Tasks That Support Problem Solving, Reasoning, and CCSS Mathematical Practices

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For more details about the CCSS-M and visual supports, please see the series of visual with audio Teaching Progressions I have made for various math domains. These can be found at karenfusonmath.com

Levels of Cognitive Process
Bloom's Cognitive Process Dimensions

Depth of knowledge

Cognitive complexity

Higher-level demands

Rich tasks

Performance assessment

Rigor

Tasks with higher-level demands

PARCC and Smarter Balance
Tasks with higher-level demands

To prepare for the tests, how much do you need to add on higher-level tasks to your math program?

First, let's examine NCTM's Principles to Actions about higher-level demands.

Then we will return to the task types and task claims in PARCC and Smarter Balance and answer this question.

Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics engages students in

solving and discussing tasks that promote mathematical reasoning and problem solving

and allow multiple entry points and varied solution strategies.

Principles to Actions: Ensuring Mathematical Success for All

NCTM, 2014, page 10.

Support students to do reasoning and problem solving

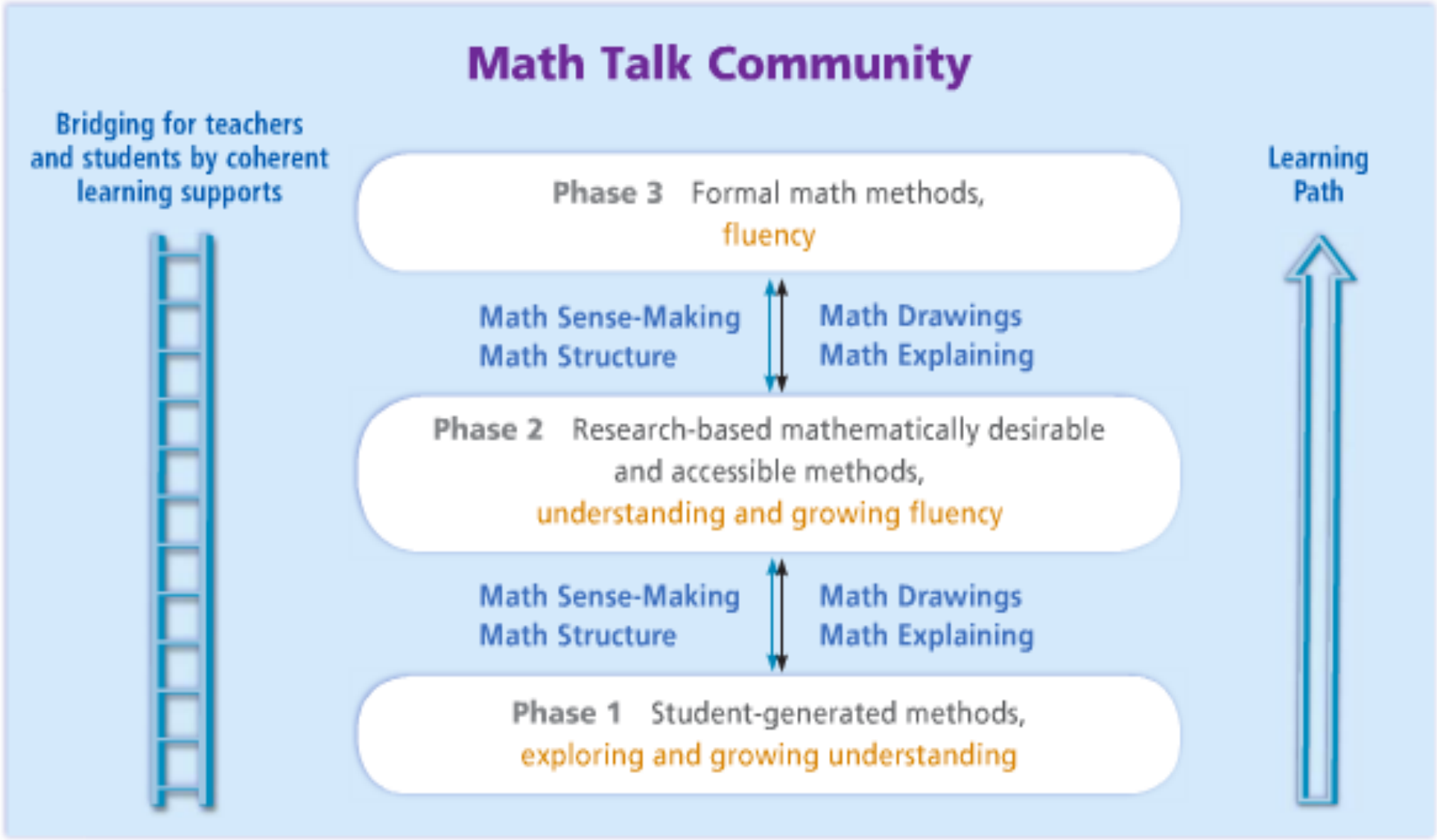
Effective teaching of mathematics engages students in

solving and discussing tasks that promote mathematical reasoning and problem solving

and allow multiple entry points and varied solution strategies.

Any task will do the above if it is in the learning zone of the students and if students are expected and supported to reason about it.

We need to shift from "Implement tasks that promote" to "support students to do" reasoning and problem solving.


Math Talk Community
Bridging for teachers and students by coherent learning supports
Phase 3 Formal math methods, fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 2 Research-based mathematically desirable and accessible methods, understanding and growing fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 1 Student-generated methods, exploring and growing understanding
Learning Path

Common Core Mathematical Practices Used in a Math Talk Community

Math Sense-Making: Make sense and use appropriate precision 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings: Model and use tools** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure: See structure and generalize 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining: Reason, explain, and question** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.

Figure 2

The Math Practices in action

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

Can I do some part of this better tomorrow?

The Solve, Explain, Question, and Justify Classroom Structure

Step 1 Solve: **All students solve.**

Step 2 Explain: **One student explains and then asks, "Are there any questions?"**

Step 3 Question: **Other students ask questions to clarify or extend.**

Step 4 Justify: **The original explainer responds to the questions by explaining more (justifying the original explanation).**

Any student at any time **can ask for help** from anyone.

Typically another student explains, so the class loops through 2, 3, and 4 again.

The discussion can now also **contrast and compare** the first and second solutions as well as others in the past.

Explaining can be done in **the whole class** or **in small groups** so that more students have a chance to explain.

Both versions have advantages.

What is new in the CCSS?

One major new aspect is the importance of visual models used by students for

- sense-making,
- problem solving, and
- supporting student explaining and Math Talk.

Students make math drawings/diagrams for

- NBT Number and Operations Base Ten computations,
- OA word problems and single-digit computations, and
- NF Number and Operations Fractions.

Support students to do reasoning and problem solving

Effective teaching of mathematics engages students in

solving and discussing tasks that promote mathematical reasoning and problem solving

and allow multiple entry points and varied solution strategies.

Ensure that students understand visual models that show the quantities or the problem situation before posing problems in that domain.

Use the mathematical practices: Help students to do math sense-making about math structure using math drawings to support math explaining.

Tasks with high cognitive demands are the most difficult to implement well and are often transformed into less demanding tasks during instruction.

Mathematics Teaching Practices

Use and connect mathematical representations

Facilitate meaningful mathematical discourse

Pose purposeful questions

Support productive struggle in learning mathematics

Elicit and use evidence of student thinking

Principles to Actions: Ensuring Mathematical Success for All

NCTM, 2014, p. 17 and p. 10.

Teach to promote reasoning and problem solving

What are students doing?

Persevering in exploring and reasoning through tasks.

Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.

Using tools and representations as needed to support their thinking and problem solving.

Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Principles to Actions: Ensuring Mathematical Success for All

NCTM, 2014, p. 24.

Teach to promote reasoning and problem solving

What are teachers doing?

Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.

Using problems that provide multiple entry points through the use of varied tools and representations.

On a regular basis, requiring a high level of cognitive demand when solving problems.

Supporting students in solving problems without taking over student thinking.

Encouraging students to use varied approaches and strategies to make sense of and solve tasks.

Principles to Actions: Ensuring Mathematical Success for All NCTM, 2014, p. 24, Fuson changed *tasks* to *problems*.

Teach to promote reasoning and problem solving

What are teachers doing?

Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. [Use phases 1, 2, 3 and learning progressions]

Using problems Selecting tasks that provide multiple entry points through the use of varied tools and representations. [Almost all problems can]

Posing tasks On a regular basis that require a high level of cognitive demand when solving problems. [Especially in phases 1 and 2 for a topic]

Supporting students in solving problems exploring tasks without taking over student thinking. [Support students to support each other]

Encouraging students to use varied approaches and strategies to make sense of and solve tasks problems. [They will do this if you let them.]

Principles to Actions: Ensuring Mathematical Success for All NCTM, 2014, p. 24.

The OA problem types form a learning progression that includes quite difficult problems.

We do not need to go outside of the standards to get difficult problems that promote reasoning.

The other CCSS domains also have concepts that are demanding for their grade level. Just teaching the CCSS using visual models and the mathematical practices promotes reasoning.

The 6 Situations

K — Add To / Take From

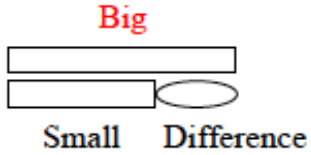
Start + Change = Result
Start – Change = Result

(becomes)

K — Put Together / Take Apart

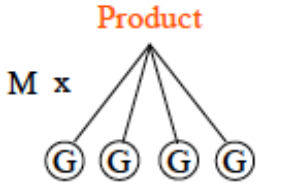
T = A + A
$\overset{i}{=}$ (identical)

Gr1 — Additive Comparison



Small + Difference = Big
Big – Difference = Small
Big – Small = Difference
$\overset{n}{=}$ (same number)

Gr3 — Equal Groups



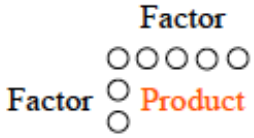
G + G + G + G = P
M times

M x G = P

(becomes)

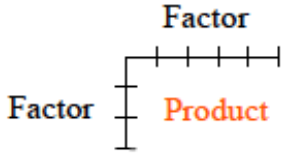
Gr3 — Rectangular Everything Times Everything

Array



(Long Division Format)

Area



Table

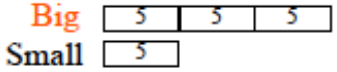
F x F = P
P ÷ F = F

(table)

$\overset{i}{=}$ (identical)

Gr4 — Multiplicative Comparison

Big = 3 x Small



Small = $\frac{1}{3}$ x Big

Big ÷ 3 = Small

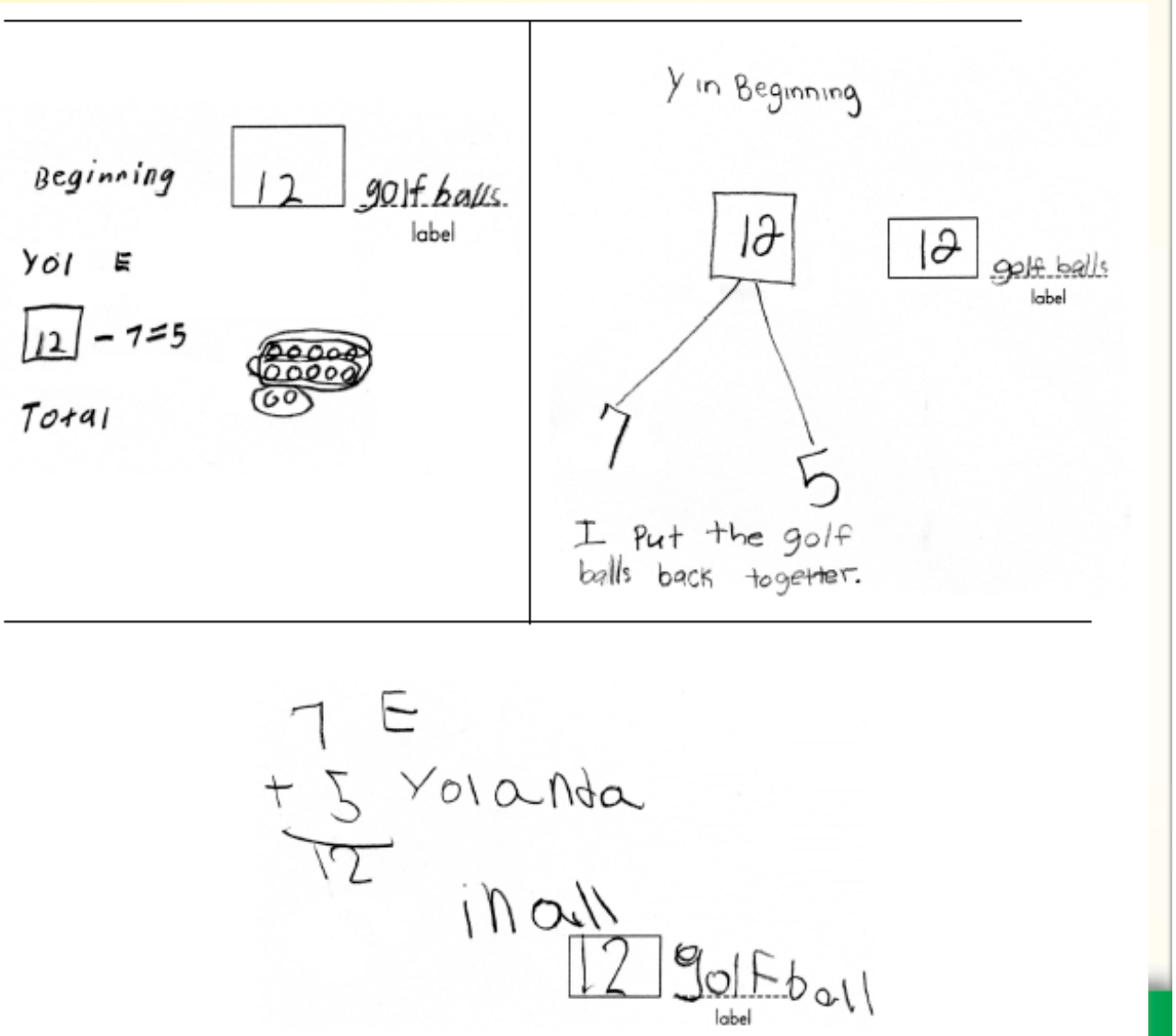
$\overset{n}{=}$ (same number)

CCSS Addition (top row) and Multiplication (bottom row) Word Problem Situations and *Math Expressions* Diagrams for Each

Grade 2 Labeled Math Drawings for a Start Unknown Problem

Yolanda has a box of golf balls. Eddie took 7 of them. Now Yolanda has 5 left. How many golf balls did Yolanda have in the beginning?

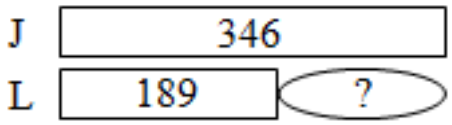
The key to solving story problems is understanding the situation. Students' equations often show the situation rather than the solution. Students drawings should be labeled to show which numbers or objects show which parts of the story situation.



Grade 3 Solution Approaches to an Additive Comparison Problem

In the summer Jana trimmed 346 bushes. Lisa trimmed 189 bushes. How many fewer bushes did Lisa trim than Jana?

Comparison Bar Drawing of Quantities



$$\begin{array}{r} {\scriptstyle 13} \\ 2\,\not{1}4\,16 \\ \not{3}\not{4}\not{6} \\ -\,1\,8\,9 \\ \hline 1\,5\,7 \end{array}$$

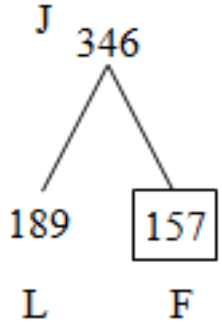
157 bushes

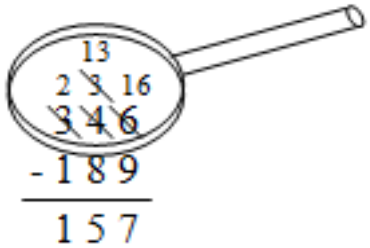
Situation Equation

L	d	J
189	+ 157	= 346
189	+ 11	= 200
200	+ 146	= 346
	157	

157 bushes

Numerical Relationships Shown in Math Mountain





157 bushes

Solution Equation

Jana	Lisa	fewer
346	- 189	= 157

$$\begin{array}{r} {\scriptstyle 13} \\ 2\,\not{1}4\,16 \\ \not{3}\not{4}\not{6} \\ -\,1\,8\,9 \\ \hline 1\,5\,7 \end{array}$$

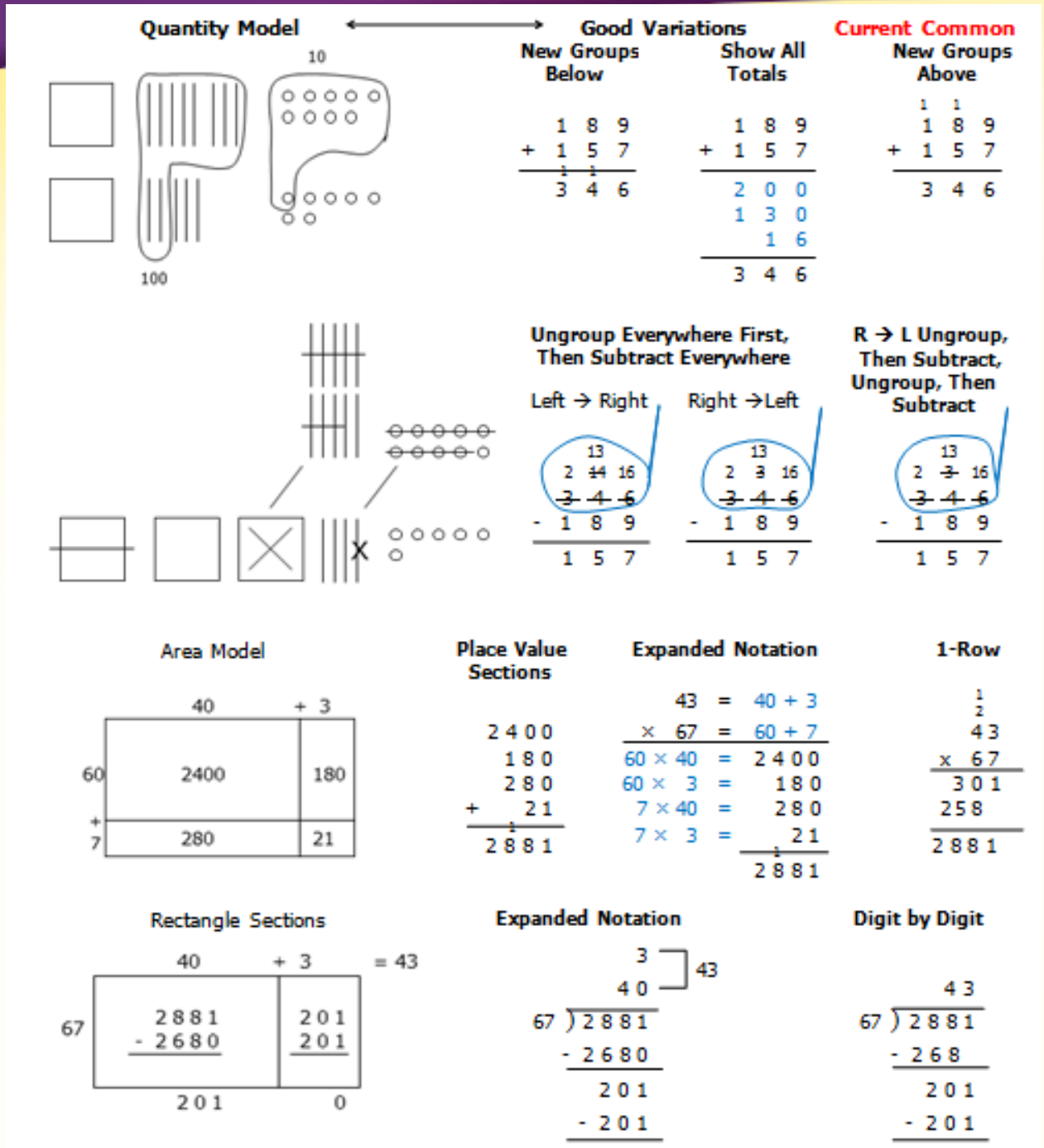
157 bushes

Tasks that promote reasoning and problem solving do not have to be set in a context

It is important to note that not all tasks that promote reasoning and problem solving have to be set in a context or need to consume an entire class period or multiple days. What is critical is that a task provide students with the opportunity to engage actively in reasoning, sense making, and problem solving so that they develop a deep understanding of mathematics.

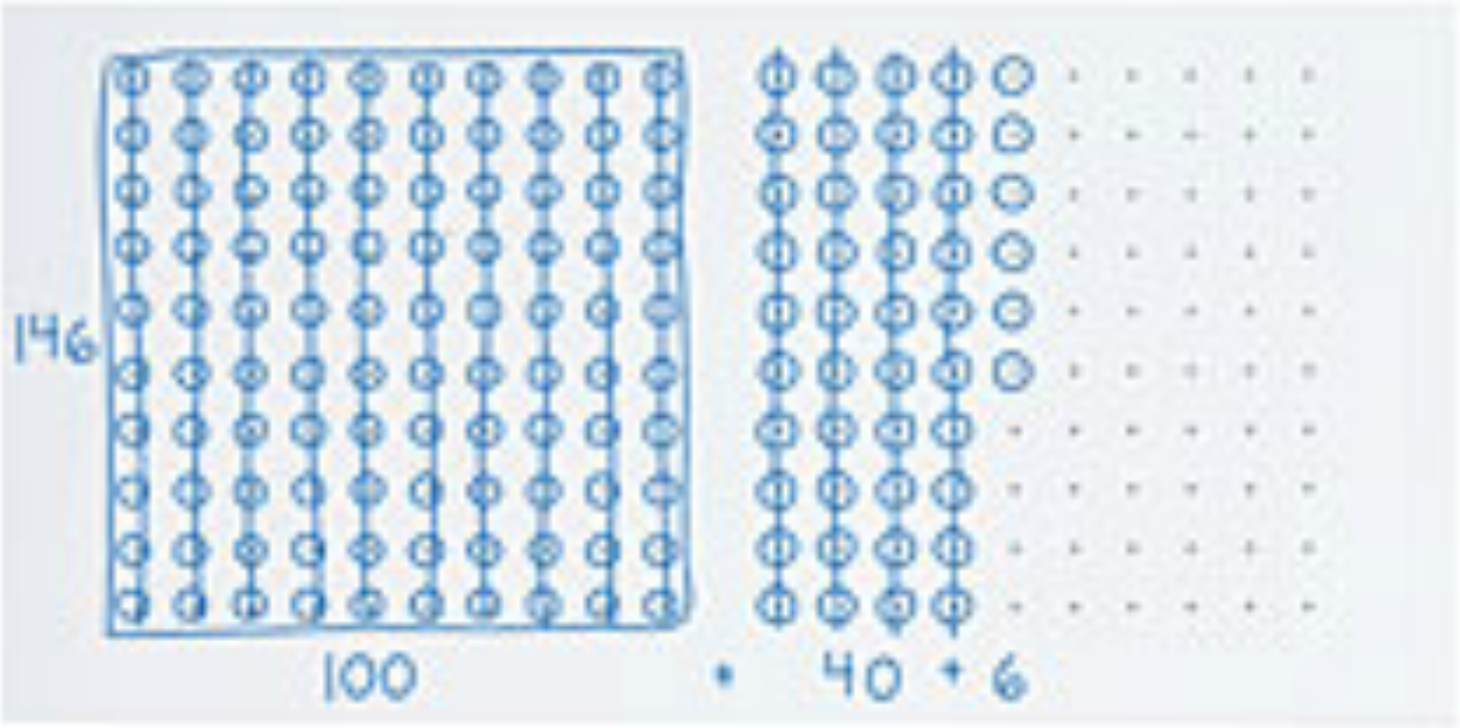
Principles to Actions: Ensuring Mathematical Success for All
NCTM, 2014, p. 17.

Drawings and Written Variations of Standard Algorithms

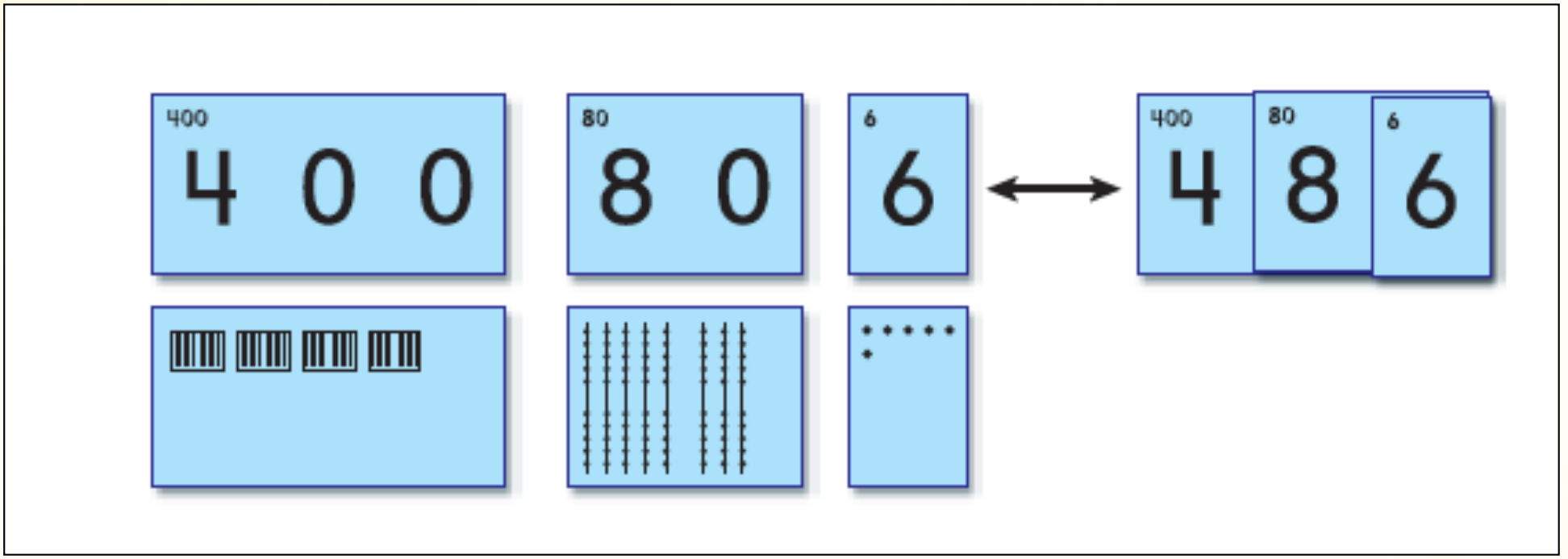


G2 Place Value Drawings 2.NBT.1 and 3

Hundreds **Tens** **Ones**



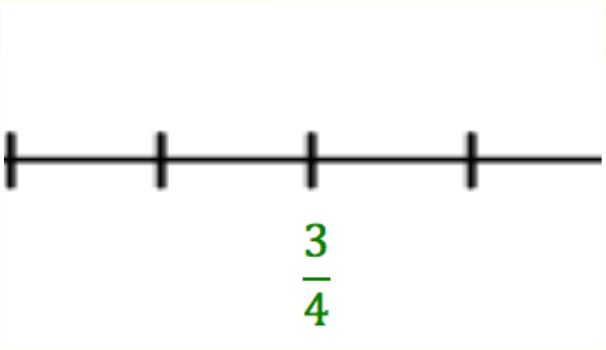
G2 Secret-Code Cards for 486 2.NBT.1 and 3



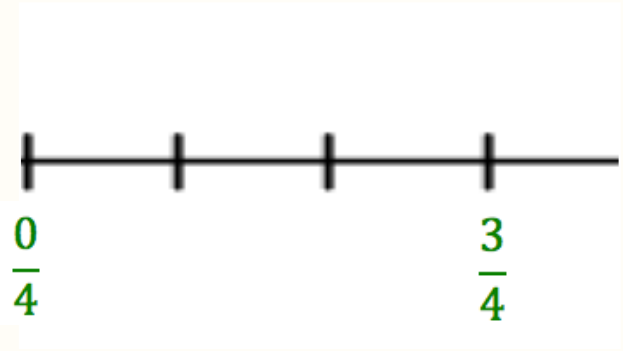
Not Enough Unit Lengths

Errors when drawing or using number lines

Error: Not enough unit lengths-
student counts marks rather than lengths.

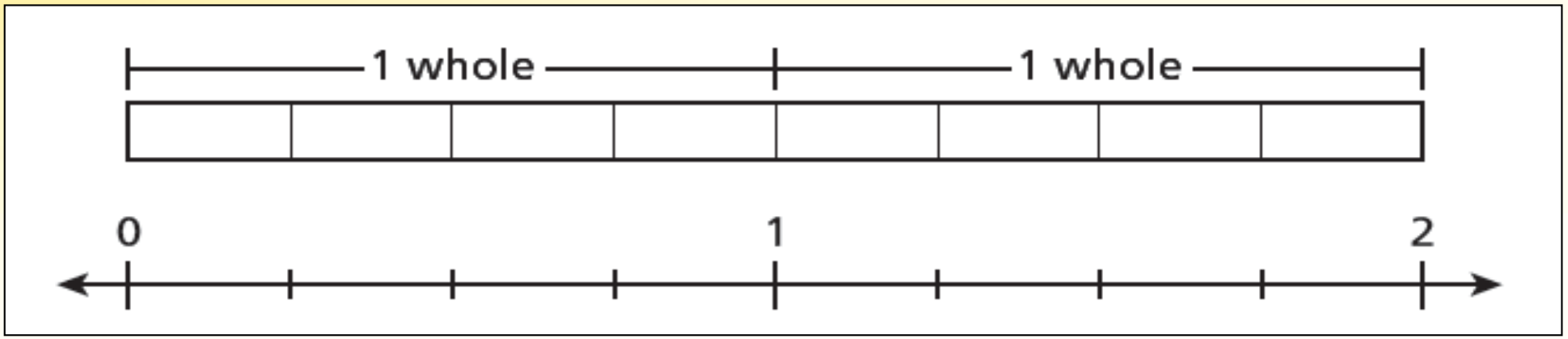


Correct: Count 3 unit lengths

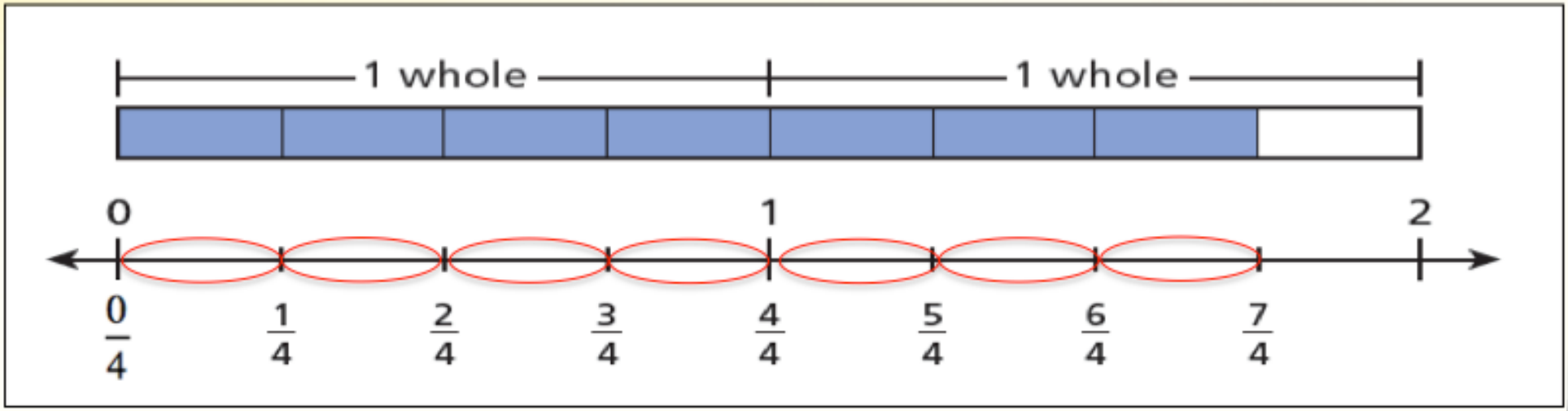


Seeing the Fraction Lengths

Step 1: Make the 4 unit fractions ¼ within each 1 whole.



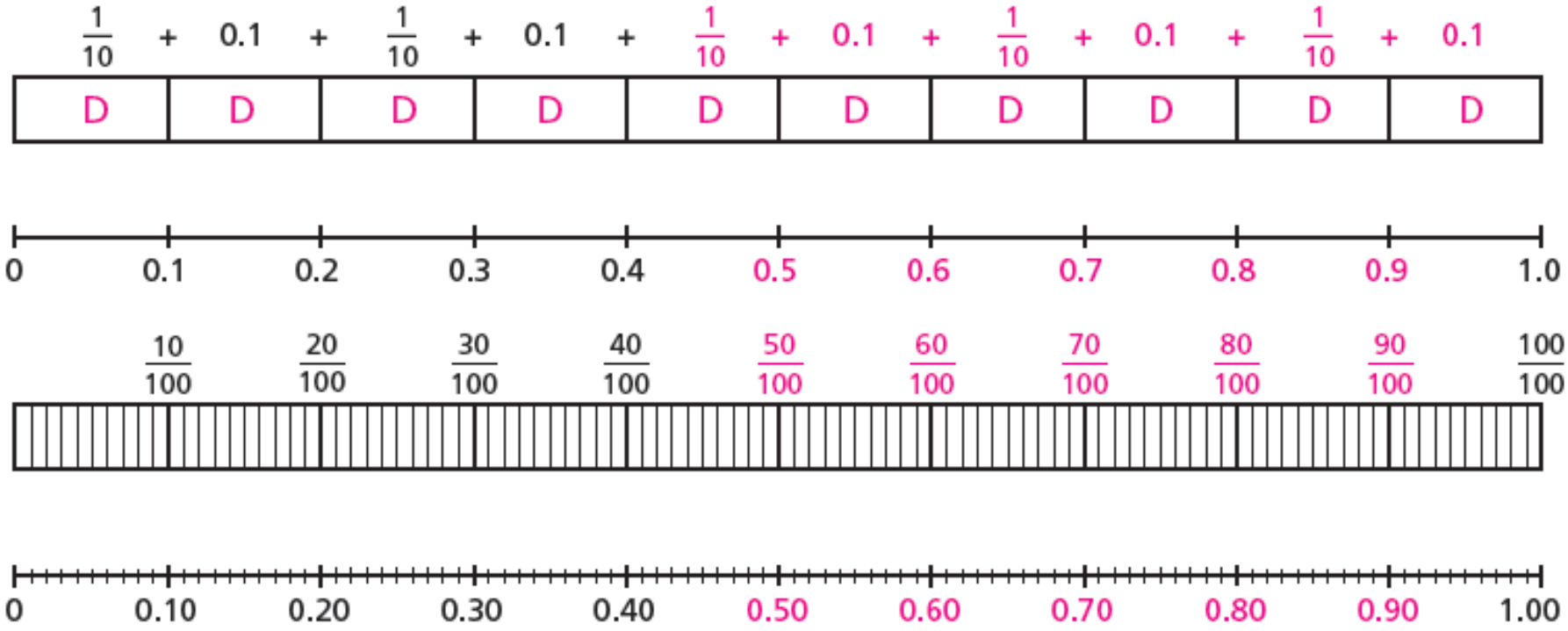
Step 2: Shade or encircle 7 unit fractions and label the number line.



The Number Line Numbers Tell the Number of Units So Far

▶ Understand Tenths and Hundredths

Answer the questions about the bars and number lines below.



Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings
that students can make and use in their own ways
in problem solving and explaining of thinking.

We want classrooms to be using the mathematical practices:
Students focus on math sense-making about math structure
using math drawings (visual models) to support math explaining.

Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

A small glitch in Principle to Actions:
The Tools and Technology Principle (pp78 to 88) should have explicitly rather than just implicitly included visual models and student drawings/diagrams as tools.

Manipulatives and drawings/diagrams are problem solving tools in the CCSS, and drawings/diagrams are specified in some standards.

Tasks do not make a CCSS math program.

You must have deep coherent learning progressions in all domains and use the mathematical practices.

Tasks do not make a CCSS test prep.

You must have deep coherent learning progressions in all domains and use the mathematical practices.

What test prep would be helpful?

PARCC Type I tasks: Concepts, skills, and procedures: Balance of conceptual understanding, fluency, and application

Smarter Balance Claim 1 tasks: Concepts and procedures: Explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency

Conceptual preparation for these test items will be met by the approach outlined above.

Test prep will be needed to help students use technology and understand unfamiliar test item formats.

PARCC Type III tasks: Modeling/applications

Smarter Balance Claim 2 tasks: Solve complex, well-posed problems in pure and applied mathematics

Conceptual preparation for these test items will be met by the approach outlined above, especially by extensive use of the CCSS OA word problem types throughout the program with coherent related research-based diagrams to support problem solving and explaining.

Test prep will be needed to help students use technology and understand unfamiliar test item formats.

PARCC Type II tasks: Expressing mathematical reasoning

Smarter Balance Claim 3 tasks: Construct viable arguments to support reasoning and critique the reasoning of others

Conceptual preparation for these test items will be met by the approach outlined above especially the use of the math practices to support the math talk community: solve, explain, question, justify.

There is a problem with the tests at this time: The technology cannot support the depth of explaining that teachers can support in their classrooms. So I suggest that districts record students explaining and post these on district websites for parents.

Districts Record Students Explaining These Key Milestones with Drawings and Share with Parents

Kindergarten: Ten in teens

G1: 2-d addition with new groups

G2: 3-d subtraction (e.g., 163 – 89)

G3: 3-d addition (e.g., 387 + 259)

G4: 2-d x 2-d (e.g., 37 x 65)

G5: 3/4 + 2/5

G6: 3/4 ÷ 2/5

Subtraction WP (e.g., 9 – 5)

Unknown addend WP (8 + ? = 14)

Start unknown WP (e.g., ? – 6 = 8)

3-d subtraction (e.g., 802 – 356)

with no drawing (fluency level) but use place value words for explaining

3-d ÷ 1-d with remainder (e.g., 293 ÷ 8)

3/4 x 2/5

division with decimals (e.g., 1.984 ÷ 0.32)

PARCC Performance-Based Assessment: Applying skills, concepts, and understandings to solve multi-step problems requiring abstract reasoning, precision, perseverance, and strategic use of tools

Smarter Balance Claim 4 tasks: Modeling and data analysis for complex scenarios **(low prominence in K to G5)**
and **Performance Tasks:** Connected components that assess a range of mathematical knowledge and skills

Conceptual preparation for these test items will be mostly met by the approach outlined above.

Some test prep may be needed to help students be confident in solving multi-step problems or extended connected components.

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