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3 Children's early counting: Saying the number-word sequence, counting objects, and understanding cardinality

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Young children in all parts of the world learn to say the number-word sequence used in their own culture. Where body parts (Saxe, 1982) or finger patterns (Zaslavsky, 1973) form the number sequence, children learn to point to these body parts or to make this sequence of finger patterns. Children then use their number-word sequence to count entities; this requires making a correspondence between the number words and the counted entities. Counting is used in three mathematically different situations to determine the quantity in that situation (see Fuson, 1988). The last counted word is used in a *cardinal* situation of discrete entities to tell the *manyness* of the whole set of counted entities, in an *ordinal* situation of discrete entities to tell the *relative position* of one entity with respect to an ordering on all of the discrete entities, and in a *measure* situation of a continuous quantity to tell the *manyness of the units* that cover (fill) the whole quantity. In English and in many other languages, the words from the number-word sequence are used for counting in all of these situations and are also used to tell the manyness in cardinal and measure situations. However, modifications of these number words are used to tell the relative position in ordinal situations; these words are 'first, second, third, fourth, fifth, sixth, etc.'. Thus, at a birthday party, a child might count:

- 'one two three four five' candles on the birthday cake and announce that there are five candles (cardinal);
- 'one two three four five' children in line for a game and announce that 'I am fifth in turn' (ordinal); or
- 'one two three four five' cups of juice poured from a big bottle and announce that 'there are five cups of juice in the juice bottle' (measure).

However, many children entering school do not even know the first five ordinal words (see Beilin, 1975, for a US sample), and primary school children still show difficulties in simple measure situations (Carpenter, 1975; Fuson and Hall, 1983; Miller, 1984).

Although children have much to learn about number during the school years, they are likely to have experienced and used number words in various contexts already, as indicated in Chapter 1. Recognizing these preschool experiences and what children have learned from them is fundamental to understanding how they approach the early stages of formal mathematical education. This chapter presents an overview of the main contexts in which preschoolers use number words. These are *learning the number-word sequence* (i.e. saying the sequence of number words without counting anything), *learning to count* (i.e. saying the number-word sequence in correspondence with entities), and *initial understandings of cardinality* (i.e. using a number word to tell how many entities there are in a set and connecting this cardinal meaning to counting). Preschool children also learn to say number words when seeing the written number symbols (1, 2, 3, . . .) (this is discussed by Sinclair, this volume). After summarizing aspects of early progress in these three areas, I will consider how children come to relate these separate meanings and how these meanings are drawn upon by children in their early mathematical work in school. These developments are important and impressive, and I conclude the chapter with recommendations to parents of preschool teachers concerning the kinds of support they can offer.

Learning the number-word sequence

The number-word sequence is originally learned as a rote sequence much as the alphabet is learned. Many of the number words have no meaning initially. The kinds of errors made in learning the sequence seem to depend upon the structure of the sequence of number words (see Fuson and Kwon, this volume). The structure of the English sequence of number words to one hundred is:

(a) a rote list of twelve words;
(b) words 'thirteen' through 'nineteen' repeat the early number words 'three, four, . . ., nine' but the irregular 'thir-' and 'fif-' obfuscate this pattern;
(c) a decade pattern of x-ty, x-ty one, x-ty two, . . ., x-ty-nine in which the x words are regular repetitions of the first nine words for 'four' and 'six' through 'nine' but are not regular for two, three or five (i.e. for 'twenty', 'thirty' and 'fifty').

The irregularities in (a) and (b) result in many children learning the words to 20 by rote. Children show understanding of the decade pattern 'x-ty to x-ty-nine' quite early but take a long time to learn the correct order of the decade words (Fuson *et al.*, 1982; Siegler and Robinson, 1982). Most middle-class children between $3\frac{1}{2}$ and $4\frac{1}{2}$ can say the words to ten and are working on the words between ten and twenty. A substantial proportion of middle-class children between $4\frac{1}{2}$ and 6 are still imperfect on the upper teens (the words between fourteen and twenty) but many know those words and are working on the decades from twenty through seventy. Most kindergarten children are learning the decades between twenty and seventy, but a substantial number of them are learning the sequence between one hundred and two hundred.

Children's ability to say the correct sequence of number words is very strongly affected by the opportunity to learn and to practise this sequence. This opportunity may be somewhat less in working-class than in middle-class families (Saxe *et al.*, 1988) and in non-intact middle-class or all lower-class families than in intact middle-class families (Ginsburg and Russell, 1981). Children within a given age group show considerable variability in the length of the correct sequence they can produce.

The incorrect sequences produced by children before they have learned the standard sequence have a characteristic structure. For sequences up to thirty, the characteristic form of most sequences produced by children is a first portion consisting of *an accurate number-word sequence*, followed by *a stable incorrect portion* of from two to five or six words that are produced with some consistency over repeated trials, followed by *a final non-stable incorrect portion* that varies over trials and may consist of many words. The first portion simply consists of the first x words in the conventional sequence of number words and varies with age as outlined above. Almost all of the stable incorrect portions have words in the conventional order but one or more words are omitted (e.g. 'fourteen, sixteen' or 'twelve, fourteen, eighteen, nineteen'). Some examples of typical counting sequences over repeated counts are given in Table 3.1.

Crucially important number-word sequence learning continues long after the child is first able to produce the number words correctly, ranging at least from age 4 to age 7 or 8. New abilities mark qualitative changes in children's mental representation of the number-word sequence. Five levels have been differentiated (see Fuson, 1988, for a summary of these levels and Fuson *et al.*, 1982, for more details).

1 *String level*: the words are a forward-directed connected undifferentiated whole.
2 *Unbreakable list level*: the words are separated but the sequence exists in a forward-directed recitation form and can only be produced by starting at the beginning.
3 *Breakable chain level*: parts of the chain can be produced starting from arbitrary number words rather than always starting at the beginning.
4 *Numerable chain level*: the words are abstracted still further and become units in the numerical sense – sets of sequence words can themselves present a numerical situation and can be counted, matched, added and subtracted.
5 *Bidirectional chain level*: words can be produced easily and flexibly in either direction.

These different levels are marked by increasingly complex sequence abilities: becoming able to start and to stop counting at arbitrary number words, to count up a given number of words, to count backwards starting and stopping at arbitrary number words, and to count down a given number of words. Children also increase their ability across these levels to comprehend and to produce order

Table 3.1 Examples of children's incorrect number-word sequences with accurate portions, stable incorrect portions, and non-stable incorrect portions

Example 1: Age 3 years 6 months
1 . . . 13 19 16 13 19
1 . . . 13 16 19
1 . . . 13 16 14 16 19
1 . . . 13 16 19 16 13 14 19 16 19
1 . . . 13 19 16 14
1 . . . 13 19 16 14 19 16 19 16
1 . . . 13 19 14 16 14
1 . . . 13 19 16 14 19

Example 2: Age 3 years 10 months
1 . . . 12 14 18 19 15 19
1 . . . 12 14 18 19 16 17 18
1 . . . 12 14 18 19 15 17 18 19 17
1 . . . 12 14 18 19 15 16 17 18 19 15 17
1 . . . 12 14 18 19 16 17 12 14 18 19
1 . . . 12 14 18 19 16 17 18 19 17 14 18
1 . . . 12 14 18 19 16 17 18 19 16 17 18 19 16
1 . . . 12 14 18 19 16 17 18 19 16 17 18 19 17 18

Example 3: Age 4 years 2 months
1 . . . 14 16 . . . 19 30 1
1 . . . 14 16 . . . 19 30 40 60
1 . . . 14 16 . . . 19 30 31 35 38 37 39
1 . . . 14 16 . . . 19 30 40 60 800
1 . . . 14 16 . . . 19 40 60 70 80 90 10 11 10 30
1 . . . 14 16 . . . 19 60 30 800
1 . . . 14 16 . . . 19 60 30 800 80 90 30 10 80 60 31 38 39 32 31 34 35 thirty-ten 31
1 . . . 14 16 . . . 19 30 800 60
1 . . . 14 16 . . . 19 30 1 80 90 60 30 90 80 30

Example 4: Age 5 years 2 months
1 . . . 29 60 . . . 69 50 . . . 59 30 . . . 39 90 . . . 99
1 . . . 29 60 . . . 69 50 . . . 59 30 . . . 37

Example 5: Age 5 years 2 months
1 . . . 29 80 . . . 99 90 . . . 97
1 . . . 49 40 . . . 49 40 . . . 49 80 . . . 89 80 . . . 89 90 . . . 97 99 40

Example 6: Age 5 years 9 months
1 . . . 49 30 . . . 39 50 . . . 59 20 . . . 39
1 . . . 39 50 . . . 59 20 . . . 39 50
1 . . . 49 30 . . . 39 50 . . . 59

Example 7: Age 5 years 10 months
1 . . . 59 80 . . . 89 100 . . . 109 70 . . . 79 80 . . . 89 50 . . . 59

Note: . . . means that the intervening words were said correctly.

relations on the words in the sequence (see Cowan, this volume; Fuson, 1988). Children's construction of these developmental representational levels depends less on particular features of the sequence than does learning the correct number words, so these levels seem to appear fairly widely: in children in the Soviet Union speaking Russian (Davydov and Andronov, 1981), in Israeli children speaking Hebrew (Nesher, pers. comm. 1980), in New Guinea Oksapmin children with a body number-word sequence (Saxe, 1982), and in US children speaking English (Fuson *et al.*, 1982). However, instructional practices may affect the manifestation of these levels (e.g. Hatano, 1982).

Learning to count

In order to count objects distributed in space, one must match each object with a number word. To do this, some sort of indicating act (some variation of pointing or of moving objects) that has both a temporal and a spatial location is needed. The indicating act serves as a mediator and simultaneously creates two correspondences: the correspondence in time between a spoken number word and the indicating act, and the correspondence in space between the indicating act and an object. The required matching between the spoken word and the object in space – the word–object correspondence – then derives from the separate temporal and spatial correspondences involving the indicating act. For counting to be correct, the *local correspondences* associated with each indicating act must be one-to-one: one word must correspond to one indicating act and one indicating act must correspond to one object. The *global indicating act–object correspondences* over the whole counted set must also be one-to-one: every object must be indicated and no object can be indicated more than once. This requires some method of remembering which objects have been counted. Finally, for the counting to be correct, the words said must be the standard number-word sequence.

The kinds of correspondence errors 3- to 5-year-old children actually make in counting and the variables that affect these errors are described in Fuson (1988, chs. 3–6; see also Gelman and Gallistel, 1978). Of the 13 types of errors made by children when counting objects in rows, only 5 are made very frequently. Two of these violate the *word–point* correspondence: in *point–no word* errors, children point at an object without saying a word and, in *multiple words–one point* errors, they say two or more words while pointing once at an object. Two other common errors violate the *point–object* correspondence: in *object skipped* errors, an object is skipped over without being counted and, in *multiple count* errors, an object is counted and then immediately counted again (it receives a word and a point and then another word and a point). The fifth common error violates *both* correspondences: in *multiple points–one word* errors, an object is pointed at two or more times while one word is said.

Children aged 3–3½ make a considerable number of these common kinds of errors. The rate of most errors drops at least somewhat at age 3½ and continues to drop at every half year after that. Five-year-olds have quite low rates of local correspondence errors when counting objects in rows but do still make errors that violate the global indicating act–object correspondence when counting complex non-linear arrays (they recount some objects and fail to count some objects). The percentage of children making at least one error of a given type generally drops more slowly than does the rate of making that error, indicating that learning to count primarily involves gradual improvement rather than sudden insights in which a given error type drops entirely out of a child's repertoire.

Children sometimes move their finger along a row of objects saying words without really pointing at objects (a *skim* error) or produce a flurry of words and of points directed generally but not specifically at the objects (a *flurry* error). Both of these types of errors, as well as *skipped objects* errors, are particularly affected by the effort with which children count. When children are asked 'to try really hard', these errors decrease considerably.

Both parts of counting – saying the words and pointing – begin to undergo internalization at the end of the preschool years. This internalization continues into the primary grades and may be accelerated by a teacher who actively discourages children from counting to find addition or subtraction answers. Children begin to count silently, though lip movements are still often evident, and they point from a distance and finally may use eye fixation instead of pointing. Especially initially, children may make more counting errors when counting is internalized. Because counting is crucial for children's understanding of addition and subtraction of small numbers (see Fuson, in press, for a review), it is important for teachers to allow and even encourage children to count.

Counting is a very complex activity. However, even by the age of 3, counting is very organized and usually exhibits the general structure of mature effective counting: it has a recognizable structure of word–point and point–object correspondences, and most of the frequent errors violate only one of these correspondences. Young children can count objects in rows quite accurately: children aged 3–3½, 3½–4 and 4–4½ had correct correspondences on 84%, 94% and 97% of the objects in rows of 4–14 objects (Fuson, 1988). Accurate correspondence does fall considerably for large disorganized arrays, because most young children do not use adequate strategies for remembering which objects they have already counted and so global as well as local correspondences are violated. Gelman and Gallistel's (1978) characterization of preschool children's early counting as governed from the beginning by the One–One Principle (every item in a set must be assigned a unique tag) is a simplification of the nature of counting and of children's counting correspondence behaviour (there are many different aspects of correspondence to understand and to carry out), but their general position that counting is structured and organized and displays considerable competence is supported.

Initial understandings of cardinality

Children initially count without any cardinal outcome for the counting. If asked 'How many candles?' just after counting five birthday candles, many 3-year-olds and some 4-year-olds will count again (they may recount many times to repeated how-many questions), or give a number word other than the last word said in counting, or say several number words. These children do use number words with cardinal meaning in non-counting situations. They subitize (immediately recognize and label) very small numbers of one, two, and possibly three or four objects, and they may imitate and independently repeat cardinal uses such as 'I have five people in my family'. But they cannot yet connect counting to these cardinal meanings: they cannot make a count-to-cardinal transition in word meaning from a counting meaning of the last counted word as indicating the last counted object to a cardinal meaning of that word as telling how many objects there are in all. Most 2- and 3-year-olds do remember the last counted word, and therefore the failure is a conceptual and not just a memory problem.

Children make this fundamental connection in different ways (see Fuson, 1988, for a review of different positions on how children come to make this connection and for data concerning the validity of these different positions). Initially for many children, this connection may reflect only a last-word rule that the last counted word is the answer to a how-many question; that word may not have a cardinality meaning referring to the whole set or to the cardinality of the whole set. Once a child answers a how-many question with the last counted word, that child will continue to do so fairly consistently across trials and across different set sizes. Some children may count a small set, also subitize that set, and see that the subitized word and the last counted word are the same (Schaeffer *et al.*, 1974; Klahr and Wallace, 1976). It may take only one such example for children to generalize this pattern to a last-word rule and use it on non-subitizable larger sets. Children may also make this last-word connection if they experience some event that makes the last counted word particularly salient (makes them notice the last counted word) or if they are told that the last counted word tells how many. Finally, preschool children's recency bias (their tendency to answer a multiple-choice question with the last provided choice) or an auditory 'echoing' of the last word may prompt the child to hazard the last counted word as a guess to the how-many question.

Many 4- and 5-year-old children do make count-to-cardinal transitions after counting a set of objects: when they say 'One, two, three, four, five. There are five candles,' the five refers to all the candles and tells how many candles there are. However, this cardinal meaning is still not mature. Children may still be misled by appearances in equivalence situations and say such things as 'This five candles (a longer row) is more than this five candles') for discussions of equivalence and order situations, see Cowan, this volume; Bryant, 1974; Fuson, 1988; Piaget, 1952). They also may not be able to solve certain kinds of addition and subtraction situations (see Baroody and Ginsburg, 1986; Briars and Larkin, 1984; Carpenter and Moser, 1984; Fuson, 1988, in press; Riley *et al.*, 1983; Steffe *et al.*, 1983; Steffe and Cobb, 1988). Children's understanding of cardinality continues to grow throughout the primary grades (see Fuson, 1988, in press, for reviews of this increasing understanding; Sinclair, this volume, discusses the implications of cardinality for learning to write numbers).

Early relationships among sequence, counting and cardinality knowledge

Gelman and Gallistel (1978) identified three principles that direct young children's counting: the stable-order principle ('the tags used must be drawn from a stably order list'), the one–one principle ('every item in a set must be assigned a unique tag') and the cardinality principle ('the last tag used in a count represents the cardinal value of the set') (definitions are from a recent statement of the principles by Gelman and Meck, 1986). They reported that the developmental relationship among these principles was: stable-order before one–one before cardinality, i.e. children learn a stable list of number words before they can make correct word–point–object correspondences before they can give the last counted word as the answer to a how-many question. Research concerning the relationships among these three aspects of counting is reported and reviewed in Fuson (1988). No support was found for the Gelman and Gallistel relationship except for very small sets. Instead, this relationship was found to depend very heavily on the size of the set counted:

- for sets of 2, 3, and 4: stable-order and one–one before cardinality;
- for sets of 4 through 7: stable-order and one–one before cardinality *or* stable-order and cardinality before one–one;
- for sets between 7 and 16: stable-order before cardinality before one–one; and
- for sets above 16: cardinality before stable-order before one–one.

These relationships are sensible because it is easier to learn a stable list of words or to make correct word–point–object correspondences for a small set than for a very large set but the cardinality principle is a rule that can easily be applied across different set sizes and does not have to reflect understanding of cardinality (and thus can be independent of a correct one–to–one correspondence between the words and the objects). The Gelman and Gallistel result is widely discussed and accepted but the order of their principles seems to be the result of limitations in their testing procedure (Frye *et al.*, 1989).

The integration of sequence, counting and cardinal meanings

Children enter school with strong counting capabilities. The sequence, counting and cardinal meanings of number words form their conceptions of numbers. They use these meanings to make sense of numerical situations in school maths. They approach addition and subtraction situations by counting out objects for the first number, counting out objects for the second number and adding them to or taking them from the first objects, and then counting the total or the remaining objects. These counting abilities enable children to solve a wide range of addition and subtraction situations (see the discussion by De Corte and Verschaffel, this volume), although as Davis shows (this volume), children still have much to learn about the contextual demands of such operations.

The different qualitative levels in children's mental representation of the number-word sequence discussed above indicate important changes in children's understanding of addition and subtraction situations. These changes manifest the child's increasing integration of counting, sequence and cardinal meanings of number words (Fuson, 1988; Steffe *et al.*, 1983; Steffe and Cobb, 1988). At the *unbreakable list* level possessed by most children entering kindergarten, children are able to use counting to determine how many entities there are in a given situation and are able to add by counting all the entities representing the addends and subtract by separating some objects from all the objects. At the *breakable chain* level, children are able to count on with entities, starting the sum count of entities by beginning with one of the addends (e.g. for 8 + 5, counting five more words from eight – '8, 9, 10, 11, 12, 13' – instead of beginning with one), and they can subtract by counting up to (e.g. for 13 – 8, counting '8, 9, 10, 11, 12, 13' to find the answer 5 words counted up from 8 to 13), by counting down from (e.g. for 13 – 8, counting down 8 words from 13 to end at the answer 5), and by counting down to (e.g. 13 – 8, counting down from 13 to 8 to find the answer 5 words counted down). At the *numerable chain* level, the number words can represent the addends and the sum for addition and subtraction situations, and children thus can solve addition and subtraction problems by using the efficient sequence procedures counting on, counting down from, counting up to, or counting down to without needing precounted sets of entities to keep track of the counting; they keep track of the words counted up or down by extending fingers, or using auditory or visual patterns, or by double counting ('nine is one, ten is two, . . ., thirteen is five'). At this level sequence, counting, and cardinal meanings become integrated within a single mental representation of the number-word sequence, as they are for readers of this chapter.

Practical implications

It is important for teachers and parents to realize how very separate and contextually bound all of the different number-word meanings are initially for young children, what an impressive intellectual feat children achieve in constructing and then relating these separate number-word meanings, and how very crucial counting is to children's understanding of number. Children can best be prepared for school maths by having preschool teachers, parents and caretakers who help children take joy and pride in their noticing and labelling numerical aspects of their environment and in their learning of the number-word sequence and who facilitate children's enthusiastic counting of all sorts of things in many different situations. Children can move from counting single sets to counting two sets in order to add them. They can match the objects in two sets or count them to find out which has more and how many more it has. These numerical activities can arise in children's everyday activities ('We have four plates on the table. How many cups do we need if each plate gets a cup?'), in games ('I get to move six spaces!'), and in classroom or family routines ('Let's say the numbers out loud together while we wait for this traffic light' or 'while we line up to go outside'). The focus should be on facilitating children's seeing and talking about many different numerical situations and not on formal number 'lessons' or on pages and pages of workbook problems. Some examples from diaries I wrote to my own two daughters about their early years are given in Table 3.2 to provide a flavour of the range of situations in which numerical conversations can arise (more examples are given in Fuson, 1988, and also see Durkin *et al*., 1986b; Saxe *et al*., 1988).

Because so much important numerical learning occurs after children can count correctly, numerical activities are as crucial after children have learned to say the correct sequence and to make accurate counting correspondences as before. The number-word sequence and the counting of objects needs to become very overlearned in order to move through the developmental levels of counting, cardinal and sequence relationships. Many counting activities for many different addition and subtraction situations, and reflection about, and discussion of, alternative solution procedures for these situations can help children move through these developmental levels. Children need to be exposed to many different meanings of addition and subtraction, not just addition as getting more and subtraction as taking away (see De Corte and Verschaffel, this volume, for these alternative meanings). Because counting forward is much easier than counting backward for children, subtraction needs to be given a counting up 'how many more?' meaning ('One meaning of 6 – 4 is how many more do I have to add to 4 to get 6?') and not just a take-away meaning. Children move from the usual take-away meaning to counting down for subtraction and make many more mistakes in counting down than in counting up. Finally, many interpreters of Piaget's work on number conclude that children must understand conservation of number (see Cowan, this volume) before they can understand addition and subtraction and thus suggest delaying work on addition and subtraction until after children understand conservation of number. However, understanding conservation of number is a relatively late development that follows the *numerical chain* sequence level. Thus, much sequence, counting and cardinal understanding, and solution of many different kinds of numerical problem situations, can precede conservation of number.

Conclusion

Young children initially learn number words in several mathematically and functionally different situations and thus the same number word comes to have different meanings. The major number-word situations during the preschool and

Table 3.2 Examples of young children's uses of number words in the home

Age	*Diary entry*
1:11	Tonight we counted steps going up to bed (we usually do). We were on nine and you said, 'One two fee four five six'. First time so many were correct.
2:0	'One two three eight jump'. (You are into counting and jumping off the hassock or even the couch.)
2:6	Two tomatoes were on the table. You said and acted out the following: 'One tomato from two tomatoes leaves one. Two tomatoes from two tomatoes leaves no.' I asked you what no tomatoes from two tomatoes was: 'Two tomatoes.' 'Sesame Street' does things similar to the first two sentences.
2:7	Putting prunes back into a box, you correctly counted them up to nine. When asked how many prunes: 'three' (your standard 'how many' answer at this point: three eyes, etc.)
2:10	I cut your peanut butter sandwich in half and then into half again. You watched and said, 'Two and two make four.' You just asked for four olives (you love olives!). Your father gave them to you, and you said, 'Two and two make four.'
2:11	I want manier than five.
3:5	You were typing and saying as you typed the numbers (they are in order on the typewriter): 'one, two, three, four, five, six, seven, eight, nine [pause]. I need a ten.'
3:10	'What's four and four?' I said to count on your fingers, 'One, two, three, four. One, two, three, four. One, two, three, four, five, six, seven, eight. Four and four make eight.'
4:2	Walking to my office with you, I asked you how many chairs were in my office. You said, 'Four'. (This was the use of mental imagery; you said you counted the chairs in your head. There were four chairs in my office, one at my desk and three at a table.)
4.9	When we were preparing bags of popcorn and peanuts for your (early) school birthday, you counted kernels of coloured popcorn. In great excitement and wonder: 'Ooh! I counted up to one hundred and two!' You refused to make piles of ten. You finally quit at 150. You played this game with your sister (then 2:4): You put coloured beads or other objects out on a cloth and then asked her questions. 'Erica, what does it make adding one and one?' Erica (without looking at the beads): 'Two'. You would put out the correct number of objects. Erica answered somewhat randomly, rarely looking at the objects.
5:7	Conversation with you in the bathtub: You: 'How much is seven and seven?' I held up seven fingers and had you hold up seven fingers. You counted. You still usually will not count on from the first number; you need to count the fingers for the first number. You asked and we did nine and nine, four and four, six and six, five and five, and then ten and ten (you did that with your fingers and toes). You said twenty. I said it was called that because it was twin tens – two tens. You said, 'I know' and thought for a bit. Then you said, 'There's a zero to make the ten and the 2 to make the [pause] two tens.' You asked what twenty and twenty were. We used all our fingers and toes. You

Table 3.2 (*Continued*)

Age	*Diary entry*
	counted on from twenty by ones. Got forty. I asked you how many tens were in forty (pointing to our fingers and toes) – it took a bit of focusing for you to see the tens. You said, 'Oh, the zero for ten and the 4 for four tens.' You drew a 2 in the air and asked, 'Is it this way or the other way [backward]?' You still make many of your numbers backward – but not letters. I asked you how you knew where to move your arm to draw a 2. 'I don't know.' 'Do you see a picture of the 2?' 'Yes, I see an invisible 2, and I just draw it.' You brought home a Montessori numeral sheet today. It was divided into 1-inch squares and was ten squares by eight squares. The teacher had written the symbols in order from 1 through 10 across the top row. You then wrote seven more rows of numerals under them. Your 1, 3, 4, 5 and 7s were mostly quite good. You were struggling with 2s; they turned more and more into Zs as they went down the page. You had trouble with the loop in the 6. The reverse direction cross-overs in the 8 were really difficult for you, so you tried different strategies on the 8s. You tried partial cross-overs and overlapping loops and then settled for a circular top and a U fastened to the bottom of the loop. Your 10s were too scrunched because you started the zero too close to the 1; you went to the right and then when you looped back to the left you ran into the 1. You also started the zero at the bottom. But overall quite a good job; everything was recognizable.
6:0	You know most of the double sums (3 + 3, 7 + 7, etc.) right away. I asked you some tonight and then asked nine plus nine. You closed your eyes and scrunched up your face and thought and then said, 'Eighteen.' I asked how you had figured it out. 'Well, I knew eight plus eight was sixteen, and I knew there had to be one in the middle, so it was eighteen' (i.e. you knew the double sums went up skipping every other number). I then asked you five plus seven. You thought for a while and then said, 'Now don't ask me to describe it because it is very difficult.' You closed eyes, etc., and after a while said, 'Twelve'. I was surprised. 'Could you give me a clue?' 'Well, I had one five and there was another five in seven with two left over, so that made one was eleven and two was twelve.' Big smile. Me too.

Note: The age is given in years and months, so 1:11 is 1 year 11 months old.

primary school years are sequence, counting and cardinal situations. Children learn a great deal about each of these kinds of situations and gradually integrate sequence, counting and cardinal meanings within a single powerful and flexible mental representation of the number-word sequence. Use of this integrated number-word sequence permits them to understand and solve addition, subtraction, multiplication and division situations involving numbers less than one hundred.

Further reading

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