have conceptual supports for constructing ten-structured conceptions of the multiunits in multidigit numbers, children can use unitary counting-on or counting-up methods and still demonstrate accurate multidigit addition and subtraction and conceptual understanding of trading (Fuson, 1986a; Fuson & Briars, 1990). Thus, the first alternative proposed here, supporting the usual unitary methods invented by children in the United States, may be perfectly adequate and more culturally appropriate. It still remains to be seen how easily children in the United States can learn ten-structured methods and how these methods can support multiunit thinking (see Baroody, 1990, and Fuson, 1990b, for further discussion of the alternatives described here).

Solution procedures are always supported by the culture within the classroom or by the larger culture outside the classroom; they never are invented by a child in a cultural or experiential vacuum. We need seriously to address questions concerning which methods should be supported rather than accepting the methods that are presently invented as the "natural" child methods. The methods invented by children only arise naturally from the children's experiences. Understanding alternative methods that are supported in other cultures can help us examine our own cultural practices more closely and perhaps even suggest methods that might be worth supporting in our own culture.

REFERENCES

Baroody, A. J. (1984). Children's difficulties in subtraction: Some causes and questions. *Journal for Research in Mathematics Education, 15,* 203–213.

Baroody, A. J. (1987). The development of counting strategies for single-digit addition. *Journal for Research in Mathematics Education, 18,* 141–157.

Baroody, A. J. (1990). How and when should place-value concepts and skills be taught? *Journal for Research in Mathematics Education, 21,* 281–286.

Baroody, A. J., & Ginsburg, H. P. (1986). The relationship between initial meaningful and mechanical knowledge of arithmetic. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 75–112). Hillsdale, NJ: Erlbaum Associates.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15,* 179–202.

Fuson, K. C. (1986a). Roles of representation and verbalization in the teaching of multi-digit addition and subtraction. *European Journal of Psychology of Education, 1,* 35–56.

Fuson, K. C. (1986b). Teaching children to subtract by counting up. *Journal for Research in Mathematics Education, 17,* 172–189.

Fuson, K. C. (1987). Research into practice: Adding by counting on with one-handed finger patterns. *Arithmetic Teacher, 35*(1), 38–41.

Fuson, K. C. (1988a). *Children's counting and concepts of number.* New York: Springer-Verlag.

Fuson, K. C. (1988b). Research into practice: Subtracting by counting up with one-handed finger patterns. *Arithmetic Teacher, 35*(5), 29–31.

Fuson, K. C. (1990a). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction and place-value. *Cognition and Instruction, 7,* 343–403.

Fuson, K. C. (1990b). Issues in place-value and multidigit addition and subtraction learning and teaching. *Journal for Research in Mathematics Education, 21,* 273–280.

Fuson, K. C. (in press-a). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. T. Putnam, & R. A. Hattrup (Eds.), *The analysis of arithmetic for mathematics teaching.* Hillsdale, NJ: Erlbaum.

Fuson, K. C. (in press-b). Research on whole number addition and subtraction. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning.* New York: Macmillan.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education, 21,* 180–206.

Fuson, K. C., Fraivillig, J., & Burghardt, B. (in press). Relationships children construct among English number words, multiunit base-ten blocks, and written multidigit addition. In J. Campbell (Ed.), *The nature and origins of mathematical skills.* North Holland: Elsevier Science.

Fuson, K. C., & Fuson, A. (in press). Instruction supporting children's counting on for addition and counting up for subtraction. *Journal for Research in Mathematics Education.*

Fuson, K. C., & Kwon, Y. (in press). Korean children's understanding of multidigit addition and subtraction. *Child Development.*

Fuson, K. C., & Kwon, Y. (1991a). Chinese-based regular and European irregular systems of number words: The disadvantages for English-speaking children. In K. Durkin & B. Shire (Eds.), *Language and mathematical education,* (pp. 211–226). Milton Keynes, GB: Open University Press.

Fuson, K. C., & Kwon, Y. (1991b). Learning addition and subtraction: Effects of number words and other cultural tools. (Systemes de mots-nombres et autres outils culturels: Effets surles premiers calculs de l'enfant.) In J. Bideaud, C. Meljac, & J. P. Fischer (Eds.), *Pathways to number (Les chemins de nombre)* (pp. 351–374). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. (Villeneauve d'Ascq, France: Presses Universitaires de Lille).

Fuson, K. C., Richards, J., & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. Brainerd (Ed.), *Progress in cognitive development: Children's logical and mathematical cognition,* Vol. 1 (pp. 33–92). New York: Springer-Verlag.

Fuson, K. C., & Secada, W. G. (1986). Teaching children to add by counting on with finger patterns. *Cognition and Instruction, 3,* 229–260.

Fuson, K. C., Stigler, J., & Bartsch, K. (1988). Grade placement of addition and subtraction topics in China, Japan, the Soviet Union, Taiwan, and the United States. *Journal for Research in Mathematics Education, 19,* 449–458.

Fuson, K. C., & Willis, G. B. (1988). Subtracting by counting up: More evidence. *Journal for Research in Mathematics Education, 19,* 402–420.

Miller, K., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development,* 2, 279–305.

Miura, I. T., Kim, C. C., Chang, C., & Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representation of number: Cross-national comparisons. *Child Development, 59,* 1445–1450.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp 229–293). Hillsdale, NJ: Erlbaum.

Song, M. J., & Ginsburg, H. P. (1987). The development of informal and formal mathematical thinking in Korean and U.S. children. *Child Development, 58,* 1286–1296.

Song, M. J., & Ginsburg, H. P. (1988). The effect of the Korean number system on young children's counting: A natural experiment in numerical bilingualism. *International Journal of Psychology, 23,* 319–332.

Steffe, L. P., & Cobb, P. (1988). *Construction of arithmetical meanings and strategies.* New York: Springer-Verlag.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application.* New York: Praeger Scientific.

Steinberg, R. (1984). A teaching experiment of the learning of addition and subtraction facts (Doctoral dissertation, University of Wisconsin-Madison, 1983). *Dissertation Abstracts International, 44,* 3313A

Steinberg, R. (1985). Instruction on derived facts strategies in addition and subtraction. *Journal for Research in Mathematics Education, 16,* 337–355.

Thornton, C. A. (1978). Emphasizing thinking strategies in basic fact instruction. *Journal of Research in Mathematics Education, 9,* 214–227.

Thorton, C. A. (1990). Solution strategies: Subtraction number facts. *Education Studies in Mathematics, 21,* 241–263.

Thornton, C. A., & Smith, P. J. (1988). Action research: Strategies for learning subtraction facts. *Arithmetic Teacher, 35*(8), 8–12.

AUTHORS

KAREN C. FUSON, Professor, School of Education and Social Policy, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208-2617

YOUNGSHIM KWON, Lecturer, Education Department, School of Education, Yonsei University, 130 Shinchon-dong Seodaemun-gu, Seoul, Korea