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An Analysis of Addition and Subtraction Word Problems in American and Soviet Elementary Mathematics Textbooks

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Current theories of how elementary school children solve addition and subtraction word problems have emphasized semantic characteristics of the problems as the major factor influencing ease of solution. The present study assesses the potential impact the instructional environment (textbooks, in particular) might have on the relative difficulty of different types of addition and subtraction word problems by comparing the presentation of word problems in four American textbook series with the corresponding presentation in one Soviet textbook series. In general, the four American text series were found to resemble each other but to differ markedly from the Soviet text series. Several important findings emerged: (a) Distribution of word problems across the various problem types was extremely uneven in the American texts, with two thirds of all problems being of only three simple one-step problem types. The Soviet problems were distributed over many types, including more complex two-step problems; (b) most of the problems in the American texts are those that American children find easiest to solve; (c) Soviet textbooks also provide a more variable and a more distributed method of presentation than do the American textbooks. The implications of this study are discussed in terms of theoretical models of word problem solving and in terms of practical ideas about textbook construction.

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How children solve simple addition and subtraction word problems has been of interest for a very long time. In recent years researchers have been investigating children's solutions of a wide range of such problems (e.g., Carpenter & Moser, 1983; Carpenter, Moser, & Romberg, 1982; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983). Central findings have been that the ease with which children solve a particular problem varies according to the semantic structure of the problem, the position of the unknown quantity, and the precise way in which the problem is worded. Classification schemes have been developed and refined (e.g., Carpenter & Moser, 1983; Riley et al., 1983; Usiskin & Bell, 1983) to categorize problems along the relevant dimensions, and, indeed, performance varies dramatically across the different categories of simple problems.

Although these studies have been useful in relating children's performance to specific characteristics of the problems themselves, they have not addressed either the question of how these problems are distributed through the elementary mathematics curriculum or the effect that this distribution might have on children's performance. A child solving a word problem certainly is influenced by the structure of a given problem. But this child also is influenced by the environment in which the skills and understandings necessary for solving such a problem were acquired. Such variables as how often similar problems have been encountered, and the context in which these problems were encountered, also will affect performance. It is these variables that constitute the focus of the present study.

Current theories of how children solve addition and subtraction word problems involve at least two components (Briars & Larkin, 1984; Kintsch & Greeno, 1985; Riley et al., 1983). One component is the identification and representation of the problem. Students are hypothesized to have or to be able to construct schemata corresponding to different types of problems. These conceptual schemata are activated or constructed during the initial representation of a problem. The nature of the hypothesized representation differs in the different theories, and according to the developmental level of the problem solver. The representation may be of a concrete object or a more abstract representation; it may be constructed word by word or from whole sentences. A second component common to all models is the selection of an appropriate action schema for solving the problem. This action schema is directly activated by the conceptual representation, although additional knowledge or processing may be required in order to select this action schema.

In all the present models, difficulty is primarily a function of problem characteristics that affect problem representation. Therefore, improvements in the ability to represent more difficult word problems should result in an increased ability to solve such problems. Longitudinal data from Carpenter and Moser (1984) also indicate clearly that children get better at the second component: Children in Grades 2 and 3 show increasing flexibility in their choice of solution procedures for given problems. Although the models describe increasingly sophisticated problem representations that children are hypothesized to use to solve word problems, very little research has been aimed at understanding the processes by which schemata for representing word problems develop through time.

Furthermore, to date there is inadequate recognition in the formal models themselves that solving a word problem may be quite different when the problem is unfamiliar than when the problem has become familiar. Although initially a child may complete the painstaking process of constructing a problem representation each time a particular type of problem is encountered, eventually the child may simply use cues in the problem text to retrieve the appropriate problem schema from memory. And while the availability of a problem schema in memory will in part be a function of how easily representations of problems of that type were constructed in earlier encounters, it surely also will be a function of the number of times such problems have been encountered. This would seem to be equally true concerning the selection of an action schema for solving a problem once an adequate representation has been constructed. The automaticity and reliability of this selection also will be related to the number of times problems of a particular type have been solved.

In other words, the problem solver brings more knowledge to bear on solving word problems than only that which is constructed upon reading the problem. It is important to tie problem-solving performance to an analysis of the task environment, as other authors have done. But it is also important to extend analysis to the broader environment in which schematic knowledge of the problem domain was acquired. Both kinds of analysis will be necessary before we can have a complete picture of how children solve word problems.

In the present study, we analyzed primary school mathematics textbooks in order to evaluate the roles that characteristics of problem presentation might have in children's developing abilities to solve word problems. Two characteristics of problem presentation were hypothesized to relate to children's abilities. First, the frequency with which children are exposed to problems of different types should relate to the ease with which problems are solved. Even though certain types of word problems have been shown to be difficult for children to solve, almost no research has investigated and reported the relative frequency with which problems of those types are encountered in elementary mathematics textbooks. One study (DeCorte, Verschaffel, Janssens, & Joillet, 1984) did report an analysis of addition and subtraction word problems in first-grade Belgian textbooks, revealing that the range of problems presented was quite restricted and in general limited to the easiest types. Learning whether this finding also applied to American texts and whether it extends to grades beyond the first is important for understanding the constraints operating on children who are building problem representations.

The second characteristic of problem presentation that is of interest here is the amount and kind of variability in problem presentation. Although problems of a given problem type may need to be grouped together during the initial formation of representational schema for that type, children also need to practice on sets of mixed problem types in order to learn to differentiate these types. Mixed problem types also need to be used because only such problem sets ensure that children are actually reading, representing, and solving the problems rather than just adding (or subtracting) all the pairs of numbers in the problems, which children can do when all the problems on a page are of a given type. Furthermore, the extent to which children need any massed practice at all on certain simple types seems debatable, for considerable data indicate that children come to school already possessing representational schemas for many of the problem types (Carpenter & Moser, 1983). Thus, for these types of problems, children might do best by moving directly into mixed practice.

The approach we have taken is to analyze textbooks according to these hypothesized factors. By comparing the way in which problems of different types are presented within several American text series, we can see if such factors are in fact related to American students' performance on problems of different types.

We also wished to compare the number, range, and organization of problems presented to American children with the number, range, and organization of problems presented to children in the Soviet Union. The Soviet Union has a long tradition of considerable emphasis in the mathematics curriculum on the solving of word problems, and Soviet researchers have considerably scrutinized this area. The Soviet Union also is presently viewed as having a very successful mathematics curriculum at the elementary school level (Wirzup, 1986). Thus it seemed of considerable interest to see how American and Soviet textbooks might differ in their treatment of word problems.

In summary, the goal of the present study was to analyze the presentation of simple addition and subtraction word problems in several popular series of American mathematics textbooks and in the Soviet elementary mathematics textbooks. The problems were categorized according to the dimensions suggested by the literature cited above, and three specific questions were asked: (a) How could the presentation of problems be described in terms of variability, frequency, and spacing? (b) How do these characteristics of problem presentation relate to the literature on American children's performance on the different types of word problems? (c) How does the analysis of problem presentation in the American text series compare to a similar analysis performed on Soviet textbooks?

METHOD

Selection of Texts

Elementary mathematics textbook series were selected for analysis at the first-, second-, and third-grade levels. The American curriculum was represented by four widely sold and widely used standard textbook series: Addison-Wesley (1983), Harper & Row (1985), Houghton Mifflin (1985), and Scott, Foresman (1985). The Soviet textbook series was officially mandated by the Soviet government and was used throughout the Soviet Union (Moro & Bantova, 1980; Moro, Bantova, & Beltyukova, 1980; Pcholko, Bantova, Moro, & Pyshkalo, 1978). A complete English translation of the Soviet textbooks was prepared and provided to the researchers by the University of Chicago School Mathematics Project. All analyses were based on the English translations.

Coding of Problem Types

Many coding schemes have been developed for classifying addition and subtraction word problems into types. The coding scheme used in this study was that employed by Carpenter and Moser (1983). This coding scheme, although slightly more detailed, is consistent with other classification schemes found in the literature (Carpenter & Moser, 1982; Greeno, 1980; Nesher & Katriel, 1977; Riley et al., 1983; Usiskin & Bell, 1983). The scheme categorizes problems according to their semantic structure (i.e., type of story action) and according to the position of the unknown in the equation representing the story. A summary of the scheme, as presented by Carpenter and Moser, is presented in Table 1.

The coding scheme identifies 20 types of addition and subtraction story problems. These 20 types are grouped into four major categories: change, combine, compare, and equalize. Although equalize problems occasionally appear in American elementary school texts (Romberg, Harvey, Moser, & Montgomery, 1974) and more recently in experimental programs in the Soviet Union (Davydov, 1982) and in Japan (Gimbayashi, 1980), only the Type 15 equalize problem was found in any of the texts analyzed for the present study. Consequently, the other types of equalize problems were ignored in subsequent analyses, and only Codes 1 to 15 appear in the graphs and tables presented here.

An important limitation of the above coding system was that it only could be applied to problems solvable with a single step. A few problems in the American text series and many problems in the Soviet series required the successive use of two steps. Such two-step problems were coded by applying the

TABLE 1
Classification of Word Problems

Join	*Separate*
Change	
1. Connie had 5 marbles. Jim gave her 8 more marbles. How many marbles does Connie have altogether?	2. Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?
3. Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	4. Connie had 13 marbles. She gave some to Jim. Now she has 8 marbles left. How many marbles did Connie give to Jim?
5. Connie had some marbles. Jim gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?	6. Connie had some marbles. She gave 5 to Jim. Now she has 8 marbles left. How many marbles did Connie have to start with?
Combine	
7. Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	8. Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?
Compare	
9. Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?	10. Connie has 13 marbles. Jim has 5 marbles. How many fewer marbles does Jim have than Connie?
11. Jim has 5 marbles. Connie has 8 more than Jim. How many marbles does Connie have?	12. Jim has 5 marbles. He has 8 fewer marbles than Connie. How many marbles does Connie have?
13. Connie has 13 marbles. She has 5 more marbles than Jim. How many marbles does Jim have?	14. Connie has 13 marbles. Jim has 5 fewer marbles than Connie. How many marbles does Jim have?
Equalize	
15. Connie has 13 marbles. Jim has 5 marbles. How many marbles does Jim have to win to have as many marbles as Connie?	16. Connie has 13 marbles. Jim has 5 marbles. How many marbles does Connie have to lose to have as many marbles as Jim?
17. Jim has 5 marbles. If he wins 8 marbles, he will have the same number of marbles as Connie. How many marbles does Connie have?	18. Jim has 5 marbles. If Connie loses 8 marbles, she will have the same number of marbles as Jim. How many marbles does Connie have?
19. Connie has 13 marbles. If Jim wins 5 marbles, he will have the same number of marbles as Connie. How many marbles does Jim have?	20. Connie has 13 marbles. If she loses 5 marbles, she will have the same number of marbles as Jim. How many marbles does Jim have?

coding scheme separately to the two parts of the problems, thus creating new categories containing all combinations of one-step problems. Take, for example, the following problem: "Two swimmers swam toward one another in a swimming pool lane. One swam 27 meters before meeting the other, and the other swam 4 meters less. How long is the pool?" This problem was coded 14/7. The first step toward solution compares the known referent set (27 m) with the unknown compared set and notes the difference (4 m less); this step is thus coded Type 14. In the second step, the just-calculated second swimmer's swim of 23 m is combined with the original referent set of 27 m to yield an answer of 50 m; this step is coded Type 7.

To keep the number of different problem types to a manageable size, we did not differentiate two-step problems requiring the same two problem types according to which operation would be done first. Thus Type 7/14 included both those problems in which one did the Type 7 problem before the Type 14 problem and vice versa. Furthermore, in some American text series, there were some Type 7 problems in which three (and very occasionally four, five, or six) numbers were to be added. These problems do not have to be considered as two-step problems, for to do them one merely sets up one arithmetic problem: the column addition of all the numbers in the problem. Because these are complex addition problems, however, they were included and were coded as 7/7.

Procedure

All story problems that involved only addition and/or subtraction were coded according to the above scheme. To work out the coding procedure, two coders, working independently, coded each problem in the Soviet and in the Scott, Foresman series. After the initial coding, the two coders agreed on 84% of the problems in the Soviet texts and on 94% in the Scott, Foresman texts. Each problem for which the coders did not agree then was discussed until agreement was reached on how the problem would be coded. The lower agreement for the Soviet problems occurred mainly because there were many more two-step problems in the Soviet than in the Scott, Foresman texts.

The three other American series were then coded by two other coders. These coders agreed on 97% of their codings. Again, disagreements were discussed and resolved by the two coders. To ensure that the two sets of codings were consistent, one member from the second coding team coded all the problems in the Scott, Foresman series and most of the problems in the Soviet series. This coding agreed with the arbitrated final coding of the first team on 97% of the problems. Problems designated in the American series as particularly difficult (e.g., "think" or "challenge" problems) were included in the coding because they were available for all children to do, even though the teacher might not assign them. Some series had an appendix of problems to accompany certain pages. Word problems from these appendices were not included because they were considered optional.[1]

The most difficult decision in coding the American books was deciding what constituted a word problem. Presentation in the American texts ranged from addition and subtraction problems displayed entirely with pictures to problems given entirely in words, with many types of intermediary forms that combined words and pictures. Some texts also presented groups of identical problems in a condensed form in which part of a problem would be presented once at the top of a page (e.g., the question "How many in all?") followed by a series of statements, each of which could complete the problem (e.g., "six balls and two kites" and below that "4 oranges and 3 oranges" and below that "three dogs and two cats," etc.). Although the child must give an answer in response to each additional statement, it is necessary only to figure out how to solve one problem (the first one). After that, one can simply apply the same arithmetic operation to each of the additional statements.

Because we were interested in describing the opportunities children have to engage in word problem solving, we defined a word problem as consisting of two premises (the given information) and a question. To be coded, a problem had to present two or more premises and a question. Thus the above "collapsed problems" were coded as only one problem. For problems presented with a combination of words and pictures, the number could be presented with either a numeral or a word, and an entity (e.g., a ball) could be presented either by a picture of a ball or by the word *ball.* However, each premise and the question had to be presented in a verbal form or in an iconic form isomorphic to a verbal form. Thus 3 O (numeral 3 and picture of a ball) was acceptable as a premise but OOO (a picture of three balls) was not. Some texts also contained problems in which a complex picture of a situation was presented, and several word problems were given with the picture. The word problems contained blanks for the numerical information in the premises. The child had to fill in the blanks by using the picture (e.g., the price or number of some entity) and then solve the word problem. These problems were coded as word problems because each was a complete word problem as soon as the child filled in the blanks.

RESULTS

A detailed tabulation of the frequency of presentation of each problem type, broken down by grade level and text series, is presented in Table 2.[2] Inspection of the table reveals several interesting findings. First, the total num-

[1]An additional analysis in which these problems were coded yielded results similar to those reported in this article.

ber of addition and subtraction word problems across all three grade levels varies considerably from series to series. The Soviet series has the most problems (493), whereas the American series have somewhat fewer problems (ranging from 328 to 430).

Second, there are very many more two-step problems in the Soviet books than in any of the American books. Across all American text series, two-step problems comprised only 7% of the total number of problems. The only two-step problem present to any extent in the American books is Type 7/7. Furthermore, many of the few two-step problems in the American texts were the special "challenge" problems not necessarily targeted to all children. In contrast, 44% of the Soviet problems were two step. There is a rich range of such two-step combinations represented in the Soviet texts, and many such problems (half the total) are given in the first year of school.

Third, the distribution of problems across the three grades also varies considerably from series to series. The number of both one-step and two-step problems drops precipitously across grade levels in the Soviet series, whereas it rises precipitously after first grade in three of the American series and rises gradually in the fourth (Harper & Row). The Soviet first-grade text contains 3 times as many addition and subtraction word problems as the American first-grade text with the most problems and 10 times as many such problems as the American first-grade text with the least problems.[3]

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[2]A difference between the United States and the Soviet Union in the age at which children start school presents us with a potential methodological problem. In the Soviet Union, children begin the first grade at age 7, whereas American children begin first grade at age 6. In making this cross-cultural educational comparison, it is not clear whether one ought to compare equivalent chronological age (because age will influence learning capacity) or equivalent years in school (because school years will influence opportunity to learn). The most valid comparison probably lies somewhere between these two. For this reason, most data are reported broken down by grade level so that both comparisons can be made. For the sake of brevity, we do not always discuss both such comparisons, but the reader will be able to make the other comparison by examining the tables.

Fortunately for the ease of making both age and grade comparisons, the data indicate that very few addition and subtraction word problems were contained in the third-grade Soviet book. (It is important to note that this does not mean that the third-grade Soviet book contains no word problems; rather, the bulk of third grade word problems in the Soviet series involve multiplication and division, and so are not included in the present analyses.) Thus any comparison of totals across texts will be a valid comparison for both years in school and age (because the Soviet 1–2 total is equal to or greater than the American 1–3 totals).

[3]It seemed possible that the American texts might present the more difficult problems in Grades 4 through 6, thus escaping our analysis. We checked the series that presented the largest range of problems in the first three grades (Scott, Foresman) and found little evidence to support this notion. Although these upper-level texts did not emphasize the three easiest problems to the same extent as in the lower grades, there was not an increased emphasis on the less common problems, nor a substantial increase in two-step problems.

TABLE 2
Number of Problems by Type, Grade, and Series

	Soviet				Addison-Wesley				Harper & Row				Houghton Mifflin				Scott, Foresman			
	1	*2*	*3*	*Total*	*1*	*2*	*3*	*Total*	*1*	*2*	*3*	*Total*	*1*	*2*	*3*	*Total*	*1*	*2*	*3*	*Total*
1	7	1	2	10	8	14	9	31	11	9	13	33	13	16	4	33	13	13	30	56
2	11	3		14	24	28	14	66	43	29	25	97	32	20	14	66	47	38	21	106
3	2	1	2	5			1	1											1	1
4	12	5	1	18			2	2		4	1	5						1	4	5
5	2		1	3							4	4							4	4
6	13	6		19						2	1	3								
Change	47	16	6	69	32	42	26	100	54	44	44	142	45	36	18	99	60	52	60	172
7	16	6	4	26	32	61	56	149	30	26	39	95	35	53	46	134	20	41	41	102
8	14	10	3	27		4	7	11	8	9	24	41		3	12	15	4	4	25	33
Combine	30	16	7	53	32	65	63	160	38	35	63	136	35	56	58	149	24	45	66	135
9	20	8	1	29		71	24	95	3	9	7	19	1	49	42	92		17	38	55
10	17	2	1	20			1	1			7	7	1		3	4		2	2	4
11	18	11		29			1	1		3		3	1		1	2			22	22
12	14	8	2	24																
13	16	8	3	27			1	1							1	1				
14	16	7		23															1	1
15	1			1													4	14	1	19
Compare	102	44	7	153		71	27	98	3	12	14	29	3	49	47	99	4	33	64	101
One-step total	179	76	20	275	64	178	116	358	95	91	121	307	83	141	123	347	88	130	190	408

1/1	1			1											2	2		1	3	4
1/2	10	1		11															1	1
1/6		1		1															1	1
1/7	3	2		5							1	1								
1/14	1			1																
2/2	9	4	1	14											1	1			1	1
2/6																				
2/7	30	11	3	44						1	2	3			3	3				
2/9		4		4																
2/10		4	1	5																
4/6			1	1																
6/6																				
6/7	1	5		6																
6/11			1	1																
7/7	2	1	1	4	3	3	7	13		2	15	17	2	3	33	38		2	12	14
7/8	2	12	7	21																
7/9		1		1									1		3	4				
7/11	18	15	4	37			1	1							1	1				
7/12		1	5	6																
7/14	29	14	1	44																
8/9																				
10/11																				
11/11		1		1															1	1
11/14		6	1	7																
12/13			1	1																
14/14		2		2																
19/20							1	1												
Two-step total	106	85	27	218	3	3	9	15		3	18	21	3	3	43	49		3	19	22
Total	285	161	47	493	67	181	125	373	95	94	139	328	86	144	166	396	88	133	209	430

Distribution of One-Step Problems

Because almost all the research that has been conducted in children's word problem solutions has used the type of one-step problems outlined in Table 1, it is interesting to compare the presentation of one-step problems across the different text series. For this purpose, we average across grade levels.

An overall view of the distribution of one-step problems across the 15 different types that appeared is presented in Figure 1 for each of the five textbook series, summed across all grade levels. (The American series are depicted by solid lines and the Soviet series by a dotted line.)

It is immediately obvious in Figure 1 that the four American text series bear a strong resemblance to each other and differ markedly from the Soviet text series. The Pearson correlation coefficients between all pairs of the four American text series, calculated across the 15 problem types, ranged from .79 to .998, with a mean of .88. By contrast, the correlations of each American curve with the Soviet curve ranged from .12 to .29, with a mean of .20.

In general, the line representing the Soviet books is relatively flat, depicting an even distribution of problems according to type. The lines representing the American books, by comparison, are peaked and uneven, reflecting a highly uneven distribution of problems across the 15 types. Soviet children thus are exposed to a much greater variety of addition and subtraction word problems than are American children.

The uneven distribution of problems in the American texts is further described by the following six points:

1. There are roughly equal numbers of the six kinds of compare problems in the Soviet books, whereas the Type 9 compare problem is the only

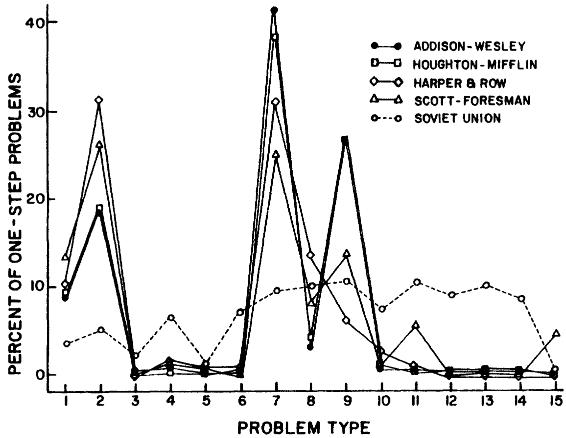

FIGURE 1 Distribution of one-step word problems according to type in one Soviet and four American elementary textbook series.

type of compare problem present in American books to any considerable extent.

2. The Soviet series presents equal numbers of the two kinds of combine problems, whereas the most equally balanced American text presents only half as many Type 8 (missing part) as Type 7 (missing whole) problems, and the least balanced American text presents only one tenth as many.
3. The Soviet texts present all six kinds of change problems, with fairly equal distribution across four of the six types. In contrast, the American texts present a very large number of two of the six change problems and either none or a very small number of the remaining four types.
4. The Soviet texts present 10 or more problems for 12 of the 15 problem types, whereas three American texts present 10 or more problems for only 5 of the 15 problem types, and the fourth American text presents that many for only 7 of the 15 problem types.
5. In the Soviet books, the most frequent problem (7/14) comprises only 9% of the total. The American books vary in which problem type is the most frequent (either Type 2 or Type 7), but in all four series the most frequent type comprises almost a third of the problems.
6. In the Soviet books, the three most numerous problem types are all two-step, and together they comprise 26% of all the addition and subtraction word problems. In the American books, the three most numerous problem types are all single-step, and they comprise 83%, 72%, 72%, and 62% of the addition and subtraction word problems in the four American series (corresponding to their ordering as in Table 2). If one considers the four (as opposed to three) most numerous types, these represent 91%, 82%, 80%, and 75% of the problems in the American series (ordered as above), whereas only 32% of the Soviet problems fall into the four most common problem types.

What is the nature of the problems that are emphasized to such an extent in the American series? Examination of the four most numerous problem types shows that all of the high-frequency problems in the American texts have semantic structure equations that are identical to their solution procedure equations. For example, the semantic structure equation for the change join (Type 1) problem in Table 1 is $5 + 8 = ?$, and one adds the two given numbers to find the answer. Similarly the semantic structure equation for the change separate (Type 2) problem is $13 - 5 = ?$, and one subtracts the two given numbers to find the answer. The semantic and solution equation for problem Type 7 is $5 + 8 = ?$ and for Type 9 and Type 10 is $13 - 5 = ?$. For each American text series, all of the four most numerous types of problems (with the exception of Type 8 problems in Harper & Row) and the vast bulk of all problems are of this simplest type: ones in which the arithmetic solution procedure directly parallels the semantic structure of the problem.

Of the 15 types of one-step problems, these most frequent examples are by far the easiest for American children to solve, according to the research literature (e.g., Carpenter & Moser, 1983, 1984; Riley et al., 1983). A rough confirmation of this observation is provided by calculating the Pearson correlation coefficient across the first 14 problem types between the summed frequency of presentation across all four American series and the difficulty of solution (provided by Riley, 1981, for first-grade children). The correlation coefficient is .66. The corresponding correlation of frequency of presentation in the Soviet textbooks with performance of American children is −.35. Clearly, there is a bias in the American textbooks toward presenting the problems that American children find easiest to solve.

In addition to the frequency with which different types of problems were presented, we also were interested in the way problems were sequenced within a given textbook. Were problems of a single type presented together, or were problems mixed in presentation according to type? To answer this question, all problems presented in each text series were divided into consecutive groups of 10 problems, and these 10-problem groups then became the unit of analysis. Two analysis were carried out on these 10-problem units.

The first analysis was conducted as follows: Each 10-problem unit was coded for the number of different types of problems occurring in the unit. Thus, for each group of 10 problems, coding could range from 1 (if all problems were of the same type) to 10 (if all problems were of different types). The results of the analysis are depicted in the upper panel of Figure 2, averaged

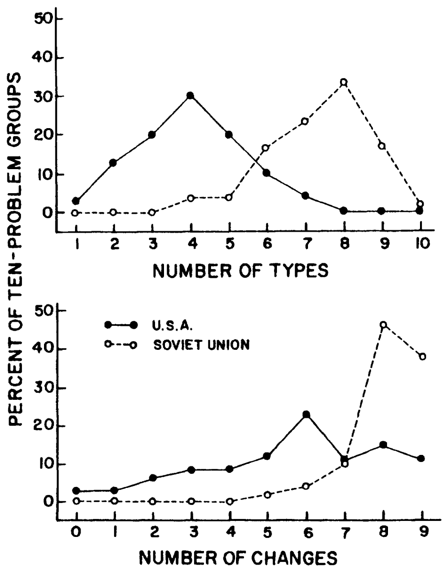

FIGURE 2 Number of types and changes in types in 10-problem groups for Soviet and American textbooks.

across the four American text series and across the three grade levels in both the American and Soviet texts. In this figure, the distribution of 10-problem units is plotted across the number of different problem types.

Clearly, the Soviet texts, even within series of 10 problems, present students with a more varied sequence of word problems than do the American texts. The modal 10-problem set in the Soviet texts contained eight or more different types of problems, whereas in the American texts the modal set contained only four different types. In the Soviet texts, 52% of all 10-problem units contained eight or more different types of problems, whereas across all the American texts, none of the 10-problem units contained eight or more different types.

Results of the second analysis of 10-problem units are displayed in the lower panel of Figure 2. In this analysis, each 10-problem group was coded for the number of changes, from one problem type to another, that occurred between adjacent problems in the group. Thus a group would be coded 0 if all 10 problems were of the same type or a maximum of 9 if all pairs of adjacent problems differed from one another. This is somewhat different from the preceding analysis: If, for example, a 10-problem set contained 5 Type 2 problems and 5 Type 4 problems, it could contain anywhere from 1 (2,2,2,2,2,4,4,4,4,4) to 9 (2,4,2,4,2,4,2,4,2,4) changes, depending on the ordering of the problems.

Again, it appears that the Soviet texts are offering students far more variability in the sequencing of problems than are the American texts. Although a full 84% of 10-problem groups in the Soviet texts contained 8 or 9 changes in problem type, the corresponding average figure for the American texts was only 26%. By contrast, although 28% of 10-problem groups in the American texts contained 4 or fewer changes, no groups were so coded in the Soviet texts.

In general, the overall findings on variability in problem types hold up across grade levels and across the different American text series; data broken down by grade and text series thus are not presented here. Although the American textbook series do differ in terms of the amount of variability in presentation they provide, none of the texts approaches the level of the Soviet books in either types or changes.

A final issue concerns the distribution of word problems throughout a given text. Even a casual glance at the Soviet and American texts indicates that they were constructed under different premises: In the Soviet texts, word problems were distributed a few on a page throughout the whole text, whereas in the American texts, they were concentrated on a few problem-solving pages. This is reflected by the fact that the proportion of first-grade text pages on which there was at least one word problem was 64% for the Soviet text series and only 10% averaged over the American series. The corresponding figures for the second grade were 35% and 14%.

DISCUSSION

The above findings have implications for theories of how children learn to solve addition and subtraction word problems. As pointed out earlier, most theories account for differences in problem difficulty by reference to characteristics of the problems themselves. We do not wish to argue that such problem characteristics do not affect problem difficulty. However, if frequency of presentation is related to difficulty, as it is, then it is important to assess the independent impact that sheer frequency of exposure might have on the relative difficulty of problems of different types.

Some theories could quite easily include frequency of exposure without radically altering the theory. In the model proposed by Riley et al. (1983), for example, problems are difficult if the child is not able to activate easily a schema to represent the problem. Although the model focuses on more complex semantic characteristics as the source of difficulty in representing the more difficult problem types, one could just as easily theorize that availability of a problem schema is a joint function of problem characteristics, frequency of exposure, and characteristics of the instructional environment (i.e., textbook presentation and instructional strategies).

Briars and Larkin (1984) did not explicitly address the role experience might play in the construction of a problem representation. They did seem to acknowledge, however, that problem representation might change with experience; they suggested that children's representations are always strongly associated with concrete objects, even later when children no longer require objects to solve problems. They also discussed how certain solution procedures that children learn (e.g., known addition facts) can make the solution of certain difficult problem types much easier. Thus this model also would seem to be able to incorporate experiential factors, such as frequency of exposure, without too much difficulty.

Although the Briars and Larkin theory seems to us to be a good explanation of how a novice would solve a word problem, it does not describe the processes an expert would go through each time a problem is solved. It seems likely to us that somewhere in the course of learning, students shift from a procedural/constructive approach to problem representation (e.g., that described by Briars & Larkin) to a more retrieval-based approach (as described by Riley et al.). We propose that both task characteristics and instructional environment (including frequency of exposure) affect this transition from a procedural/constructive to a retrieval-based representation as well as produce changes within each of these kinds of representation, as discussed above.

Another possible interpretation of the relation between problem difficulty and frequency of presentation found in this study is that difficulty influences frequency of presentation and not vice versa. This is also quite plausible, as textbook makers probably are motivated to give teachers material that will not be too difficult to teach; hence they overrepresent the easy problem types in their texts. Perhaps an even better interpretation is that causality, in this case, is operating in both directions, leading to a snowball effect in which children are asked to solve easier and easier problems, which in turn makes the difficult problems even more difficult and thus less likely to appear in the elementary mathematics textbooks.

This leads naturally into a discussion of the practical implications of this study. One thing that is clear from Figure 1 is that the American textbooks are constructed haphazardly with respect to the dimensions we have analyzed in this study. Clearly, the designers of the text series did not attempt to provide an even distribution of word problems across the various types, nor did they attend to principles that suggest that variation, as well as repetition, is crucial for learning.

Ironically, it is the Soviet, not the American, texts that look as if their authors consciously used a category system such as that used in recent years by American researchers. In fact, it is difficult to imagine that the distribution of problems in the Soviet series occurred by chance. It must have been generated by some kind of analysis, most probably an analysis of the kinds of addition and subtraction problems present in the real world. That this Soviet analysis generated a category system similar to that generated by American researchers seems to give increased validity to the parameters used to define these categories.

We can only speculate as to how important these weaknesses in textbook construction are for the development of word problem-solving skills among primary school children in the United States. A limitation of this study is that we did not analyze children's learning in direct response to the text characteristics, and so we cannot know the cost these characteristics might have for children's performance. Nevertheless, it would seem that textbooks should be constructed in accordance with what we know about the nature of learning. Techniques of text construction can never be refined unless we construct texts in principled ways and then evaluate the validity of the principles.

We are not suggesting that the presentation of word problems in the Soviet series is ideal and that American textbooks ought immediately to follow this model. We have been unable to find any data on how well Soviet children solve various kinds of word problems or on how effective the present distribution of word problems in Soviet texts is. We are hoping, however, that this article will initiate research into the effects on learning of the different presentation variables identified here.

For example, although it is not clear at what age Soviet children begin to solve the more difficult one-step problems or the two-step problems on their own (without the support of the teacher), the inclusion in the first grade of so many problems that seem quite difficult to the American eye must imply that children are capable of solving more difficult problems than we typically believe they can solve. (In fact, Fuson, 1986, showed that when taught, first graders can solve Type 9 compare problems as easily as they can solve Type 2 change/less take-away.) Furthermore, the data are quite clear that American children entering first grade already can solve the simple kinds of addition and subtraction word problems on which the American texts spend so much time. It would seem preferable to build on this foundation by moving on to more difficult problems rather than merely continuing to reinforce for 3 years the problems on which children already show competent performance. We believe the time has come to begin research in earnest on how to teach children to solve the nontrivial forms of word problems.

In conclusion, we hope that we have demonstrated that analysis of the instructional environment is important and must be considered with task analysis in theories of how children learn. Children's difficulty with solving word problems probably results both from developmental limitations and from restricted opportunities for learning and practice. The relative importance of each of these factors remains to be seen.

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