BRIEF REPORT

Grade Placement of Addition and Subtraction Topics in Japan, Mainland China, the Soviet Union, Taiwan, and the United States

KAREN C. FUSON, *Northwestern University*
JAMES W. STIGLER, *University of Chicago*
KAREN BARTSCH, *University of Michigan*

There has been considerable recent concern about the mathematics achievement of children in the United States compared to that of children in the Soviet Union and in countries of the Far East (Benderson, 1984; Fiske, 1987; Husen, 1967; Stevenson, Lee, & Stigler, 1986; Stigler, Lee, Lucker, & Stevenson, 1982; Wirszup, 1986). One possible source of the superior achievement in these other countries is the placement of topics within the mathematics curriculum. If topics are presented earlier in these countries, children would be given an opportunity to cover more topics and thus might be able to learn more mathematics by comparable grade levels. The present study examined the grade placement of topics in basic addition and subtraction computation with whole numbers across five countries: Japan, mainland China, the Soviet Union, Taiwan, and the United States.

METHOD

Mainland China, Taiwan, and the Soviet Union have national curricula with a single textbook series used for the whole country. (In mainland China, the same language, Mandarin Chinese, is used in schools all over the country, so the textbooks are identical throughout the country. In the Soviet Union, the language of mathematics instruction in the elementary grades is usually the local language, so the single textbook series is translated from Russian into many different local languages.) Copies of the mathematics books used in mainland China and in Taiwan were obtained as part of a larger project investigating children's mathematics achievement in different countries (Stevenson et al., 1986); the analyses of these books were done by fluent readers of Chinese. The Soviet mathematics textbooks for the elementary grades were translated by the University of Chicago School Mathematics Project; these translations were used for this study. In Japan, the placement of

This research was funded by a grant from the Amoco Foundation to the University of Chicago School Mathematics Project.

topics within the mathematics curriculum is prescribed by the central educational authority; all textbook series conform to the specified placement. A prominent Japanese textbook series (*New Mathematics,* published by Tokyo Shoseki, 1984) was used in the analysis; the analyses of these books were done by fluent readers of Japanese. In the United States, the grade placement of topics is theoretically quite variable, but the national textbook market and statewide controls in some large states limit the variability. However, five standard and widely used textbook series were analyzed to ensure that conclusions were not based on one or two possibly atypical series. The study used the 1983 series from Addison-Wesley and the 1985 series from Harper & Row; Holt, Rinehart & Winston; Houghton-Mifflin; and Scott, Foresman.

The measure selected as being both most reliable for use in a cross-cultural textbook analysis and most pertinent to the concerns of this study was the first time a given topic was taught to any substantial extent. The operational measure of being taught to a substantial extent was the presence of at least five problems of the specified kind. These problems could appear anywhere in the main part of the text (e.g., three could be on a practice page, one in a story problem, and one in a chapter review). However, a problem was not counted if it was in some sense optional: that is, if it was starred (as a special "difficult" problem for that grade level), if it appeared in an optional section of the text, or if it appeared on special practice pages at the end of a text that a teacher might or might not use. Topics were placed by grade according to the system used by Stigler et al. (1982). If a topic appeared in the first half of a text for grade X, it was given the score $X.0$. A score of $X.5$ was given for a topic occurring in the second half of a text.

To facilitate the interpretation of the comparative grade-placement data, it is helpful to know the comparative ages of children at each grade level. In Japan, mainland China, Taiwan, and the United States, children enter first grade at age 6. In the Soviet Union children entered first grade at age 7 at the time these textbooks were used.

RESULTS

Addition and Subtraction of Two Single-Digit Numbers

Grade placements for important topics in addition and subtraction computation for whole numbers are listed in Table 1. All the countries introduce in the first half of the first grade the addition of two single-digit numbers whose sum is 10 or less. Subtraction of these numbers (minuends through 10) begins in the first half of the first grade for the other countries and for two of the United States textbooks and in the second half of the first grade for the other three United States textbooks. The texts for mainland China continue on to the addition and subtraction of all pairs of two single-digit numbers (i.e., numbers with sums of 11 through 18 and the corresponding subtraction

combinations with minuends of 11 through 18) in the first half of the first grade. The texts for Japan, Taiwan, and the Soviet Union do so in the second half of the first grade. These more difficult addition and subtraction problems first appear in the United States textbooks at quite varying times between Grades 1.5 and 2.5. (The Addison-Wesley; Holt, Rinehart & Winston; and Houghton-Mifflin series did have an optional chapter on the more difficult single-digit sums and differences to 18 at the end of the first-grade textbook. The Table 1 entries indicate the placement of the first treatment that was not optional.)

Addition and Subtraction of Two Multidigit Numbers

In all five countries, addition and subtraction with a one- or two-digit number and a two-digit number with no trading (i.e., the inclusion of combinations in which the sums in each column are less than 10) appears in the second half of the first grade. (The one exception was that in Taiwan two-digit minus one-digit problems with no trading were introduced at the beginning of second grade rather than at the end of first grade.) In China and the Soviet Union, the curriculum continues directly on during that period to the addition and subtraction of such numbers with trading (e.g., $25 + 7$, $32 - 25$). In Japan and Taiwan, such problems with trading first occur in the first half of the second grade. In the United States, problems with trading (regrouping, carrying) are deferred until the second half of the second-grade textbooks. (The Scott, Foresman series did have two-digit plus two-digit problems with trading at the end of the first-grade book, but these problems all had pictures that supported the solutions. Such pictures provide a good introduction to the topic, but all entries in Table 1 were for problems presented in symbolic form without pictures to support concrete solution procedures.) Thus, textbooks in the United States place the first treatment of addition and subtraction with trading later than all other countries and also show the biggest lag (a whole year) between the addition and subtraction of multidigit numbers without trading and the addition and subtraction of such numbers with trading.

Adding or subtracting two numbers having three, four, five, or six digits is placed considerably earlier in other countries than in the United States. All other countries presented problems of this size in the first half of the third-grade text. (The texts in Taiwan did not present addition or subtraction problems with five or six digits at any grade level.) The first appearance of such problems in the texts of the United States was distributed over a 2.5- to 3.5-year period after the introduction of two-digit problems with trading.

A traditionally difficult topic in subtraction is that of multidigit problems with zeros in the minuend. There is a difference between the United States and the other countries in the grade at which more difficult subtraction problems (four-digit problems) with two or more zeros in the minuend first appear. In the United States all textbooks but one first place this topic at Grade 4.0

Table 1
Grade Placement of Addition and Subtraction Calculation

Topic	Mainland China	Japan	Taiwan	Soviet Union	United States				
					Addison-Wesley	Harper & Row	Holt, Rinehart	Houghton-Mifflin	Scott, Foresman
Add/sub of two single-digit numbers									
1 digit + 1 digit, all sums ≤ 10	1.0	1.0	1.0	1.0	1.5	1.0	1.0	1.0	1.0
1 digit − 1 digit, all minuends ≤ 10	1.0	1.0	1.0	1.0	1.5	1.5	1.0	1.0	1.5
1 digit + 1 digit, all sums ≤ 18	1.0	1.5	1.5	1.5	2.0	1.5	2.0	2.0	1.5
2 digits − 1 digit, all minuends ≤ 18	1.0	1.5	1.5	1.5	2.0	1.5	2.5	2.0	1.5
Add/sub of two multidigit numbers									
2 digits ± 1 or 2 digits									
No trading: 15 + 3, 15 + 23	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
2 digits ± 1 or 2 digits									
with trading from ones: 15 + 8	1.5	2.0	2.0	1.5	2.5	2.5	2.5	2.5	2.5
3 digits ± 2 or 3 digits									
with trading from tens	2.5	2.5	2.5	2.5	3.0	3.0	3.0	3.0	2.5
4 digits ± 3 or 4 digits									
with trading from hundreds	2.5	2.5	3.0	3.0	4.0	4.0	3.0	3.0	3.0
5 digits ± 4 or 5 digits									
with trading from thousands	3.0	3.0	—	3.0	6.0	4.0	4.0	4.0	5.0
6 digits ± 5 or 6 digits									
with trading from ten thousands	3.0	3.0	—	3.0	—	6.0	5.0	5.0	6.0
4 digits − 4 digits with ≥ 2 zeros									
in the minuend	2.5	2.5	3.0	3.0	4.0	4.0	4.0	4.0	3.0
Addition of three or more numbers									
Add 3 one-digit numbers									
with trading: 7 + 3 + 4	1.0	1.5	1.5	1.0	2.5	1.5	2.0	2.5	1.5
Add 3 two-digit numbers									
with trading from ones	1.5	2.0	2.0	1.5	3.0	3.0	3.0	2.5	2.5
Add 3 four-digit numbers									
with trading from hundreds	2.5	—	3.0	3.0	5.0	5.0	4.0	—	3.0
Add 4 five-digit numbers									
with trading from thousands	3.0	—	—	—	—	—	6.0	—	—

Note. — means that the topic did not appear in the first six grades. Problems in the second row include the special case of two-digit − one-digit numbers in which the two-digit number is 10.

(mean placement of 3.8), whereas it appears at Grades 2.5 and 3.0 in the other countries (mean 2.8).

Addition of Three or More Numbers

The addition of three single-digit numbers with trading (sum over 10) appears about a year later in the United States than in the other countries (a mean of 2.2 vs. 1.3), and the addition of 3 two-digit numbers also appears about a year later in the United States (a mean of 2.8 vs. 1.8). The time between the introduction of two-digit problems and that of the considerably more difficult four-digit problems ranges in other countries from 1.5 to 3 years (mean 1.7) and in the United States from 1.5 to 3.5 years (mean 2.3). The combination of the late start and the greater mean lag results in the mean placement of the addition of 3 four-digit numbers in the United States much later than in the other four countries (4.3 vs. 2.8).

Column addition of many large numbers (e.g., adding 4 five-digit numbers) does not meet the criterion of five problems for most of the texts sampled in the United States or for the texts of Japan, Taiwan, or the Soviet Union. Children who attend the special abacus schools in Japan, mainland China, and Taiwan do such problems on the abacus, but evidently the solution of such problems by all children is not a goal in most countries surveyed.

Further Analysis of Single-Digit Addition and Subtraction

Being able to solve single-digit addition and subtraction problems (sums and minuends through 18) is required for the solution of other more advanced addition and subtraction topics. Two features concerning single-digit sums and differences that differentiated the textbooks in the other countries from those in the United States were the amount of the second-grade textbook devoted to such topics and the inclusion in the first-grade textbooks of other countries of a particular method for solving such single-digit problems.

The second-grade textbooks in the other countries devoted 0%–3% of their pages to such problems, and many of these problems occurred in the sections on multiplication where addition was being reviewed as part of the multiplication procedure. In the United States, between 19% and 31% (mean 23%) of the pages of the second-grade textbooks contained single-digit sums and minuends through 18. There was no similar between-country difference in the percentages of pages in the first-grade texts devoted to this topic. Such coverage ranged from 29% to 42% (mean 38%) in the other countries and from 31% to 44% (mean 36%) in the United States.

In all four of the non-U.S. countries the first half of the first-grade text contains exercises involving the decomposition of a single-digit number into all of the various pairs of numbers whose sum is the given number. This knowledge is then used to teach a particular method of mental calculation for finding the sum of two single-digit numbers between 10 and 19: the *over ten*

method, in which one addend is decomposed into the particular pair of numbers such that the first number of the pair will add to the other addend to make 10 (e.g., $7 + 5 = 7 + 3 + 2 = 10 + 2 = 12$). The reason that teachers in mainland China are able to move to this mental calculation of sums over 10 in the first half of the first grade may be that preschools there emphasize the learning of composition and decomposition of numbers below 10 (personal communication with the director of teacher training for preschool education in mainland China, 1983); thus much of this foundation may not need to be covered in first grade. The Chinese and Japanese languages facilitate this over-ten method because the part over 10 directly gives the answer. Counting on from seven is said, "Eight, nine, ten, ten one, ten two," so the thinking in the over-ten method is "Seven plus five equals seven plus three plus two equals ten two." In contrast, English-speaking children end up with "twelve" (or with the other irregular teen words), and many beginning second graders do not automatically know the tens pattern for these teen words, for example, that $10 + 2 = 12$ or $10 + 5 = 15$ (Steinberg, 1983/1984). Thus, the over-ten method is more difficult in English.

The texts of the non-U.S. countries also included exercises to teach minuends through 18 as *up over ten* or *down over ten* to find the missing addend. In up over ten, the student begins with the smaller number and adds mentally (a) the number added to that smaller number to make 10, and (b) the ones part of the larger number. For example, to do the problem $13 - 6$, one would ask, "Six plus what to make ten?" "Four." "So four (from six to ten) plus three (from ten to thirteen) is seven. So thirteen minus six is seven." In down over ten, the number being subtracted is decomposed into the part that when taken away from the minuend will give 10, and then the other part is taken away from 10 (e.g., $13 - 5 = 13 - 3 - 2 = 10 - 2 = 8$). To show this method, it is useful to write two successive subtractions without parentheses ($13 - 3 - 2$), a form that is "illegal" in the United States, where such problems would have to be written as $(13 - 3) - 2$. Such problems are the inverses of adding three numbers. Problems like these with minuends less than 21 (i.e., the inverses of problems in row 12 of Table 1) appear without parentheses in the first half of the first grade in mainland China and the Soviet Union and in the second half of the first grade in Japan and Taiwan. Similar problems involving 3 two-, three-, and four-digit numbers appear in the texts of mainland China and of the Soviet Union from Grades 1.5 to 3.0, but problems of this size do not appear in textbooks in Japan or Taiwan.

In contrast to the effort in other countries to teach a particular means of adding and subtracting single-digit numbers with sums and minuends over 10, in the United States no particular method is emphasized or even provided in most textbooks. (The Harper & Row series was an exception, for it did contain exercises supporting some methods: counting on by one, two, or three; doubles plus one, such as $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$; the nines by compensation to ten, such as $9 + 7 = 10 + 6$. However, no method that

would work for all problems was supported in the text.) In the U.S. texts pictures of objects are included with the early problems and then numerals alone appear. Various methods for teaching such addition and subtraction, including the over-ten method, may be discussed in materials for preservice and in-service teachers and are sometimes recommended in the teacher's manuals for textbooks. But in general, it is up to individual teachers to teach or not to teach any given method, and they must teach it without any special practice exercises or support given on the text pages.

DISCUSSION

There is remarkable uniformity in the grade placement of addition and subtraction calculation topics in the curricula of Japan, mainland China, the Soviet Union, and Taiwan. Substantial differences exist between the placements for these countries and the placements for the United States. In the textbooks from other countries, problems with sums and minuends to 18 appear earlier and disappear earlier than in texts from the United States. Both the simplest and the most difficult multidigit addition and subtraction appear earlier (from 1 to 3 years earlier) than in the United States. The addition of three or more multidigit numbers also appears from 1 to 3 years earlier. The earlier disappearance of problems with sums and minuends to 18, and thus the presumed ability to solve such problems at an earlier grade, may be linked to the inclusion in all texts, except those of the United States, of activities supporting a specific method of solving problems with sums and minuends to 18: adding up over ten and subtracting up over ten or down over ten. Because the solution of such problems is necessary for solving multidigit problems with trading, the presumed ability to solve such problems at an earlier grade may also enable multidigit addition and subtraction with trading to appear as early as it does in these other countries.

An analysis of the grade placement of topics such as the one presented in this paper obviously has limitations. It is limited to the intended curriculum as manifested in textbooks. Any curriculum has a complex relationship to what actually occurs in classrooms. Teachers may always choose to teach topics at different times, and even in different grades, than those times and grades in which they occur in the curriculum. However, given the strong centralized character of the educational systems in Japan, mainland China, Taiwan, and the Soviet Union, it seems unlikely that departures from the curriculum would be huge. It seems likely that very large differences do exist between these countries and the United States in the grade at which work on particular topics in addition and subtraction is concentrated.

REFERENCES

Benderson, A. (1984). Recapturing the lead in math and science. *Focus*. Princeton, NJ: Educational Testing Service.

Fiske, E. B. (1987, January 11). U.S. pupils lag in math ability, 3 studies find. *The New York Times*, pp. 1, 10.

Husen, T. (1967). *International study of achievement in mathematics: A comparison of twelve countries.* (Vols. 1–2). New York: Wiley.

Steinberg, R. M. (1984). A teaching experiment of the learning of addition and subtraction facts (Doctoral dissertation, University of Wisconsin—Madison, 1983). *Dissertation Abstracts International, 44,* 3313A.

Stevenson, H. W., Lee, S. Y., & Stigler, J. W. (1986). Mathematics achievement of Chinese, Japanese, and American children. *Science, 231,* 693–699.

Stigler, J. W., Lee, S. Y., Lucker, G. W., & Stevenson, H. W. (1982). Curriculum and achievement in mathematics: A study of elementary school children in Japan, Taiwan, and the United States. *Journal of Educational Psychology, 74,* 315–322.

Wirszup, I. (1986). The current crises in mathematics and science education: A climate for change. In J. VanTassel-Baska (Ed.), *Proceedings of the 9th Annual Research Symposium of the Phi Delta Kappa Chapter of Northwestern University: The nature of mathematics and science education in a global society* (pp. 3–11). Evanston, IL: Northwestern University.

AUTHORS

KAREN C. FUSON, Professor, School of Education and Social Policy, Northwestern University, Evanston, IL 60208

JAMES W. STIGLER, Assistant Professor, Department of Behavioral Sciences, University of Chicago, 5730 S. Woodlawn Avenue, Chicago, IL 60637

KAREN BARTSCH, Graduate Student, Department of Psychology, University of Michigan, 3433 Mason Hall, Ann Arbor, MI 48109