2. The Acquisition and Elaboration of the Number Word Sequence

Karen C. Fuson, John Richards, and Diane J. Briars

In this chapter we describe children's acquisition and elaboration of the sequence of counting words from its beginnings around age two up to its general extension to the base ten system notions beyond one hundred (around age eight). This development occurs, in our view, in two distinct, though overlapping, phases: an initial acquisition phase of learning the conventional sequence of number words and an elaboration phase, during which this sequence is decomposed into separate words and relations upon these pieces and words are established. During acquisition, the sequence begins to be used for counting objects. Near the end of the elaborative phase, the words in the sequence themselves become items which are counted for arithmetic and relational purposes.

Learning the ordered sequence of counting words up to twenty is essentially a serial recall task: The words in the sequence must be recalled and they must be pro-

Thanks to James W. Hall, Walter Secada, and Leslie Steffe for helpful comments on an earlier draft of this paper, to Lloyd Kohler for endless hours of painstaking coding and data reduction, and to the many undergraduates who so patiently and enthusiastically worked on the data collection or analysis: Kathy Amoroso, Holly Arnowitz, Patty Bloom, Steve Cieslewicz, Chris Daley, Del Flaherty, Steven Meyers, Joanne Murabito, Sharon Nussbaum, Krista Peterson, Debbie Rubins, Arlene Siavelis, Karen Simon, and Toya Wyatt. Special thanks to Betty Weeks, Leona Barth, and Dee Mihevec for arranging and coordinating our data gathering efforts and to the teachers at the National College of Education Demonstration School, the Walt Disney Magnet School, and the School for Little Children for their very considerable help. Thanks to Jolene Hocott and Mary Ann Hinkle for allowing us to use their students in several studies. This material is based upon work supported by the National Science Foundation and the National Institute of Education under Grant No. SED 78-22048 and Grant No. SED 78-17365. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation or the National Institute of Education.

C. J. Brainerd (ed.), *Children's Logical and Mathematical Cognition*

duced in the correct order. "Learning" and "test" trials (e.g., "Show Grandma how you can count, dear") are presented in a haphazard fashion over a period of as much as 3 years. The acquisition of the sequence from twenty to one hundred is also a serial recall task, but one of a list with a repeating pattern. In the acquisition phase, the sequence functions as a single, connected, serial whole from which interior words cannot be produced independently. In the elaboration phase, the links between individual words become strengthened, and contiguous words (with their connecting link) can be separated and produced apart from the total sequence. Each word in the sequence then can serve as the stimulus for the recall of the next word—each word is a "bead" connected only to the immediately preceding and immediately following words. Therefore, in naturally occurring serial lists such as the number word sequence, the latter elaborative phase has the structure of an associative chain, but the former acquisition phase does not. Evidence supporting this view will be discussed in the "Elaboration of the Sequence" section of this chapter.

Several years are required for the acquisition and elaboration of the sequence of number words. Consequently, different parts of the sequence may be in different phases of development at the same time. For example, relations may be established between words at the beginning of the sequence at the same time that the child is acquiring words later in the sequence. Thus, statements in this chapter about particular phases or levels of development refer to some portion of the sequence rather than to the whole sequence. Typically, the most advanced development is at the beginning of the sequence, with progressively less advanced development toward the end.

Young children hear number words in a variety of contexts. The number words vary in meaning according to the contexts in which they are used, and early in their learning of these words, children build up separate, context-specific areas of meaning. As children age, these areas begin to connect. Fuson and Hall (in press) have reviewed the literature on some of these meanings and uses, namely, sequence meanings (arising from the number words in their conventional sequence), counting meanings (arising from the use of the conventional sequence in counting entities), cardinal meanings (arising from the use of a number word to refer to the numerosity of some group of entities), ordinal meanings (arising from the use of a number word to refer to the relative position of some entity), measure meanings (arising from the use of a number word to refer to the numerosity of the units in some quantity), and quasi- or nonnumerical meanings (e.g., street addresses and telephone numbers). In this chapter we outline the development of meanings for sequence words and, where appropriate, relate this development to other meanings and uses of number words. We use the terms above (sequence, counting, cardinal, ordinal, and measure number words) to refer to the use of a number word in the specified context (e.g., a number word used in a cardinal context we term a cardinal number word). The details of the meaning of any such use depend upon the meaning the user and the listener construct. By the use of these terms we do not impute to a child an adult or mature understanding of sequence, counting, cardinal, ordinal, or measure numbers. Rather, we use these terms to emphasize the different contexts in which number words are used.

The developmental sequence presented in this chapter has resulted from successive interactions between empirical and conceptual analyses. The empirical work has ranged from pilot work with a few subjects to full-scale studies. Much of this work is preliminary. We fully expect that the levels of development that we describe now will be modified in various ways both by ourselves and by others as additional data are generated.

Acquisition of the Sequence

Distinction between Sequence and Nonsequence Words

Children seem to learn very early the distinction between counting words and noncounting words, and the words they produce in counting contexts (e.g., when asked to "count" or to "count these blocks") are confined almost entirely to counting words. In our counting experiments with 3-, 4-, and 5-year-old, subjects have never used anything but counting words. With over 30 middle class 2-year-olds, two children have used letters from the alphabet (mixed in with number words) on one trial each. Gelman and Callistel (1978) also reported very infrequent use of noncounting words by 2- to 5-year-olds. The noncounting examples given by them were two 2-year-olds who used the alphabet on some trials. Thus, the identification of counting words and counting contexts and the restriction of words used in counting contexts only to counting words seems to be easily and successfully accomplished. The only intrusions seem to be of other "words," the letters of the alphabet, which are learned in a very similar way: as an arbitrary, long sequence having a conventional order and which adults and other children seem to love to ask one to recite.

Overall Structure of Sequences

The most common form of sequences up to thirty is the following: an initial group of words that is some beginning part of the conventional sequence (e.g., "one, two, three, four, five"), a next group of words, which deviates from the conventional sequence but which is produced with some consistency by a given child (e.g., "seven, nine, ten, twelve"), and a final group of words, which has little consistency over repeated productions (e.g., "fourteen, eighteen, thirteen, sixteen, twenty"). Identifying these three groups of words (the stable conventional, stable nonconventional, and nonstable portions) in the sequence of a given child requires repeated counting trials from that child. An example of such repeated trials is in Table 2.1. In this example, the stable, conventional sequence portion is "one two three four" and the stable nonconventional portion is "four six eight nine" (the linking member in each portion is recorded so that the structure with respect to omissions, reversals, etc. of the nonconventional portion is clear). The nonconventional portions vary from trial to trial and consist of the words following the "nine."

Table 2.1 Example of One Child's Repeated Counting Trials

one	two	three	four	six	eight	nine	fourteen	sixteen	thirteen	five
one	two	three	four	six	eight	nine	twelve	fifteen	sixteen	thirteen
one	two	three	four	six	eight	nine	fourteen			
one	two	three	four	six	seven	eight	nine	eleven		
one	two	three	four	six	eight	nine	fifteen	thirteen	eleventeen	
one	two	three	four	six	eight	nine	sixteen	eight	four	twelve
one	two	three	four	six	eight	nine	thirteen	two	six	
one	two	three	four	six	eight	nine	ten	thirteen	sixty	

Data Samples and Tasks

Data will be presented below concerning each of these sequence portions. The data come from two samples. The longitudinal sample consists of 33 3-, 4-, and 5-year-old middle class children attending an educational demonstration private school. At the first interview, six children in each half-year age group were included; three children had moved at the time of the second interview, so the sample dropped from 36 to 33. Word sequence data were collected twice (with a 5-month interval) on three different tasks: rote (nonobject) counting ("Count as high as you can for me"), counting a pile of 50 blocks ("How many blocks are in this pile?"), and counting a row of blocks that was lengthened on successive trials by the addition of one or two blocks ["I put down 1 (2) more block(s). How many blocks are there now?"]. On the final task the row was lengthened successively from 4 to 33 blocks. The cross-sectional sample consisted of 87 children aged 3 years 6 months to 5 years 11 months: the children in the Time 2 interview of the longitudinal sample who were of this age (27 of them) and 60 additional children–12 children balanced by sex in each half-year age group. These additional children attended a Chicago public school whose population was computer selected to match the population of the city racially and economically. They received the same tasks that had been used with the longitudinal sample.

The first data collection for the longitudinal sample was videotaped. Two coders transcribed these tapes; disagreements were resolved by a third person. The data collection from the cross-sectional sample was done by various pairs of trained collectors. Disagreements during training and during data collection were rare (less than 1% disagreement).

The Conventional Portion

Effects of Sex. Each sequence measure in each section below was examined for effects of sex and for interactions with this variable using analyses of variance. No main effects of sex and no interactions with this variable were found for any measure.

Cross-Age Variability. As might be expected, the conventional portion of the sequence increases considerably over this age range. Means, standard deviations, and ranges of the best rote count (i.e., no objects present) sequence produced by a child are given by half-year age groups for the cross-sectional sample in Table 2.2. One-way analyses of variance across age groups on these scores revealed a significant effect of age [$F(4, 81) = 5.93, p < .0003$]. Pairwise contrasts using the Newman-Keuls procedure indicated that the sequences of children in the two youngest groups differed significantly from those of the two oldest groups, whereas the sequences of the middle group (old fours) did not differ significantly from any of the others. Thus, the second half of the fourth year appears to be a time of considerable extension of the number word sequence.

The first five rows of Table 2.3 present the percent of each age group with sequences of given lengths. These data indicate that the largest percentage of the two youngest groups have sequences between ten and fourteen, the 4½- to 5-year-

Table 2.2 Means, Standard Deviations, and Ranges by Age for the Last Word Reached Accurately in the Conventional Sequence

	Counting rows of blocks		No object counting	
Age	100% of trials	Single best trial	Single best trial	Best trial with one omission[a]
3 years 6 months to 3 years 11 months				
Mean	8.00	14.06	14.17	16.56
SD	4.75	6.20	6.51	6.51
Range	(2-19)	(4-29)	(4-29)	(9-29)
4 years to 4 years 5 months				
Mean	9.47	14.00	17.18	18.71
SD	7.63	6.94	8.71	8.52
Range	(0-27)	(6-33)	(10-39)	(11-39)
4 years 6 months to 4 years 11 months				
Mean	19.23	20.77	29.59	36.47
SD	8.79	8.45	28.19	26.94
Range	(10-34)	(14-34)	(12-100)	(13-100)
5 years to 5 years 5 months				
Mean	22.38	27.63	40.19	44.81
SD	9.79	7.84	25.76	23.13
Range	(10-34)	(14-34)	(11-100)	(13-100)
5 years 6 months to 5 years 11 months				
Mean	25.00	26.94	38.17	43.00
SD	8.49	6.95	22.44	19.64
Range	(13-35)	(13-35)	(13-90)	(13-90)

[a] Sequence could omit one word; this sometimes was fairly far from the end of the otherwise accurate conventional sequence, for example, 1, 2, . . . , 13, 14, 16, 17, . . . , 29.

Table 2.3 Percentage of Age Groups Producing Accurate Sequences of Various Lengths

Age/grade[a]	$n<10$	$10\leqslant n<14$	$14\leqslant n<20$	$20\leqslant n<30$	$30\leqslant n<72$	$72\leqslant n<101$	$101\leqslant n<201$	$201\leqslant n$
3 years 6 months to 3 years 11 months	17	44	22	17	0	0	0	0
4 years to 4 years 5 months	0	41	35	12	12	0	0	0
4 years 6 months to 4 years 11 months	0	12	47	18	12	12	0	0
5 years to 5 years 5 months	0	6	25	13	44	13	0	0
5 years 6 months to 5 years 11 months	0	6	22	17	44	11	0	0
Kindergarten	0	7	11	30	26	4	22	0
First grade	0	0	3	14	7	21	48	7
Second grade	0	0	0	0	8	3	31	58
Third grade	0	0	0	0	0	4	25	71

[a] The first five groups are from our cross-sectional sample. The last four are from Bell and Burns beginning of the year interviews.

olds have sequences between fourteen and twenty, and the two oldest groups have sequences between thirty and seventy-two. Table 2.3 also contains data from Bell and Burns (Notes 1 and 2) on the sequences of older children (kindergarten through second grade). These data come from a heterogeneous sample of children from a small city bordering Chicago. Children were asked to count to thirty, and they then were stopped and their sequence production was checked at certain key points (63-72, 98-101, 196-201, and even higher). These data indicate considerable sequence production ability by the first and second graders, even though teachers indicated that they did not teach such higher counting and that all of the children's computational work was with numbers less than 100.

An examination of the sequences produced by children revealed that some children would omit a single word in a sequence and then continue to produce many more correct words. These children thus seemed to be much more able than those who produced no correct portion past their first error. To examine this capability, a more lenient measure, "best with one omission," was devised; it is the last word in a sequence that is correct except for a single omission. The means, standard deviations, and ranges for this measure for the rote counting sequences are also given in Table 2.2. This measure indicates improved sequence production, especially for the three oldest groups. Thus, many of these children had productive knowledge about the sequence beyond the point of their first error. As before, a one-way analysis of variance revealed a significant effect of age on this measure [$F(4, 81) = 8.99$, $p < .0001$], but here the means for the two youngest age groups were significantly different from those for the three oldest age groups (Newman-Keuls $p < .05$).

Within-Age Variability. The very large ranges and standard deviations in Table 2.2 indicate considerable variability within age groups, also. Some 3-year-olds have longer conventional portions than do some 5-year-olds. This rather large within-age variability is indicated in more detail in the first five rows of Table 2.3, and the final three rows of Table 2.3 indicate that this extreme variability continues into the early grades of the elementary school (Bell & Burns, Notes 1 and 2).

Decade Structure. The big jump (from 17 to 30 to 40) in the means in Table 2.2 for the young 4-year-olds (age 4 years to 4 years 5 months), the old 4-year-olds (age 4 years 6 months to 4 years 11 months), and the young 5-year-olds (age 5 years to 5 years 5 months) and the similar jump in the percentage of 4-year-olds and 5-year-olds with sequences over thirty (Table 2.2) is the result of some old 4-year-olds and many young 5-year-olds at least partially solving what we termed in earlier articles (Fuson & Mierkiewicz, Note 3; Fuson & Richards, Note 4) the "decade problem." This problem arises from the repetitive decade structure of the sequence between twenty and one hundred. Many older children in our samples gave evidence that they understood this repetitive structure. Above the twenties their sequences showed the pattern of "*x*-ty, *x*-ty-one, *x*-ty-two, . . . , *x*-ty nine" followed by a different "*x*-ty to *x*-ty-nine" chunk. However, most of them had not yet learned the order of the *x*-ty words, the multiples of ten. The sequence would move, for example, from the twenties to the fifties, to eighties, to thirties, to the fifties again, to twenties, etc. As Tables 2.2 and 2.3 indicate, the full solution of this problem is not attained by almost all children until the beginning of second grade, though a significant portion of kindergarten children have solved it.

Siegler and Robinson (in press) asked children to produce a number word sequence once in each of four sessions. They differentiated three groups of children by the place in the sequence where word production stopped: the first group stopped between one and nineteen, the second, between twenty and ninety-nine, and the third, above one hundred. Siegler and Robinson (in press) reported that the nature of the stopping points differed in the three groups: no obvious stopping-point regularities for the first group, an absolute majority of children in the second group who stopped at a word ending in "nine" and a few who stopped on a word ending in "0", and for the third group many counts ending in "nine" but even more ending in "0". When we examined the stopping points in the cross-sectional sample on the two rote counting trials (administered at the beginning and at the end of the interview), we found somewhat different results. The stopping points for our first group were distributed fairly evenly over the words from one through seventeen, but one-third of the stopping points were at "eighteen" or "nineteen." For this group, our percentage of counts stopping at a word ending in "nine" was 26% compared with Siegler and Robinson's 14%. As did Siegler and Robinson, we found low percentages of counts and of children in this group with stopping points ending in "0" (4% and 8%), and we found a similar percentage of children ending a rote count with "nine" (38% compared to their 40%). In our second group (those with sequences between twenty and ninety-nine), we found much lower percentages of rote counts and percentages of children ending with a "nine" (31% vs. their 69% and 45% vs. their 96%) but higher percentages of rote counts and of children ending with a "0" (31% vs. their 4% and 43% vs. their 14%). Thus, our two groups of children do not differ in their rates of stopping at "nine," but they do differ in their rates of stopping at "0." We had only four children in the third group. They all stopped at one hundred on each trial.

Siegler and Robinson examined stopping points as a way to indicate children's knowledge of the decade structure. They inferred from their findings that the first group of children understood neither the structure of the teens nor that of the decades and that the many children in group two stopping at a "nine" word indicates that they know the decade structure but not the next decade word (and so they stop producing words). Our finding that as many of our children stopped at a 0 word as at a "nine" word contradicts the latter inference. However, we consider the use of stopping-point data to indicate knowledge of structure to be somewhat risky. The point at which children stop producing words in sequence is influenced by factors other than whether they, in fact, could produce additional words. They may make assumptions about stopping points preferred by the experimenter; they may tire; they may seek variety. In our sample, only 22% of the children stopped at the same word in their two rote counts, and the differences between the stopping points were often large. This variability is much larger than would be indicated by the consistency level differences that we found for the conventional sequences, and so other factors would seem to be influencing these stopping points. Some of Siegler and Robinson's (in press) findings of stopping-point differences in the three groups of children (especially those concerning words ending in "nine") do not seem to generalize to other samples. We do not interpret these differences, however, as necessarily contradicting their models but rather as indicating some other factors that might be affecting choice of stopping points.

The extent to which the "decade problem" is easily amenable to practice and to direct instruction is not clear at the moment. Three different training methods seem possible. One method would emphasize linking the first member of a cycle to the last member of the preceding cycle (e.g., practicing "thirty-nine, forty"). Another method would focus upon learning the list of decades as a new rote sequence ("ten, twenty, thirty, . . . , ninety") and then using this list to select the correct next cycle. Our informal interviewing of adults suggests that some adults use this method when learning a number word sequence in a foreign language. Finally, decade words might be connected to their corresponding digits (twenty to two, thirty to three, etc.) and the order of digits used to order the decades. The relative effectiveness of these alternatives might be examined in future research.

Within-Child Variability in a Single Session. We examined the extent to which a portion of the sequence, once learned, is reliably produced over trials. This was separated into reliability over short periods of time (variability within a single session) and over long periods of time. The latter is addressed by the longitudinal data in a later section. To assess the within-child variability at one session, children's repeated sequence productions in the rows task (in which blocks were added on each trial to make the row longer and longer) were examined. The number of sequences produced by a child varied from 3 to 24, with the 3 year 6 month to 4 year 11 month age groups producing a mean of about 16 trials, and the 5 year- to 5-year-11-month age groups producing means of about 10 trials. The lower number of trials for the older children resulted from their sometimes using the numerosity of a previous row to respond to the "How many?" question after one or two blocks had been added to the row (e.g., There were 13 blocks and 2 were added: "Fourteen, fifteen. There are fifteen now.").

Because we did not know at what level of consistency changes might be observed, several levels of sequence production were analyzed. The measure at each level was the last word in a correct portion of a sequence. For example, in the sequence "one, two, three, four, five, six, eight, nine, thirteen, nineteen," that measure would be "six." From highest to lowest consistency, the levels chosen are:

100%: the sequence was produced correctly up to that word on 100% of the sequence.
80%: the sequence was produced correctly up to that word on 80% of the trials on which the row was long enough to allow production of that sequence.
60%: . . . on 60% of . . .
40%: . . . on 40% of . . .
Best: the sequence was produced correctly up to that word on at least one trial.

The analyses of the levels 80%, 60%, and 40% above revealed some fluctuations by level and by age group within the extreme 100% and Best level performances, but these were fairly minor. Therefore the data presented here will be confined to the two extreme levels, 100% and Best. Additional data can be found in Fuson and Mierkiewicz (Note 3).

Means, standard deviations, and ranges for the Best and 100% scores of the cross-sectional sample are presented in Table 2.2 by age group. A 2 (Consistency Level) by 5 (Age) analysis of variance revealed significant main effects of Level [$F(1,81) = 49.96, p < .0001$], and of Age [$F(4, 81) = 16.73, p < .0001$], and a significant Level by Age interaction [$F(4, 81) = 2.77, p < .04$]. The interaction is a result of much closer means (two- to three-word difference) for the 100% and Best sequences for the old 4-year-olds and old 5-year-olds than for the other groups (about six-word differences). Thus, across this whole age range within-child variability in the sequences produced in one session clearly exists. The age differences in variability that appeared here (i.e., the interaction) should probably be replicated before any interpretation is made. Pairwise contrasts using the Newman-Keuls procedure indicated significant differences ($p < .05$) between the 100% sequences of the two youngest and the three oldest age groups and significant differences between the Best sequences of the two youngest, the middle, and the two oldest age groups.

Longitudinal Data: Age 3-5. The within-child variability of sequences produced over a 5-month period was examined at two extreme consistency levels: 100% and Best Overall (the single best sequence produced on any task). A 2 (Consistency Level) by 2 (Time) by 5 (Age) analysis of variance was conducted on scores consisting of the last word in the accurate portion of the sequence. Significant main effects were found for Consistency Level, an overall mean of 30.0 for 100% and 36.7 for Best Overall [$F(1, 26) = 15.80, p < .0005$]; for Time, an overall mean of 29.4 at Time 1 and of 37.4 at Time 2 [$F(1,26) = 15.47, p < .0006$]; and for Age, overall means of 9.7, 13.7, 18.2, 29.3, 44.5, and 77.8 [$F(5, 26) = 6.93, p < .0003$]. A significant Consistency Level by Time interaction was also found [$F(1, 26) = 3.96, p < .05$] with a larger increase over the 5-month interval in the single Best Overall scores (from 31.0 to 42.4) than in the 100% consistent scores (from 27.7 to 32.4). For most age groups the 100% score at Time 2 was approximately equal to the Best Overall score at Time 1. Thus, the process of acquisition of longer correct sequences seems to have at least two aspects: extension of the sequence and consolidation of this extension so that it is always produced. In the five-month interval, the most recent extension (the Best Overall score) seems to become consolidated (becomes the 100% score) at the same time that a new longer extension is being made.

Stable Nonconventional Portions

Nature of the Stable Nonconventional Portions. Stable, nonconventional portions of a sequence consist of a group of two or more words that deviate from the conventional sequence and are produced consistently by an individual over several trials within a given session. In a later section we shall examine the extent to which these within-session stable portions remain stable over longer periods of time. As with the conventional portions, we did not know where important differences might occur in stable portions, and so we examined several consistency levels (stable over 40%, 60%, 80%, and 100% of the trials). In the example given in Table 2.1, the stable nonconventional portion "four, six, eight, nine" is stable over 80% of the trials (seven out of eight). None of the words following "nine" occurs more than 40% of the time. If "fourteen" had occurred after "nine" two more times, then the portion "four, six, eight, nine, fourteen" would have been a stable portion at the 40% consistency level (occurring four out of eight times). Similarly, if the "seven" had occurred within the stable portion three more times, the portion "four, six, seven, eight, nine" would have been a stable portion at the 40% level (four out of eight times). These examples illustrate the two major ways in which fluctuations in the stability of the nonconventional portion of words result: (a) the occasional insertion of correct words within the stable portion, and (b) the addition of a word or words at the end of the stable portion.

The nature of the stable portions produced by children is exactly what one would expect in a serial recall task: Almost all of the stable portions have the words in the conventional order, but they contain omissions. Of the stable portions in the two samples (longitudinal and cross sectional) of children aged 3 through 5, 88% contained omissions, 3% contained repetitions, and 9% contained reversals. All examples of the stable portions containing reversals and repetitions are given in Table 2.4. Two of the reversals involve "six" or "sixteen," three involve "seven" or "seventeen," two involve "eight" or "eighteen," and one involves "fifteen." The two repetitions are substitutions for the word "fifteen." Table 2.4 also contains the distribution of words that were omitted across all stable portions with words "twenty" and below. In those stable portions which consisted of two words (the last word in the conventional portion and a later word), "fifteen" was omitted more than all other words put together. This may be because of its irregular construction as "fifteen" rather than "fiveteen." In those stable portions consisting of three of more words, almost all words are represented in the omissions.

For the words between ten and twenty, the distribution of omissions resembles that of a typical serial position curve except that it is not bowed (i.e., its high point is not pushed toward the end of the distribution); rather it is quite symmetrical about the midpoint word, "fifteen." However, this symmetry may be an artifact of two different factors operating at each end of the teens distribution. First, because most of the youngest children in our samples could produce correct sequences up into the teens at least once (mean Best score for the 3½- to 4-year-olds was 14), data from younger children would be needed to reflect accurately omissions of "ten," "eleven," and "twelve." Second, for a word to appear in Table 2.4, some word following it in the sequence must have been produced. For example, each "eighteen" omission must have had a "nineteen" or a "twenty" or a "twenty-one," etc., consistently produced. However, Table 2.4 does not imply, as a serial position display does, that each of the words at the far right (the recency portion of the list) was produced. Data on the rate at which each word between ten and twenty was

Table 2.4 Number of Occurrences of Errors in Stable Sequence Portions

	Consistency level[a] (%)	Omissions																
Word omitted		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Two-word portions	40[b]		1		1				1				16	3	1			1
	100		1		1				1				13	2	1			1
Three or more-word portions	40[b]		3	2	2	1	0	2	2	3	10	10	16	9	10	5	3	5
	100								1	1	3	5	8	4	6	2	1	2

Reversals: Portion	Reversals: Consistency level[a] (%)	Repetitions: Portion	Repetitions: Consistency level[a] (%)
. . . , 15, 17, 19, 18	67	. . . , 12, 14, 14, 16, . . . , 29	100
. . . , 4, 9, 10, 11, 8	71	. . . , 14, 16, 16, 17	60
. . . , 12, 14, 18, 19, 16, 17	83		
. . . , 14, 18, 19, 17	62		
. . . , 4, 8, 9, 6, 7	53		
. . . , 16, 18, 19, 15	75		

[a] Percentage of child's trials on which error was produced.
[b] Between 40% and 100%.

produced across the sample of 36 children producing stable portions between ten and twenty are given in Fig. 2.1. As in the omission data, there is a huge drop off for "fifteen," and here there is also a considerable dropoff from "nineteen" to "twenty." In the sequences produced 100% of the time, production of all of the teen words other than "fifteen" is approximately the same and somewhat less than that of "ten" through "twelve." In those produced less consistently, somewhat more fluctuation occurs among the teen words and the word "twelve."

The data from Table 2.4 and from Fig. 2.1 taken together seem to indicate that during the acquisition of the teen portion of the sequence, children initially produce stable, nonconventional portions with multiple word omissions most frequently in the thirteen to seventeen range. Some of these stable multiple word omissions contain the words "eighteen" and "nineteen" and others do not. Relatively few of these

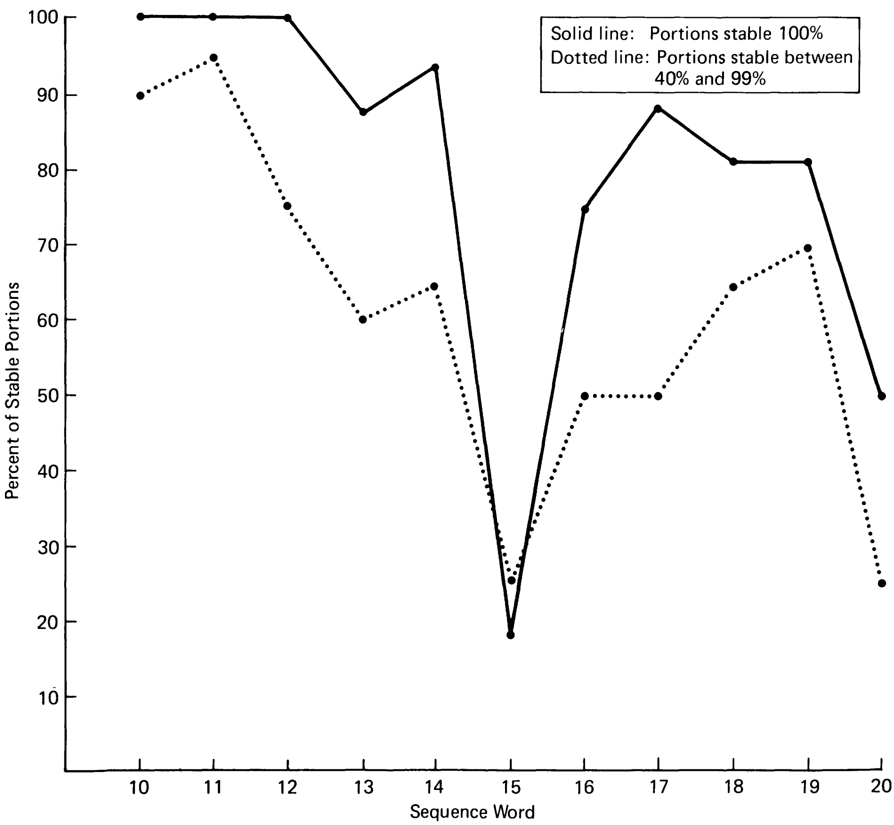


Figure 2.1 Percentage of conventional and stable portions containing words between ten and twenty. This figure includes words from the conventional portion preceding a stable portion (e.g., for 1, 2, . . . , 10, 11, 12, 13, 16, 18, 19, 1→13 is conventional and 13, 16, 18, 19 is stable; all words ten and above would have been entered in the table).

stable portions contain the word "twenty." Over time these multiple omissions become filled in with the correct words until the only remaining omission is the word "fifteen." The two children in the longitudinal sample whose stable portions moved from three or more words to two words followed this pattern. The omission of the word "fifteen" persisted in some children even after they produced a conventional sequence to "twenty-nine" or even into the thirties. It remains to be seen whether such a long-lasting omission has relatively trivial implications, that is, it is an easy one to correct, or whether it is more difficult to remedy.

Stable portions can have different lengths (i.e., contain a different number of words) and the gaps in them can be of different sizes (e.g., "twelve, fourteen" is a one-word gap, while "twelve, eighteen" is a five-word gap). For 100% stable portions of words below twenty, the mean length in the cross-sectional sample was 3.6 words, the range was 2-10 words, and the percentages of gaps of one, two, three, and four or more words were 71%, 0%, 7%, and 21%, respectively. Those figures for the stable portions at the 40% level are a mean of 3.13, range 2-10, and gap percentages of 59%, 11%, 9%, and 20%. Therefore, most of the gaps are of one word, but some gaps are of two and three words, and a sizable number (about 20%) are of four or more words.

Stable nonconventional portions containing words above twenty are of two kinds. Some (37.5% in our sample) end in a word between ten and twenty and then jump to a decade word (e.g., "eighteen, forty" or "seventeen, thirty") or to a decade-one word (e.g., "fourteen, forty-one" or "eighteen, eighty-one"). These confusions may stem from a misunderstanding of the decade structure or from acoustic confusion of "-ty" and "-teen" (e.g., fourteen and forty). Other stable nonconventional portions above twenty (37.5%) end in twenty-nine and jump to another decade ("twenty-nine, fifty, fifty-one, fifty-two, fifty-three"). Still others (25%) begin with the word "twenty" and then jump back into the teens words and produce several of them. Each of these types of stable portions seems to reflect partial knowledge of the decade word structure.

Do All Children Have Stable Portions? Our data indicate that stable, nonconventional portions are typical of sequences below thirty during the acquisition period, but our data for sequences longer than this are somewhat incomplete. Because stable, nonconventional portions occur after the conventional portion, it is necessary to obtain repeated productions of fairly long sequences. The task we used—successively adding one or two blocks to a long row of blocks (up to 34)—was successful in accomplishing this aim: Children seemed to enjoy seeing the row grow longer and longer and stuck with this repetitive and somewhat boring task fairly well. However, the ceiling of 34 blocks meant that we could not examine the existence of stable portions in those children who produced accurate sequences up to 34. Furthermore, some children made counting errors on the rows task (skipping blocks, pointing at blocks without producing words, and skimming along blocks while only producing occasional words) which meant that the last word they produced was always within the accurate portion of their sequence. Finally, a few children refused to continue the task although they were still producing entirely correct sequences. For the Time

1 interview of the longitudinal sample and for the additional children in the cross-sectional sample, 27 of the 96 children had accurate sequences up to 34, 11 made counting errors that resulted in the production only of short accurate sequences, and six stopped the task while still producing correct sequences. Of the remaining 52 children who had the opportunity to produce stable nonconventional portions, 46 or 88% of them did produce such portions, 28 at the 100% level (i.e., these stable portions were produced on every trial on which they could have been produced) and 18 at the 40%-80% levels. Thus, the production of stable nonconventional portions seems to be quite typical during the period of the acquisition of the number word sequence, at least for sequences below thirty. Children with accurate sequences to 34 also produced nonconventional portions higher than this that were stable over two or three rote counting trials, but because the number of repeated trials we have for these children is so low, we did not include them in the above stable analyses (they were included in the 27 children with accurate sequences up to 34).

Three of the six children who did not produce stable nonconventional portions produced sequences with what we characterized as "trouble spots": places in the sequence where each word in that trouble spot was produced on some trial but no trial contained all of the words and the productions varied enough that no stable portion was produced. This trouble spot pattern also characterized the sequences of some children with stable portions only at the 40% level; their other sequences contained other words from that troubled area, each with several different patterns of such production. Two examples are:

1. one, . . . , fourteen, sixteen, seventeen, eighteen
   or one, . . . , fifteen, seventeen, eighteen;
2. one, . . . , twelve, fourteen, fifteen
   or one, . . . , twelve, fourteen, sixteen
   or one, . . . , thirteen, fifteen
   or one, . . . , thirteen, fourteen.

This trouble spot pattern seems to be characterized more by an unequal and unpredictable production of certain words in the troubled area than by the consistent production of a sequence in that area.

Cross-Time Stability of Stable Portions. Are stable nonconventional portions really stable across different tasks and across several days, or are they just a temporary and misleading phenomenon resulting from a short-term fixation on certain patterns of words beyond the conventional portion of the sequence? We have tried to gather short-term (e.g., two-day or one-week intervals) data on this question, but we have had difficulty in developing a task that will quickly reach the portion of a child's sequence just beyond the conventional portion and will hold the child's interest during repeated productions of long word sequences. The only longitudinal data we have at this time is from our original longitudinal sample with a 5-month gap in interviews. Of the 11 children in this sample who had produced stable nonconventional portions at Time 1, one child was no longer at the school and four now produced sequences that were correct at the old stable portion. The other six all were still producing stable portions that were related to their old stable portions. Two children continued to produce the same stable portion that they had produced 5 months earlier ("thirteen, sixteen, twenty-one, twenty-two" and "four, six, seven, eight, nine, ten, eleven, twelve, fourteen, seventeen, eighteen"). Two children now gave stable portions that consisted of their old portions with all but one of the old omissions filled in (the stable sequence "twelve, fourteen, sixteen" had become "twelve, thirteen, fourteen, sixteen" and the stable sequence "ten, twelve, thirteen, seventeen, eighteen, nineteen, twenty-one, twenty-two, twenty-eight" had now become correct from ten to twenty-eight except for the omission of fifteen). Two children now produced a word they had omitted in the old stable portion but they also omitted the word next to it that formerly had been produced (a change from "five, six, eight" to "five, seven, eight" and a change from "twelve, fourteen, sixteen" to "twelve, fifteen, sixteen"). These data seem to indicate that the stable portions are not temporary stabilities, but rather they reflect ways in which the sequence is stored, remembered, and produced over fairly long periods of time. Words in multiple omissions get filled in over time, though sometimes an old contiguous word gets lost in this process.

The Stable-Order Principle. Gelman and Gallistel (1978) also reported stable nonconventional counting sequences produced by preschool children, but they characterized whole sequences, rather than parts of sequences, in this way, and they labeled such sequences "idiosyncratic." This word seems a bit too strong for such stable, nonconventional portions, for most of them in our data consist of the conventional sequence with omissions rather than more idiosyncratic creations. Gelman and Gallistel took the production of such stable, idiosyncratic sequences as evidence for what they called "the stable order principle." Their operational definition of this principle was the production of a stable list over repeated counting trials. However, if the stable order principle does not imply something beyond a description of the nature of the sequences produced by children, it is not clear why this is called a principle. Much of Gelman and Gallistel's discussion about this stable order counting principle seems to imply that children "honor this principle" (i.e., produce stable ordered sequences) from some understanding about the need for using a stable sequence in counting objects (e.g., to ensure that the numerosities obtained by repeated counts of the same set are the same).

Our data are in agreement with those of Gelman and Gallistel (1978) in that stable portions are typical of the sequence productions of children. However, the existence of such sequence portions does not, in our judgment, constitute evidence for possession of the stable order principle by children, if "principle" is meant to imply something more than the observation that stable, nonconventional sequences are produced. We rather consider stable, nonconventional sequences to result from the serial nature of the number-word sequence learning task. First, the existence in the nonstable sequence portions of so many forward ordered runs (see below) and the fact that most stable, nonconventional word groups differ from the conventional sequence only by omissions suggests that children learn the order of the words in the conventional counting word sequence along with the learning of the words. Second, the nonstable portions of many children's productions contain words from the earlier, produced conventional portion, and these repeated words most often are not consistent with the way those words were produced earlier. This repetition and its inconsistent nature would seem to constitute strong evidence that these children, in fact, do not understand the stable order principle. A possible caveat to this negative inference is that after "thirteen," the standard English word sequence does begin to display repetitions of parts of words (e.g., "four-teen," "twenty-five"). Until the nature of these repetition patterns become clear, they may confuse children and make it more difficult for them to observe that, in fact, each counting word is unique and that it occurs always in the same order in the sequence. In conclusion, it seems quite problematic to draw inferences about children's understanding of the need for using a stable sequence in counting only from the nature of their counting word productions. Direct evidence of such understanding is needed.

Nonstable Sequence Portions

Children did not always stop counting while in the conventional or stable portions of their word sequences. Many continued to produce words after their stable sequence, but these portions were not stable across repeated trials. Of the 46 children who produced stable portions, 28, or 61%, also produced nonstable portions. The remaining 18 children said words beyond their stable portion on fewer than four trials (usually only the rote counting and movable block tasks), making the stability of this extension of their word sequence impossible to determine. Eight of these children had conventional and stable portions sufficient for the rows task (i.e., of 33 or more words); eight others made errors in counting the rows (skipping objects, etc.) so that the last word uttered was always within the stable portion of their word sequence. Two children stopped the rows task while still within their stable portion. The data reported in this section come from those children on whom we had four or more trials of sufficient length to determine that their nonstable portions were indeed not stable.

Five examples of nonstable portions are given in Table 2.5. Nonstable portions are by definition irregular over repeated trials. However, they also possess some structure and some regularities, that is, they are not entirely random productions. Nonstable portions are composed largely of three different types of elements: (a) runs—from two to five words contiguous in the conventional sequence (e.g., "sixteen, seventeen, eighteen" or "twenty-one, twenty-two, twenty-three"); (b) runs with omissions—from two to five words in the conventional order but containing omissions (e.g., "twelve, fourteen, seventeen"); and (c) single unrelated words. The runs and runs with omissions are all forward directed (i.e., they are in sequence order), and these runs, runs with omissions, and the separate words are concatenated in a generally forward direction. For the longest nonstable portion of each child the ratio of contiguous word pairs that were in conventional sequence order ranged from 0.40 (more backward than forward) to 11.50, with a mean ratio of 4.52. The average nonstable portion therefore went forward four or five words, fell back to an

Table 2.5 Examples of Sequences with Nonstable Portions

Case L: Age 3 years 10 months	
	1→12 14 18 19 15 19
	1→12 14 18 19 16 17 18
	1→12 14 18 19 15 17 18 19 17
	1→12 14 18 19 15 16 17 18 19 15 17
	1→12 14 18 19 16 17 12 14 18 19
	1→12 14 18 19 16 17 18 19 17 14 18
	1→12 14 18 19 16 17 18 19 16 17 18 19 16
	1→12 14 18 19 16 17 18 19 16 17 18 19 17 18
B[a]	1→12 14 18 19 13
R[b]	1→12 14 18 19 17 15
Case M: Age 3 years 6 months	
	1→13 19 16 13 19
	1→13 16 19
	1→13 16 14 16 19
	1→13 16 19 16 13 14 19 16 19
	1→13 19 16 14
B	1→13 19 16 14 19 16 19 16
R	1→13 19 14 16 14
R	1→13 19 16 14 19
Case N: Age 4 years 2 months	
	1→14 16→19 30 1
	1→14 16→19 30 40 60
	1→14 16→19 30 31 35 38 37 39
	1→14 16→19 30 40 60 800
	1→14 16→19 40 60 70 80 90 10 11 10 30
B	1→14 16→19 60 30 800
B	1→14 60 30 800 80 90 30 ten-eighty 60 31 38 39 32 31 34 35 thirty-ten 31
R	1→14 16→19 30 800 60
R	1→14 16→19 30 1 80 90 60 30 90 80 30

earlier word, and then went forward another four or five words, etc., or contained some other pattern of several forward and one backward word that would lead to the 4.5 ratio (e.g., nine forward words followed by two backward ones).

Some words in the nonstable portions were favorites across children. Table 2.6 contains the total number of times a word appeared in the longest nonstable portion of each child. The words "thirteen," "sixteen," "eighteen," "nineteen," and "twenty-nine" occurred with considerably higher frequency than other words.

"Favorite" words also appeared within the nonstable portions of individual children. For some children, certain elements (runs, runs with omissions, and words) were repeated within a given nonstable portion, but these repetitions did not form regular patterns: Random elements were inserted in between others, and the elements themselves were sometimes modified slightly (by omission or intrusion of a word). In addition, these "favorite" words or groups of words were not necessarily so favored in a later sequence production, though they frequently appeared once in other nonstable portions.

Table 2.5 (*continued*)

Case O: Age 4 years 1 month	
	1→12 15 18 19
	1→10 12 14 18 19 16 11-teen
	1→10 14 15 19 1 2 3
	1→12 14 18 19 17 16
	1→10 12 14 18 19 16 11 12
B	1→7 18 19 12 14 16 11-teen 12 15 17 18 19 12
B	1→7 18 19 12 17 18 19 12 15 19 12 16 11-teen 14 18 19 14
R	1→10 17 16 18 19 14 18 13 17 18 16
R	1→10 11-teen 17
R	1→6 11-teen 6 14 17 18 19 12 16 18 19 12 4 14
Case P: Age 4 years 4 months	
	1→11 13 16 18 40 5 6
	1→11 13 16 18 40 5→8
	1→11 13 16 18 14 5→13
	1→11 13 16 18 14 5→13 16 18
	1→11 13 16 18 14 15 16 18 19 23 26 11 13 16 18
	1→11 13 16 18 40 16 18 10 11 13 16 18 24 26 28 24
	1→11 13 16 18 14 6 9 10 11 13 16 18 24 28 26 23
	1→11 13 16 18 24 28 22 3→11 13 16 18
	1→11 13 16 18 20 1→11 13 16 18 16 18
	1→11 13 16 18 20 21 26 24 28 1→5
	1→12 10 11 13 16 18 21 22 17 16 18 21 22 26 24 26 23 28 16 14 12 13 16
B	1→11 13 16 18
R	1→11 13 16 18

[a] B=Blocks trials: while counting a pile of 50 movable blocks.
[b] R=Rote trials: no objects.

The extent to which individual children repeated a word within a nonstable portion seems to vary somewhat with the location of that nonstable portion within the word sequence. If the single longest nonstable portion of each child is considered, the mean numbers of words repeated within that portion were 1.32 and 1.38 for portions with words only above twenty or only below ten, respectively. For nonstable portions with words only between ten and twenty and for those containing words from the teens as well as words above twenty, the mean numbers of repeated words were 1.52 and 1.63, respectively. The latter comparatively high figures may indicate that children producing nonstable portions within these ranges do not yet know either the decade pattern or very many of the decade words, so they repeat the teen and twenty words that they do know.

Children also vary in the relationship that the words in their nonstable portion have to those in their conventional and stable portions. For 25% of the subjects, the nonstable portion contained words from fairly early in their conventional portions. For all but one of these subjects, this seemed to be because they knew very few words outside their conventional portions: After these new words were produced, chunks from the conventional portion were emitted alternatingly with these new words. For the other subject, the production within the nonstable portion of the

Table 2.6 Total Number of Times a Number Word Appears in the Longest Nonstable Portion

Word	Number of times word appears	Word	Number of times word appears
One	3	Twenty-one	6
Two	6	Twenty-two	8
Three	5	Twenty-three	6
Four	7	Twenty-four	11
Five	8	Twenty-five	9
Six	8	Twenty-six	14
Seven	8	Twenty-seven	13
Eight	8	Twenty-eight	11
Nine	11	Twenty-nine	23
Ten	7	Thirty	12
Eleven	14	30-39[a]	11
Twelve	14	40-49[a]	7
Thirteen	30	50-59[a]	5
Fourteen	20	60-69[a]	8
Fifteen	10	70-79[a]	6
Sixteen	25	80-89[a]	8
Seventeen	15	90-99[a]	5
Eighteen	24	Words used four or more times outside of decade pattern: sixty, sixty-two, sixty-one, eighty	
Nineteen	31		
Twenty	22		

[a] Complete decade counts appearing in the nonstable portion (e.g., 30, 31, 32, 33, . . . , 38, 39).

conventional sequence from "one" or from "five" seemed rather to represent a hypothesis about repetitions in the structure of the word sequence (see example P in Table 2.5). Each repetition followed a word that "sensibly" preceded it (e.g., those repetitions beginning with five always followed a word with a "four" in it). For another 46% of the sample, the nonstable portion contained some words from earlier portions, but these came from near the end of the conventional portion or from the stable portion. For the remaining 29% of the subjects, the words in their nonstable portion were entirely new ones; none appeared earlier in the conventional or stable portions.

The data on nonstable portions are based on sequences produced in three different tasks: counting a row of fixed blocks, counting a large pile (50) of blocks, and rote (nonobject) counting. In many of our tasks, we intentionally gave children more objects to count than words they possessed in their conventional sequence. Children therefore had to make counting errors (skip objects, etc.), quit counting, or continue producing words past their conventional sequences. Most of them did the last, and no child seemed uncomfortable in doing this or verbalized less faith in those words produced beyond the conventional portion. The rote (nonobject) sequences that were produced were generally consistent with the object sequences, with one-third of the children who produced nonstable portions doing so on the first rote trial (the first counting trial overall) when no objects existed to extend sequence production. However, the possibility still remains that on the object trials the production of "incorrect" words was perceived by some children as a lesser evil than the other options (stopping or making correspondence errors), and that in fact children had differential faith in the conventional and nonconventional portions of the sequences they produced. This possibility needs to be examined in future work.

Models of the Number Word Sequence during the Acquisition Period

Greeno, Riley, and Gelman (Note 5) and Siegler and Robinson (in press) have proposed models of children's production of the number word sequence. Greeno and co-workers model the word sequence as separate words connected by a relation "next." This word sequence is then produced as part of the counting act. The Siegler and Robinson Model I (the model for sequences below twenty) has a beginning portion of the sequence consisting of single words connected by a "next" relation (these words are in the conventional order but may contain an omission) and one (or presumably more than one) later-occurring group of words connected by a "next" relation. In this model, when the last word in the first group of words connected by the "next" relation has been produced, a random choice "from the number list" (this is undefined) is made. This model thus incorporates the conventional and stable portion notions described in this chapter but views the nonstable portions as random productions. All of these features (a beginning portion followed by stably produced words with an omission, connected groups of words later in the sequence, random production of words and connected groups of words from the later part of the child's sequence) are actually consistent with parts of an earlier version of this chapter to which Siegler and Robinson refer (Fuson & Richards, Note 4). In that article, we termed the nonstable portions "spews" and described them as essentially random productions. Our subsequent analyses, however, have indicated that in fact they are not random, though not entirely regular, either. Thus, the Siegler and Robinson model does go a step beyond viewing the production of a number word sequence as involving only a simple "next" process, but it does not account for the nonrandom though irregular nature of the final nonstable portions nor for the probabilistic nature of the production of the end of the conventional and the stable portions. Sequences produced by Model I would consist of two parts: a conventional and stable part produced identically on every trial and a later part that differed on every trial. The data reported in this chapter obviously are inconsistent with both of these model productions: the ends of children's conventional and stable portions vary somewhat over trials (for this reason we needed the different consistency trials in our analyses), and the nonstable portions are not completely random. Models of each of these aspects obviously will need to involve some probabilistic process.

Siegler and Robinson proposed a more complicated model for sequences between twenty and ninety-nine. In the earlier draft of this chapter, which Siegler and Robinson referenced (Fuson & Richards, Note 4), we reported that children's sequences above twenty-nine often showed evidence of knowledge of the "x-ty to x-ty-nine" decade structure, and we noted that many children aged 4½-6 had what we labeled there (and here) "the decade problem," that is, children produced the decades out of order, frequently showing repetitions of these decades. Siegler and Robinson found similar patterns in their counting data, and their Model II incorporates these common findings. In that model, children produce a decade word (Siegler and Robinson term this a "rule applicability" word) and then cycle through adding each digit word to this word. When children do not know the order of the decades, the model postulates a random selection of a decade word. Again, our data indicate that though not entirely regular, this choice is also not random. "Favorite" and less favorite decades exist for particular children. Therefore, a probabilistic model again is probably more appropriate than a random model.

Modeling the number word sequence during its acquisition will obviously be a challenging task. As the Siegler and Robinson models make clear, such models will need to differentiate among sequences of different lengths (those that do and do not involve the decades, the hundreds, the thousands, etc.) because additional structure is involved in the higher word sequences. Such models also will need to account for the various probabilistic aspects of the sequence. After the sequence is acquired and is consistently produced, a simple model such as the one suggested by Greeno Riley & Gelman et al. is more appropriate, though after acquisition issues concerning the nature of the elaboration of various parts of the sequence (see later sections of this chapter) become important. At present we are considering two possibilities for models during the acquisition period. The first one is composed of probability trees for each number word. A tree connects a word to each word which may follow it, and each branch of such a tree is assigned a probability. Words in the portion of the sequence produced consistently have a single branch, and those occurring later in the more inconsistent portions have several branches. The other model involves two different memory stores. One consists of a connected "string" of number words that are produced one by one in sequence consistently from trial to trial. The other contains words and runs of words (with and without omissions), each of which has a probability attached to it. These probabilities determine which word or run will be produced, and the probabilities change with the production of a word or run.

Two important ultimate goals of any models of the production of the number word sequence are to model the processes involved in both the acquisition of new words and in the change from inconsistent to consistent production of words. First, however, we must be able to model how a given sequence is produced at one point in time, and we cannot yet do that adequately.

Invented Number Words

Some of the words in the nonstable portions are invented words. Twenty-seven percent of the cross-sectional sample produced at least one invented word. The mean number of different invented words produced by each of these children was 3.85 (SD = 3.78), and the mean number of such words produced including repetitions was 5.70 (SD = 5.74). Table 2.7 lists all of the invented words from the 96

3- through 6-year-old subjects in the cross-sectional and longitudinal samples. Almost all of these words continue a given decade above nine ("twenty-ten, twenty-eleven"), and a few continue the teen structure downwards (e.g., "eleventeen"). These "errors" obviously are not random but are based upon partial knowledge about structure within the number word sequence.

Very Early Word Sequences

A final point might be made about the very early acquisition of the counting word sequence. It is not clear whether all children start with at least the first word or two of the conventional sequence or whether some children first produce totally nonconventional sequences. In 140 children involved in our counting studies and pilot work for these studies, one 3-year-old and two 4-year-olds produced only nonstable nonconventional sequences. A few other children produced such sequences when they were tired or being silly, but produced sequences beginning with a conventional portion when told to "try hard" or to "shape up." This makes us somewhat reluctant to infer that the other three subjects could not produce a sequence of counting words that began with a conventional group of words. Gelman (Note 6) reported that retarded children produce only nonconventional sequences. Whether the very earliest counting word sequences of most children begin with a conventional portion is not yet settled. However, it is clear that most such sequences produced by 2-year-olds do begin with some conventional word or words.

Summary

The acquisition of the standard sequence of counting words up to one hundred begins in middle class American children before or soon after the age of 2 years and ends for most of them in first grade. The age of acquisition is extremely variable, with some 3-year-olds producing longer correct conventional sequences than some 5-year-olds. Most middle class children 3½ years or older can produce sequences to ten and are working on the teen part of the sequence, and children 4½ to 6 or 6½ are working on solving the decade problem. During the period of acquisition, the form of the sequences produced by most children is that of a conventional portion, followed by a stable, nonconventional portion containing omissions, followed by a nonstable portion that may be characterized in different ways for different children. Now that the nature of the sequences during the acquisition phase is beginning to be established, research is needed on ways by which new words are added and on factors that affect such additions.

Elaboration of the Sequence

After the number word sequence is acquired, it first functions as a unidirectional whole structure. The number words can be produced only by reciting the whole sequence. The elaboration of the sequence is a lengthy process of differentiating the

Table 2.7 Invented Number Words

Word	Number of children[a]	Number of times[b]	Mean use per child[c]	Range of use[d]
fiveteen	1	1	1.00	–
eleventeen	3	8	2.67	1-6
twelveteen	1	1	1.00	–
fivety	1	1	1.00	–
eleventy	1	1	1.00	–
ten-eighty	1	1	1.00	–
twelve-one	1	1	1.00	–
twelve-two	1	1	1.00	–
twelve-three	1	1	1.00	–
twelve-four	1	1	1.00	–
twenty-ten	8	22	2.75	1-8
twenty-eleven	7	16	2.28	1-6
twenty-twelve	9	11	1.22	1-2
twenty-thirteen	6	11	1.83	1-3
twenty-fourteen	6	6	1.00	–
twenty-fifteen	3	4	1.33	1-2
twenty-sixteen	3	3	1.00	–
twenty-seventeen	1	1	1.00	–
twenty-eighteen	4	4	1.00	–
twenty-nineteen	4	4	1.00	–
twenty-twenty	2	2	1.00	–
twenty-twenty two	1	1	1.00	–
twenty-thirty	5	7	1.40	1-2
twenty-forty	2	3	1.50	1-2
twenty-fifty	1	2	2.00	–
twenty-sixty	1	2	2.00	–
twenty-seventy	1	2	2.00	–
twenty-eighty	1	1	1.00	–
twenty-one hundred	1	1	1.00	–

words in the sequence and constructing relations among these words. We have divided this period of elaboration into five levels (see Table 2.8): (a) string level—the words are not objects of thought; they are produced but not "heard" or reflected upon as separate words; (b) unbreakable chain level—the separate words can be "heard" and they become objects of thought; (c) breakable chain level—parts of chain can be produced starting from arbitrary entry points rather than always starting at the beginning; (d) numerable chain level—the words are abstracted still further and become units in the mathematical sense in that segments of connected words can themselves be counted or kept track of (they are countable items in the terminology of Steffe, Richards, and von Glaserfeld, Note 7); (e) bidirectional chain level—words can be produced easily and flexibly in either direction. These different levels are marked by performance differences in more complex aspects of sequence

Table 2.7 *(continued)*

Word	Number of children[a]	Number of times[b]	Mean use per child[c]	Range of use[d]
thirty-ten	5	6	1.20	1-2
thirty-eleven	1	1	1.00	–
thirty-twelve	1	1	1.00	–
thirty-seventeen	1	1	1.00	–
thirty-eighteen	1	1	1.00	–
thirty-nineteen	1	1	1.00	–
thirty-thirty	2	3	1.50	1-2
fifty-ten	3	3	1.00	–
fifty-eleven	1	1	1.00	–
fifty-twelve	1	1	1.00	–
fifty-thirteen	1	1	1.00	–
sixty-ten	1	4	4.00	–
sixty-fifteen	1	1	1.00	–
sixty-twenty	1	1	1.00	–
sixty-twenty one	1	1	1.00	–
sixty-twenty two	1	1	1.00	–
sixty-twenty three	1	1	1.00	–
sixty-twenty four	1	1	1.00	–
sixty-twenty five	1	1	1.00	–
sixty-twenty six	1	1	1.00	–
sixty-twenty seven	1	1	1.00	–
sixty-twenty eight	1	1	1.00	–
sixty-twenty nine	1	1	1.00	–
eighty-twelve	1	1	1.00	–
eighty-nineteen	1	1	1.00	–

[a] Number of children who said the word at least once.
[b] Number of times word was said overall.
[c] Mean word use per child.
[d] Range of frequency use per child.

production, in the ability to comprehend or produce relations on the words in the sequence, and in uses of the sequence of words. The abilities at each level are presented schematically in Table 2.8.

Producing relations on and using the number word sequence in other contexts require knowledge in addition to the sequence skills themselves. Placement of relations or uses on the same horizontal line in Table 2.8 implies that the sequence skill is requisite for that relation or use. Developmentally, the lag between the acquisition of a sequence skill and a relation or use may be very small or fairly large, depending on the difficulty of the additional knowledge required. In some areas we know something about the nature and the difficulty of this additional knowledge; in other areas we know very little. Vertical placement of sequence skills within levels implies developmental lags except where specifically noted.

Table 2.8 Sequence Production Levels

Sequence levels	Forward sequence skills	Backward sequence skills	Relations	Counting, cardinal, ordinal, measure context uses
String ——→ 1	Produce word sequence from one; words may be undifferentiated			Count: no intentional one-to-one correspondences can be established
Unbreakable chain ●-○-○-○-○-○→ 1	Produce word sequence from one; words differentiated			Count: intentional one-to-one correspondences can be established Card: cardinality rule can be acquired (can count to find out "How many?") Ord.: ordinality rule can be acquired (can count to find out "What position?") Meas.: measure rule can be acquired (can count to find out "How many units?") Card. Op.[a]: simple addition problems if objects for the sum just need to be counted

	Count up from one to a		And Then, And Then Before: the chain can be used to find these relations	Card.: make a set of numerosity "a"
				Ord.: find the "ath" entity
			Comes After, Comes Before: the chain can be used to find these relations	Meas.: make (find) a quantity made up of n units
				Card. Op.: Count-all and count-part procedures for addition and subtraction
Breakable chain ○-○-○ ●-○-○-○-○→ a	Start counting up from a		And Then, And Then Before: these relations can be produced immediately	Card. Op.: Count on without keeping track (addition)
			Comes After, Comes Before: these relations can be produced immediately	
			Between: partially correct solutions may be given by using And Then or And Then (Before) relations	
	Count up from a to b		Between: can produce all words between a and b going forward	Card. Op.: Count on from a to b without keeping track (subtraction)
		Count down from b		Card. Op.: Count back from b without keeping track (subtraction)
		Count down from b to a	Between: can produce all words between a and b going backward	Card. Op.: Count back from b to a without keeping track (subtraction)

Table 2.8 (*continued*)

Sequence levels	Forward sequence skills	Backward sequence skills	Relations	Counting, cardinal, ordinal, measure context uses
Numerable chain ├-├-┤ ●-├-├-├-├-├→ a	Count up n from a; give b as answer 1. $n = 1$ (And Then) 2. $n = 2, 3$ (4) 3. $n > 4$			Card. Op.: Count on with keeping track (addition)
	Count up from a to b, keeping track; give n as answer			Card. Op.: Count on from a to b keeping track (subtraction or missing addend problems)
		Count down n from b; give a as answer		Card. Op.: Count back with keeping track (subtraction)
		Count down from b to a; give n as answer		Card. Op.: Count back from b to a keeping track (subtraction)
Bidirectional chain ←├-├-├-├-├-├→ a	Can count up or down quickly from any word; can shift directions easily			

[a] Card. Op.: Cardinal Operation–an operation on cardinal number words. We include the two earliest operations–addition and subtraction–here in Table 2.8.

String Level

At the string level the individual number words are completely embedded within the sequence. As sequence-number words, they are produced only within a recitation of the known sequence as a whole. The number word sequence for the young child at this level is just like any other recitation (e.g., nursery rhymes): The child "hears" the recitation only as a single whole and, if aware of the composing words, is so only to the extent of learning the correct recitation in some wholistic way. The individual words in some parts of the sequence may be inadequately differentiated, as, for example, in other sequences, "LMNO" or sweet "landaliberty."

The sequence-number words can be used in the act of counting at the string level, but because the words are not yet heard and reacted to as separate words, only a global correspondence can be established among the word sequence, the sequence of indicating acts (usually pointing), and the items being counted. The counting act at this level consists of the production of the string of number words and of a sequence of indicating acts roughly aimed at the entities to be counted. From the adult perspective, some one-to-one correspondences may occur, but the child has not made the requisite distinctions in its own behavior to make such a correspondence. Rather, these correspondences arise fortuitiously or because of some human central nervous organization that makes it simpler to produce sequences of verbal and motoric acts in synchrony rather than completely in isolation. Over a period of two years, our experimenters have made records of over 40 2-year-olds counting various types of objects in various settings (homes, nursery schools, mother drop-in centers). We have found it difficult to obtain systematic data over various conditions (at least for object arrays of sufficient size to move beyond the child's accurate sequence) and difficult to describe the counting act at this level in any detail. Two impressions from this work might be noted for future research. First, the counting act seems to consist of the rather independent production of two separate sequences of behavior (the words and the pointing acts). Second, pointing at stationary objects seems to be a distinguishing feature of counting, for attempts to elicit imitation of counting that involved the movement of objects from an uncounted to a counted pile usually ended prematurely in some type of play with one of the piles of objects (i.e., such moving actions are part of "building with blocks," not part of "counting"). An exception to this is when the moved object is the child herself. Our observations and mother report data indicate that a frequent natural use of counting is in counting stairs as one walks up or down them.

Very few data presently exist about the string level. At the moment this level is characterized chiefly by what a child cannot do; these limitations will be more evident as the abilities on the higher levels are presented.

Unbreakable Chain Level

Differentiated Words. At the unbreakable chain level the sequence words are distinguishable, and can be "heard" or attended to, as words in the production of the sequence. However, the sequence must still be produced starting from the beginning; it cannot yet be "broken" and produced from an arbitrary entry point. Because each word has some separateness, intentional, as opposed to fortuitious, one-to-one correspondences among the words, indicating acts, and counted entities are now possible, thus laying the foundation for accurate counting (see Fuson & Mierkiewicz, Note 3, for data about age-related changes in counting accuracy). This distinguishing of words that appears at this level is simpler with counting sequences that begin with monosyllabic words. Israeli children fail initially to differentiate their sequence words and tend rather to make correspondences with each syllable of the first three two-syllable sequence words (echat, shtayim, shalosh) rather than with each word (Nesher, Note 8).

The clear differentiation of words in the unbreakable chain enables the child to establish the counting meaning of a number word (the meaning produced in the act of counting). It also enables the child to begin to establish a relationship (see Table 2.8) between the counting meaning and the meaning associated with the use of the final count word as an appropriate response to "How many?" (cardinal meanings), "Which position?" (ordinal meanings), or "How many units?" (measure meanings). This relationship between numerosity and the last word said in counting has (perhaps unfortunately) been termed the cardinality rule (Schaeffer, Eggleston, & Scott, 1974) or the cardinality principle (Gelman & Gallistel, 1978). Similar rules need to be constructed to relate counting to ordinal and measure meanings of number words (see Fuson & Hall, in press, for a more detailed discussion of this point). Such links with separate cardinal, ordinal, or measure number words are only possible when the number word sequence consists of differentiated words (e.g., one two three four five) rather than of a string of words (e.g., onetwothreefourfive).

A final use of the sequence skill of counting up from one is that children can begin to solve simple addition (and perhaps even subtraction) problems if objects representing each addend are provided and the total group of objects just needs to be counted.

Evidence for Unbreakable Chains. A chain is unbreakable if a person given a word from her chain cannot at once give the next word in the chain but must instead produce the sequence up to the given word before responding. The unbreakable chain is a whole structure that can only be produced from its starting point (or from some special starting points within the chain). Adults still have chains at the unbreakable chain level for at least the musical scale and the alphabet. For example, 19 of 20 adult self-reports in an informal study we did indicated that these adults had to say the whole musical scale (do, re, mi, . . .) up to a given word before they could tell the word that immediately followed it. Reaction time data also support such a "produce and search" process by adults with the alphabet (Hovancik, 1975; Lovelace, Powell, & Brooks, 1973; Lovelace & Spence, 1973; Klahr & Chase, Note 9). However, because of its length, the alphabet seems to differ somewhat from the musical scale: The common use of the rhyme, or song, of the alphabet tends to decompose it into unbreakable chunks (ABCDEFG HIJKLMNOP etc.). Consequently, the search process may involve only the production of one of these unbreakable chunks rather than the whole chain.

We examined whether the number word sequence of children had an unbreakable chain level in a study comparing the ability of 3- and 4-year-olds to produce the next word when given a single word versus two or three successive words from the sequence. The latter condition was designed to induce in the children a sequence recitation context similar to that induced by their own production of the sequence. The effect was to impose upon them the strategy adults use when they cannot immediately answer "comes right after" questions: They produce the sequence, stop at the given stimulus word, and then give the next word. Superiority of the sequence recitation context would indicate that the number word sequence in young children does go through an unbreakable chain level, but that many children do not think of using the sequence production strategy for questions involving the next word. Each of the 24 children in this study was given three presentation conditions which varied the number of successive number words said (one, two, and three words). Questions were of the form, "When you are counting, what word comes right after 6 (or 5, 6 or 4, 5, 6)?" The order of presentation of these conditions was completely counterbalanced. Number words of two different sizes were given: single digit (three through nine) and teens (between thirteen and eighteen). A 2 (Age) by 2 (Size of Number Word) by 2 (Number of Stimulus Words Said) analysis of variance was done on the percentage of correct responses for the two comparisons of single and multiple stimuli (one- versus two-word and one- versus three-word comparisons).

The percentage of correct responses in each condition is given in Table 2.9. For the one- versus two-word comparison, significant main effects were found for the Number of Stimulus Words [$F(1, 22) = 11.36, p < .01$], for the Size of Number, [$F(1, 22) = 5.45, p < .05$], and for Age [$F(1, 22) = 10.53, p < .01$]. The Age by Size of Number Word interaction was also significant [$F(1, 22) = 5.06, p < .05$]. More children gave correct responses to two-word than to one-word stimuli (69% vs. 45%). More correct responses were given by 4-year-old children than by 3-year-old children (73% vs. 41%). The 3-year-olds gave more correct responses for single-digit number words than for teens words (50% vs. 32%), while the 4-year-olds gave equal levels of correct responses for these different sizes (73% correct for both sizes). Almost identical results were obtained for the one- versus three-word comparison. The main effects and the interaction described above were all significant at the .01 level. The performance levels in the three-word condition were similar to those in the two-word conditions except that the 3-year-olds did slightly worse on teens in the three-word than in the two-word condition (33% vs. 48%).

The results of this experiment indicate that initially the number word sequence is in a recitation form, as a directed recited sequence, rather than as an associative

Table 2.9 Percentage of Correct Responses on Recitation Context Study

	One-word stimulus			Two-word stimulus			Three-word stimulus		
Age	Digit	Teens	Mean	Digit	Teens	Mean	Digit	Teens	Mean
3-years	39	15	27	62	48	55	63	33	48
4-years	64	63	63	82	83	82	83	81	82

chain of separable linked elements. The equal performance in the two- and the three-word conditions indicates that two words are sufficient to establish the directionality of a recitation context and enable a child to produce the next word.

Data from Siegler and Robinson (in press) also support an unbreakable chain level in the number word sequences of children. They reported that preschool children who were asked to start producing the number word sequence from a word well within their accurate counting range made a decade transition error (e.g., went from fifty-nine to seventy) or stopped at the end of a decade significantly more often than when they were producing the sequence from one. The total recitation context of the conventional sequence enabled children to produce a longer sequence than starting at some arbitrary point within it.

Additional evidence for the existence of an unbreakable chain level in the number word sequences of young children comes from reaction time studies of simple addition and subtraction problems given to 4- and 5-year-old children (Brainerd, Note 10). These problems required the children to increase or decrease the given addend by one. Such problems are quite easy to answer by using the number word sequence. Brainerd found reaction times supporting what he called a "drop back and count up" strategy. In order to produce the word following a given number word, some children produced the number word sequence starting from the beginning or very early in the sequence. In our terms, the children "drop back" to a piece of the sequence that is breakable and then count up, or they must begin at "one" in their unbreakable chain.

Counting up to "*a*". The main sequence skill to emerge at the unbreakable chain level is the ability to count up from one to a preselected word, "*a*." This is more difficult than simply producing the sequence, for the child must remember the word up to which she or he is counting and must create some way to stop counting when that word has been reached. The latter would seem to require some checking procedure. This might be instituted after each word is produced, or it might follow some estimate of where the designated word "*a*" is in the sequence and be used only when "close" to the designated word.

Emergence of "counting up to '*a*' " may be based on a combination of maturational and specific experiential factors. Case, Kurland, and Daneman (Note 11) have used a counting span task that shares characteristics with "counting up to '*a*'." In their task, a child must count a set, give its numerosity, then count a second set and give the numerosities of both sets in the order in which they were counted, etc. Thus, this task requires that a child remember a number word (the numerosity of the first set) while counting the second set. Case and co-workers found than 6-year-olds have a span of two (i.e., can count a second set and then give the numerosities of the first two sets), while 4-year-olds have a span of only one. Experience with counting does influence span. Several weeks of massive practice increased the span of 4-year-olds to that of the average 6-year-old, and adults using a new counting sequence have a span equal to that of 6-year-olds (Case, Kurland, & Daneman, Note 11). However, this additional experience must be quite extensive to have an effect, and Case and co-workers presented other data that implicated maturational factors as the chief source of this change in span. The same balance of factors would seem to be operating with the "counting up to '*a*' " skill.

The ability to count up to a prespecified word enables new counting uses of the word sequence to be made (see Table 2.8). In addition to the sequence skill, these uses all require specific knowledge about the context in which that skill is being applied. In cardinal ("How many?") contexts, a child can now find or make a group of objects of a prespecified numerosity. In ordinal contexts, children can find (or make) the "*a*th" entity. In measure contexts they can find (or make) quantities of "*a*" units.

The cardinal uses allow the child to develop general procedures for the solution of addition and subtraction problems. The count-all procedure for addition requires only the two sequence skills at the unbreakable chain level. In this procedure, items are counted out for one addend, then more are counted out for the other addend, and then all of the items are joined together and counted for the sum. For a count-part solution procedure for subtraction problems, items are counted out for the total, from which are separated items for the numerosity to be subtracted. Finally, what remains is counted (cf. Steffe, Thompson, & Richards, in press). The application of the word sequence skills to these cardinal operations of addition and subtraction requires that the child understand the relationship between counting meanings and numerosity (cardinal) meanings of number words in both directions; that is, the child must know that she or he can count a set of objects to find its numerosity and that, if a numerosity is known, a set of objects with the desired numerosity can be constructed by counting out objects. The child must also understand the fundamental meaning of addition as asking for a total of two different numerosities and of subtraction as asking for the remainder or the difference of two numerosities. Preschool children evidently have some basic understanding of "adding to" and "taking from" (Brush, 1978; Starkey & Gelman, in press), and school-aged children can use objects to model different types of addition or subtraction situations presented verbally (Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, in press; Moser, Note 12). When problems are presented symbolically, young children's abilities are not so clear, but even 4-year-olds apparently can easily learn the count-all procedure for addends of five or less (Groen & Resnick, 1977).

Relations on Sequence Words. At the unbreakable chain level the sequence skill "counting up to '*a*' " can be used to generate relationships between words in the sequence. One such relationship is "And Then," that is, " '*a*' and then ?" is the word immediately following "*a*" in the word sequence. Evidence was discussed above indicating that adults who have a chain at the unbreakable chain level (the alphabet or musical scale) do use their chain to answer And Then questions; that is, they produce the chain up to "*a*" and then give the next word as the answer. Some young children also seem to produce an unbreakable chain to find And Then relations in the number word sequence. Some children in a sample of 36 3-, 4-, and 5-year olds asked to produce words immediately following given words either said the word sequence aloud or gave visible lip movement evidence of subvocalizing the sequence before producing the required word. Far more children, however, did not use any observable procedure and simply replied quickly with a number word. These words were sometimes correct and sometimes not. The recitation context study reported earlier indicates that many children at these age levels can use the chain to answer And Then questions if successive words from the chain are spoken by the experimenter. It is not clear whether many children are not able to produce the chain from the beginning to answer an And Then question or whether children simply fail to think of using this strategy. The spontaneous use of it by some children supports the latter interpretation.

The sequence skill "counting up to '*a*' " can also be used to answer questions about the general order relation "Comes After" (as in seven comes after four). In response to a question such as, "Does eight come after five?" two such procedures using the unbreakable chain are possible. One could produce the unbreakable chain listening and stopping at the sound of either word; this requires knowing that the word heard first does not come after the other word. Or one could produce words and stop at the sound of the second word; this requires only a direct use of the meaning of "Comes After." Again, in our studies, we have sometimes seen children producing an unbreakable chain (i.e., the sequence from its beginning) in response to "Comes After" questions, but, as with the "And Then" questions, a fairly quick response (sometimes correct and sometimes not) was much more common.

A brief note is necessary here on the choices we made for the names of the sequence relations we discuss in this chapter. We have at various times used different labels for these relations. None has been entirely satisfactory. Other possibilities for the relation which takes some word from the number word sequence and gives the next word (and which we have termed "And Then") are "Immediate Successor," "Comes Just After," and "Next." The problem with the first two (and similar variants) is that their usage requires that the normal sequence order be reversed: "7 is the Immediate Successor of 6." Because the forward linking relation is such a crucial one in the sequence, and because it depends so heavily on the forward recitation context, a term that would enable the relation to be stated in its forward recitation form seemed desirable. "And Then" was chosen because it met this requirement and because it implied only execution knowledge of the sequence and no further conceptual knowledge, as some of the other choices might have been inferred to involve. The awkwardness of the inverse of this relation ("And Then Before") is also then a positive characteristic, because it so accurately reflects the much more difficult nature of isolating in a forward directed sequence the word immediately preceding a given word. The terms "Comes After" and "Comes Before" were chosen for the general order relations on sequence words, because they seemed more general than "Comes Later Than" or "Comes Earlier Than," which refer only to time.

Other Comments. The unbreakable chain level may last for a considerable period of time in children even with a short chain and daily use. After the 5-year-old daughter of one of the authors learned the sequence of the days of the week, on her own she used the sequence at least once daily to solve relational questions about the sequence (e.g., "Today is Tuesday. What will tomorrow be?"). For at least four weeks, she was unable to answer And Then questions (i.e., to give the day following a given day) without running through the whole sequence to produce the answer.

Finally, evidence does exist about the relationship between two abilities (accurate counting and responses to "And Then" questions) at the unbreakable chain level. A sample of 14 3- and 4-year-old children (ages 3 years 6 months to 4 years 8 months) was selected from a larger sample on the basis of the length of their correct word sequences (these ranged from twelve to nineteen) and on their counting ability (over several trials they made a moderate number of word-object correspondence errors with accurate correspondence on at least one trial). These children were given words from their own conventional and stable word sequences and were asked to give the word that came immediately after these words. All children received the words between four and twelve, and children with 100% conventional and stable sequences above that received words up to "nineteen." Response scoring was based on a child's own stable sequence; for example, if the sequence was ". . . , fourteen, sixteen, seventeen," sixteen was scored as correct for the word following fourteen. The mean number of accurate responses was 49%, range 0-100% correct. Thus, while these children were capable of accurate object counting, they did not reliably or universally use the strategy of producing their word sequence in order to answer the "And Then" question.

Breakable Chain Level

Forward Sequence Skills. The breakable chain is a chain of connecting links that can be entered and produced beginning at any of its links (words). There are two new skills at this level: "counting up from '*a*' " and "counting up from '*a*' to '*b*.' " The latter is more difficult because the word to which one is counting-up must be remembered during the counting. The skill of counting up from "*a*" for "*a*" below ten seems to be acquired between age 3½ and 5 at about the time when children are acquiring correct sequences through the teens. We found that of 14 children aged 3½-4½ who had correct conventional sequences ending somewhere between twelve and nineteen, 6 were unable to start counting up from various words below ten, 3 did so on 60% of their trials, and 5 did so on 100% of their trials. Counting up from "*a*" for "*a*" in the teens seems to be well established for most children by age 6. Secada, Fuson, and Hall (Note 13) found only 6 out of 63 6½-year-olds who could not count up from "*a*" when "*a*" was a word in the teens. Data about counting up from "*a*" to "*b*" are reported in the later section "Counting Down from '*b*' to '*a*.' "

Forward And Then Relation. The ability to produce an immediate response to an And Then question appears to some extent in 3-year-olds and reaches fairly high levels in 5-year-olds. In the recitation context study described earlier, the correct response rate for 3-year-olds for And Then questions ("When you are counting, what word comes right after eight?") for words between two and nine was 39%, and for words between twelve and nineteen was 15% (see Table 2.9). Similar rates for the 4-year-olds were 64% and 63%, respectively. Success rates for words below 20 rose to 72%, 86%, and 100% for a sample of 36 middle class prekindergarteners (aged 4 years to 5 years 5 months), kindergarteners (aged 5½-6½), and first graders (aged 6½-7½). Correct item response times taken by digital stopwatch dropped from a mean of 2.56 seconds for the prekindergarteners to 1.67 and 1.38 seconds for the kindergarteners and first graders. Thus, most end-of-the-year kindergarteners seem to have these responses accurately and immediately available.

The sequence skill "counting up from '*a*' " and the ability to respond immediately to an And Then relational question would seem to be closely related. If one successively produces And Then related words beginning at "*a*," one sounds as if one is counting up from "*a*," and vice versa. It is not clear whether the processes involved in these two procedures are the same, however. In the sample of 14 children for whom counting up from "*a*" data ("*a*" less than ten) were given above, data on responses to And Then questions were also gathered. Eight of the children performed better on the counting up from "*a*" task than on the And Then task; those performing moderately well on the And Then task counted up from "*a*" on all trials, and those giving only a very few correct And Then responses counted up to "*a*" on 60% of their trials. One of these eight children counted up from "*a*" in response to each And Then question about "*a*"; this child was the only one who responded accurately to every And Then question. Alternatively, four children did better on the And Then questions (success at a low–for one child–or at a moderate level–for three children) than on the count up from "*a*" task (they failed to count up on even a single trial). It is possible that this failure was a result of inadequately communicating the task to these children. However, some of them did seem to be trying to start counting up but they always finally had to begin at one. Thus, rather than these procedures developing together, it seems that children may begin to do fairly well on either one of these procedures before the other one.

These procedures also seem to be acquired in somewhat different patterns. Performance on And Then questions seems to improve continuously (scores for this sample ranged over the whole possible range), while that on "counting up from '*a*' " appears at only two levels (around half the trials or on all of the trials). All of these data seem to implicate two different processes rather than a single one used for both tasks. Understanding of the processes involved in these two procedures and of the relationship between them must await more definitive research.

Backward Sequence Skills. Two new sequences appear at the breakable chain level: (a) the ability to produce a backward number word sequence beginning from an arbitrary number word (the sequence skill "counting down from '*b*' ") and (b) the ability to start at and stop at arbitrary words ("counting down from '*b*' to '*a*' "). Each of these will be discussed in turn.

Producing Backward Sequences. Backward word sequences are sometimes learned as separate new word sequences, as in "ten, nine, eight, . . . , two, one, blast off!" However, except for the rocket example and for some nursery school songs, backward sequences are rarely required in our culture, especially above ten. They therefore seem rarely to be separately acquired but rather result from a slow and laborious production from the forward sequence. Vocalizing and subvocalizing patterns by the child subjects in the studies to be reported next indicated that many of them produced a two- to five-word forward sequence segment that included the word from which the backward sequence is to begin, then said this segment backward, and then repeated these two phases of (often subvocal) forward and overt backward chunks. Other children were not even this efficient; they always began their forward sequences from "one." This production of the backward from the forward sequence was especially evident for sequences between ten and twenty. For example, producing a backward sequence from eighteen might sound like "(silent lip movement: fourteen, fifteen, sixteen, seventeen, eighteen) eighteen, seventeen, sixteen, (lip movement: thirteen, fourteen, fifteen) fifteen, fourteen, thirteen, etc." This procedure requires the alternating use of two fairly difficult abilities: backward digit span (remembering several words and producing them in reverse order) and remembering the last word already produced in the backward sequence while finding and producing a forward segment that will end with the word just before that last word.

The ability to produce a backward sequence from twenty is relatively late to appear. More than half of a sample of 14 5-year-olds attending a heterogeneous urban school were unable to give a backward sequence beginning from a word between seven and twenty. Eleven of 32 6-year-olds were unable to produce such sequences from words between eleven and twenty, even though all of these could produce accurate forward sequences above twenty and at least one backward sequence beginning from a word below ten. The backward sequences were produced by the 6-year-olds with widely varying degrees of ease, with some children producing them smoothly and quickly and others doing so only very slowly and laboriously, with much subvocalization of forward segments. During this generation procedure by the latter children, the forward sequence was evidently so salient that occasional forward intrusions would occur within the backward sequence (e.g., "fourteen, thirteen, twelve, thirteen, eleven").

Bell and Burns (Notes 1 and 2) examined the ability of a heterogeneous sample of kindergarten and first, second, and third graders to produce backward sequences at various points from ten up to 3141. The percentage of correct performance at each testing point for each grade is given in Table 2.10. Producing a backwards sequence even from ten is a problem for a substantial number of the kindergarten sample and producing a backwards sequence from thirty remains a problem for almost two-thirds of the first graders and a third of the second graders. Performance within each of the first three grades is extremely varied, ranging over almost the entire range tested.

Bell and Burns also found a similar performance lag between backward and forward sequences for most children through third grade. In the same sample as above, they examined the production of both forward and backward sequences at seven levels: below thirty (Level 1), to thirty (Level 2), and then at certain higher key points: 68 to 72, 98 to 101, 197 to 203, 997 to 1003, and 3148 to 3151 (Levels 3-7). The range in the level differences between the forward and backward sequences of individual children was 0 (no difference) to 5 (forward sequences to 1003 and backward sequence less than thirty). The percentage of this kindergarten through third grade sample with no difference in the level of their forward and backward

Table 2.10 Percentage of Age Groups Producing Accurate Backward Sequences of Various Lengths[a]

	Not 10→1	10→1	30→20	72→62	101→91	203→193	1003 →999	3151 →3141
Kindergarten	41	44	7	4	4	0	0	0
First grade	7	59	10	3	17	3	0	0
Second grade	3	30	15	10	13	15	0	15
Third grade	0	17	8	4	21	13	21	17

[a] These figures are computed from raw data generously made available to us by Bell and Burns. Each percentage is for children who were successful up through that length but no higher.

sequences was 38%, and the percentages with differences of one, two, three, four, and five levels between their forward and backward sequences were 20%, 18%, 16%, 8%, and 3%, respectively. These figures indicate that considerable individual differences exist in the ability to produce a backward sequence when a forward one is known. The percentage of children at each performance level whose backward sequence was at the same level as their forward sequence increased from 25% at Level 2 (in forward counting) to 50% at Level 5, indicating that producing a backward sequence once the forward one is known becomes relatively easier for children with longer sequences. The percentage with both forward and backward levels at Level 6 dropped to 14%, indicating perhaps that producing a sequence backwards over 1000 (i.e., from 1003 to 997) is particularly difficult. The percentages of children with no differences between forward and backward sequences were relatively high at Levels 1 and 7 (44% and 71%) due to floor and ceiling effects.

The types of errors the Bell and Burns children made at the decade words were of two types: they either omitted the decade word altogether (e.g., 72, 71, 69, 68, . . . , 62, 61, 59) or they began the backwards sequence within a decade with the decade word (72, 71, 60, 69, 68, 67, . . . , 62, 61, 50, 59, 58, etc.). Thus, for these children (and perhaps for all children at some developmental point) the decade word seemed to serve as the "starting signal" for the production of a decade-digit sequence; this "starting signal" was deemed necessary for a backward as well as for a forward production. Children also displayed similar difficulties with the hundreds and thousands words. The backward sequences for the most part maintained the structure within a decade (*x*-ty-nine, *x*-ty-eight, . . . , *x*-ty-one), however, indicating that children were using their knowledge of this forward structure to generate the backward sequences. Converging evidence on this point comes from Secada (Note 14): pauses in the hand signs made by deaf children producing backward sequences from thirty sometimes come between the production of a sign for twenty and that for the digit word accompanying that twenty (thirty, twenty-nine, twenty—pause—eight, twenty—pause—seven, twenty—pause—six, twenty," etc.). These children seem to know that each word will be a "twenty-*x*" word, but they need to stop and think to produce the correct digit word.

Our data indicate that children initially produce backward sequences through the use of the echoic memory technique adults report using with the alphabet: The generation of parts of the forward sequence and the production of these parts backward while the forward part is still in short-term memory. The ability to do this would seem to be dependent upon a child's processing capacity (in the sense used by Case, Kurland, & Daneman, Note 11) and on the level of the word sequence (i.e., at least at the breakable chain level). Later, as suggested by Bell and Burns, children use their ability to produce a backward digit sequence (from nine to one) and their knowledge of the structure of the forward sequence to produce backward sequences above twenty. Children's knowledge of the forward structure dictates how they will produce the backward digit sequence. Problems occur at the transitional points in the structure and reflect either inadequate knowledge of the forward structure or a representation of this structure that is inadequate to support the backward production at that point (as the auditory short-term memory supported the earlier productions). The situation with respect to the teens is not entirely clear. There was no evidence in any of our data that children were aware of or used the digit pattern present in the teens to produce a backward sequence. Rather they exhibited alternating forward-backward partial productions, indicating that the same echoic memory process used below ten was being used in the teens. Eventual facility in producing teen backward sequences rapidly may come from one of two sources: After they can produce it, children learn the backward sequence from twenty by rote (much as they do the sequence from ten) or they may later learn the digit structure in the teens (perhaps as a result of learning the symbols for those words) and then use this structure to produce the backwards sequence rapidly. An alternative with all backward sequences, of course, is that they may be acquired independently as the forward sequence is, but this seems to occur rarely except for the sequence from ten to one.

Counting down from "b" to "a." Counting down from "*b*" to "*a*" (i.e., from one arbitrary word to another) is the backward skill analogous to counting up from "*a*" to "*b*." To assess the relative difficulty of counting up from "*a*" to "*b*" and counting down from "*b*" to "*a*," both of these tasks were administered to 16 children from a university laboratory preschool. The children ranged in age from 4 years 2 months to 5 years 6 months (mean age 4 years 8 months). The order of presentation of tasks was counterbalanced, with the counting up and counting down tasks administered on separate days to reduce interference between them. Each testing session began with an explanation of the task and two practice trials. For example, the counting up task was described as follows: "To count up from two to five, you start counting at two and count up to five, like this: two, three, four, five. Start at two; stop at five." Practice trials of counting up from three to six and four to seven followed. An analogous description and practice trials were given for the counting down task. Six counting trials were then presented. Five, five, seven, eight, eleven, and thirteen were used as starting numbers for counting up; eleven, twelve, thirteen, sixteen, eighteen, and twenty were used for counting down, with "*b*" differing from "*a*" by either seven or fourteen numbers. "Twenty-one" was the largest number appearing in these trials. Consequently, only children with conventional word strings exceeding that number were used in this study. The assessment of children's conventional word strings revealed that our subjects were divided into two groups: Nine children had conventional strings of at least fifty (rote counts were stopped at fifty), while the other seven had conventional strings below thirty-nine, with most ranging from twenty-nine to thirty-four. As a result, the length of a child's conventional sequence was included as a variable in the following analyses. This allowed an examination of the relationship between the degree of acquisition of the conventional word sequence and its elaboration, as evidence by performance on the counting up and counting down task. In general, children exhibited the behaviors observed in the counting up and counting down studies described earlier, like getting a "running start" to count up from "*a*" by surreptitiously counting from one, determining the next number when counting down by counting forward from a lower number, and intruding forward counts while counting down (e.g., "fifteen, fourteen, thirteen, fourteen, fifteen, sixteen, . . .")

The number of correct trials per child was measured by two criteria: In strict scoring, only flawless counts were considered correct, while lenient scoring included as correct those sequences with mistakes that were spontaneously corrected (such as lapsing into counting forward while trying to count backwards). A 2 (Direction of Sequence) by 2 (Conventional Sequence Length: to 50 or below 39) by 2 (Number to be Counted Up or Down: seven or fouteen) analysis of variance was done on each of the scores (strict and lenient). As expected, counting up from "*a*" to "*b*" was clearly easier than counting down from "*b*" to "*a*." The main effect of Direction of Sequence was significant for both types of scoring: $F(1, 14) = 13.73, p < .01$ for strict scoring and $F(1, 14) = 8.06, p < .02$ for lenient scoring. The mean number of correct counting up and counting down trials was 4.25 (out of 6) and 2.58 trials, respectively, for the strict scoring, and 4.56 and 3.19 for the lenient scoring. The length of the conventional sequence produced by a child had a significant effect on performance with the lenient scores, $F(1, 14) = 7.97, p < .02$, and this effect approached significance with the strict scores, $F(1, 14) = 4.13, p < .06$. Children with sequences to fifty had lenient score means of 4.56 correct trials per task compared to a lenient score mean of 3.00 for children with shorter conventional sequences; these means for strict scores were 4.06 and 2.57, respectively. This effect of conventional sequence length indicates that children with longer conventional word sequences have elaborated the earlier parts of their word sequences more than children with shorter conventional sequences as measured by the possession of these two sequence skills. The interaction of direction of count and conventional sequence length was not significant with either type of score, indicating that the backward sequence elaboration was delayed similarly in both groups of children. The main effect of number counted up or down was significant for the strict criterion scores [$F(1, 14) = 4.63, p < .05$], but not for the lenient scores, indicating that spontaneously corrected mistakes were more likely to occur on the longer sequences than on the shorter ones. With the strict scores, 60% of the trials in which seven words were counted up or down were correct, while 50% of those requiring fourteen words counted up or down were correct.

Errors made on these tasks revealed more details about the development of word sequence skills. The most common mistakes were sequence errors (selecting an incorrect next word, usually due to omission) and forgetting the stopping point, thereby producing a correct sequence that was too long or too short. In counting up from "*a*" to "*b*," sequence mistakes were made on only 5% of the trials, while 20% of the trials contained stopping-point errors. Thus, the major difficulty in counting up from "*a*" to "*b*" was stopping the word sequence at the appropriate point, with 80% of the mistakes being of this type. On the counting-down task, 27% and 28% of the trials contained counting mistakes and stopping-point errors, respectively. The difference between tasks in the number of sequence mistakes was significant [$F(1, 14) = 8.16, p < .02$], again indicating that backwards sequences are more difficult to produce. The difference between counting up and down in the number of stopping-point errors was not significant. However, it seemed possible that this was attributable to the smaller number of counting-down trials that were actually completed. To test this, a second analysis compared the percentage of completed trials containing stopping-point errors on each task. This analysis revealed significant main effects of both Direction of Count [$F(1, 14) = 5.29, p < .05$], and Length of Conventional Sequence [$F(1, 14) = 5.29, p < .05$]. The mean stopping-point error rate was 20% for counting up and 39% for counting down. The children with longer and shorter conventional sequences made stopping errors on an average of 20% and 41% of their completed counts, respectively.

These results are consistent with a model of short-term memory having limited capacity (Case, Kurland, & Daneman, Note 11). According to such a model, more difficult tasks require more processing capacity, leaving less space available for retaining other information or for executing other cognitive processes. At least two sources of stopping-point errors seem possible: difficulty in remembering the stopping point and difficulty in using an adequate checking procedure for determining when this number has been reached. Both of these require space in short-term memory. Because producing a backward sequence is more difficult than producing a forward one, as indicated by the greater number of sequence errors and the number of trials not completed, producing a backward sequence requires more short-term memory capacity. More backward stopping-point errors would then be expected because less space is available for retaining the stopping number or for executing the checking procedure. Similarly, the significant effect of length of conventional sequence suggests that the sequence skills are more effortful (or take more space in memory) for children with shorter conventional sequences. Consequently, these children are more likely either to forget the stopping-point word or to fail to execute their checking procedure than are children with longer conventional sequences.

And Then Before Relations. The first step toward the production of a backward word sequence is the ability to answer And Then Before questions for a given number word, for example, given "eight" to produce "seven." Performance on And Then Before questions ("When you are counting, what number comes just before eight?") is considerably lower than that for And Then questions ("When you are counting, what number comes just after eight?") until about age 5½. For two samples of 24 children aged 3½-4½ and 4½-5½, the percentages of correct performance on these two types of questions were 13% versus 49% and 57% versus 81%, respectively. Accuracy levels for these two relations were roughly equivalent for 12 kindergarteners aged 5½-6½ (80% vs. 86%), and performance was at ceiling for 12 end-of-the-year, middle class first graders aged 6½-7½ (100% accuracy for both relations). Response times taken by a digital stop-watch for producing correct And Then Before responses were slower for all age groups than times for producing correct And Then responses (7.9 vs. 2.6 seconds, 4.3 vs. 1.7 seconds, and 2.1 vs. 1.4 seconds for the three oldest groups described above).

Use of And Then and And Then Before Relations. The And Then and And Then Before sequence relations have analogous relations on cardinal words: One Smaller Than and One Bigger Than. It was not evident whether these sequence and cardinal relations developed independently or whether one type of relation was used to construct the other. We examined this question in a study with 72 prekindergarten, kindergarten, and first grade children. At each age children were randomly assigned to cardinal or sequence conditions and given one of the following types of questions: sequence: "When you are counting, what number comes right after (comes right before) seven?" or cardinal: "What number is one bigger than (one smaller than) seven?" Subjects in both conditions were given both after and before (or bigger and smaller) questions. In all the Age by Number Word Size (below ten and between ten and twenty) cells, responses to the And Then/And Then Before sequence questions and the One Greater Than/Smaller Than cardinal questions were approximately the same, with two exceptions. For words between ten and twenty, correct responses to the sequence questions exceeded those for the cardinal questions for the prekindergarteners in the forward direction (And Then responses 78% correct and One Greater Than responses 56% correct) and for the kindergarteners in the backward direction (And Then Before 78% and One Smaller Than 50% correct). Data reported in earlier sections indicated that 4-year-olds are just becoming able to answer And Then questions for words in their sequence, and 5-year-olds are becoming able to do so quite well for And Then Before questions. These are therefore exactly the ages at which one would expect performance on cardinal questions to be lower than that on sequence questions, if children do use the latter to answer the former. That is, one would not expect children just acquiring sequence relations to use them for cardinal relational questions. However, after the And Then or the And Then Before relations have been acquired, they could be used to respond to verbal questions involving the corresponding relations in cardinal contexts. At the latter point, performance in sequence and cardinal conditions would be equivalent. This is the pattern observed in the data. Thus children seem to use these sequence relations to determine the cardinal relations.

Comes After and Comes Before Relations. In a preliminary exploration of performance on Comes After and Comes Before relational questions, 36 middle class children aged 4½-7½ were given questions such as, "In counting, which comes later, five or nine?" or "In counting, which comes earlier, five or nine?" All pairs of words were four words aparts; words "two" through "nine" and "twelve" through "nineteen" were used. The form of the questions (Comes Later Than or Comes Earlier

Than), order of the words within the word pair, and size of the pair of words (single digits or in the teens) were all counterbalanced. Mean correct response rates for both question forms and both sizes of number word pairs fell (in no particular pattern) between 61% and 72% for children aged 4½-6½ and rose for those aged 6½-7½ to 98% and 92% for single-digit and teens responses, respectively.

Three possible derivations of the Comes After and Comes Before relations seem plausible. One of these involves the use of other sequence relations, one depends on the use of analogous cardinal relations, and the third requires the use of some mental process on some representation of the sequence. These three derivations are: (1) that the Comes After (Comes Before) relation results from the application of transitivity to successive iterations of the And Then (And Then Before) sequence relation; (2) that the Comes After (Before) relation is derived from the isomorphic Greater Than (Less Than) relation defined on the cardinal number words (or vice versa); and (3) that the Comes After (Before) relation is derived from the sequence itself by some sort of direct mental process with psychophysical properties. We shall discuss each of these in turn. During this discussion, it should be kept in mind that, in fact, it is possible that either different children use different ones of these methods or that different mixtures of these methods are used, perhaps depending upon the size of number involved.

The first possible derivation of the Comes After relation is the most mathematical and perhaps the most obvious. It predicts that a child first learns And Then for words in her or his sequence and uses these successively to find Comes After relations. An example is: "5 And Then 6" and "6 And Then 7" and "7 And Then 8" and "8 And Then 9"; therefore, 5 is followed by 9 (or 9 Comes After 5). If And Then is so used to construct Comes After, the response to an And Then question should be faster than that to the derived Comes After relation. One would also expect that some children could answer And Then questions but would not be able to answer Comes After questions.

We examined this in a preliminary way by giving to 36 first graders, kindergarteners, and prekindergarteners Comes After questions for number word pairs that differed by four ("In counting, which comes later, 2 or 5?") and And Then questions for the smallest number in the Comes After pair ("In counting, which comes later, 2 or 3?"). The scores of the first graders reached ceiling. For the kindergarteners and prekindergarteners, there were no significant differences between these conditions in the error scores (all scores were between 61% and 72% correct). Response time data taken by digital stopwatch showed different patterns for number pairs above and below ten. For the former, the mean response times of correct judgments (based only on those item pairs for which both And Then and Comes After responses were correct) for And Then responses were slower than the times for Comes After responses (1.9 vs. 1.5 seconds, 2.1 vs. 1.5 seconds, and 3.5 vs. 1.4 seconds for the first grade, kindergarten, and prekindergarten groups). For the number words less than ten, neither type of pairs was consistently better across all age groups. These response time data indicate that the first proposed derivation of the Comes After relation–from the composition of contiguous And Then relations–is inaccurate. Transitive application of the And Then relation does not seem to be the process by which the Comes After relation is determined.

The second alternative for the source of the Comes After (Comes Before) sequence relations is that they are derived from the Greater Than (Less Than) cardinal relations. Fuson and Hall (in press) discussed this possibility and presented data that were not definitive with respect to the relationship between the two types of relations for words below ten but found that for words between ten and twenty, the cardinal relations seemed to be derived from the sequence relations. In their study they compared the performance of children aged 4-7 years old on questions involving the order relations on sequence and on cardinal words [the Comes After (Comes Before) and the Greater Than (Less Than) relations]. These data supported the conclusion that below the age of 6½, the order relations on cardinal (numerosity) words less than ten develop earlier than (or at least are more accurate than) those on sequence words, that a single process for deriving the order relations on sequence words is used over the whole range of sequence words from one to twenty, and that this same sequence process is used for order relations on cardinal words between ten and twenty. By age 6½ a ceiling effect is reached for all questions of this type for words below twenty. These data thus support the opposite of Alternative 2 above for number words between ten and twenty and are not definitive with respect to it for words below ten.

Some authors in the literature have, in fact, argued for the opposite of the second alternative above, that is, they have argued for derivation of the cardinal relations from the sequence relations. Parkman (1971), for example, describes a model in which sequence words are produced covertly and very quickly and are used to decide Greater Than relational questions. Some of the kindergarten subjects on some items in the Sekuler and Mierkiewicz (1977) study overtly used such a sequence production procedure: The child would count from one until she or he reached one of the number words and then said that the other word was the bigger one (Mierkiewicz, Note 15). Although we also observed the overt use of this procedure, most children gave no evidence of using it. It may be that this is a "fail-safe" procedure, used when the more usual process fails.

The third possible source of performance on Comes After relational questions is that this relation on a given pair of number words is "read off" an internal representation of the number word sequence by a process with psychophysical characteristics. Such processes are characterized by an inverse relationship between reaction time and the distance between two stimuli (reaction time increases as the distance decreases) and are assumed to occur by some sort of analog process in which items become decreasingly discriminable as they become more similar on some physical scale. There is considerable literature on the existence of these relations in adults. The linear order literature and the digit and alphabet comparison literature are particularly relevant to the Comes After (Comes Before) sequence relations. These are reviewed by Fuson and Hall (in press) with respect to their implications for understanding relationships between the order relations on sequence words (Comes After/Comes Before) and those on cardinal words (Greater Than/Less Than). In spite of the considerable amount of research on the Greater Than (Less Than) order relations on cardinal words, the process used in producing these relations is still not clear. Nor is it clear whether the Comes After (Comes Before) order relations on sequence words below ten are derived from those on cardinal words or whether the sequence order relations involve a different (though possibly similar) representation and an independent processing of this sequence representation.

The Between Relation. The adult Between relation on sequence words (e.g., "Six is between five and eight") is equivalent to the conjunction of the Comes Before and the Comes After relations. For example, "Six is between five and eight" is equivalent to "Six comes after five and six comes before eight." The Between relation is also related (somewhat more primitively) to And Then and And Then Before relations, for the proper use of either of these will give at least one word between two given words (e.g., "five And Then" will be a word between five and eight as will "And Then Before eight"). The Between relation is also related to two word sequence skills at the breakable chain level: counting up from "*a*" to "*b*" and counting down from "*b*" to "*a*." The use of either of these will generate not only some but exactly all of the words between "*a*" and "*b*." The Between relation also possesses a spatial meaning. A request for a number word between five and nine might then elicit the use of a representation of the number word sequence that has a spatial aspect, with the resulting answer "seven," the "most" between word. If the Between relation is instead initially linked to counting up or down or to the And Then/And Then Before relations, such a request might produce the words "six" or "eight." Very little empirical work has been done on any of these aspects of the Between relation.

Using the Between Relation. The Between relation on sequence words also has a counterpart relation in cardinal or numerosity contexts. The Between relation in cardinal contexts is defined by counterparts of the Comes After and Comes Before relations on sequence words: a word is between (in the cardinal sense) five and nine if it is "Bigger Than five" and "Smaller Than nine." We examined the nature of the relationship between the Between relation on sequence words and that on cardinal words. Forty-eight kindergarten and first grade children were asked to respond to cardinal or to sequence questions that gave the boundaries for the Between relation in two separate phrases. The questions asked were of the form, "Tell me two numbers that come after three and before seven when you are counting" (sequence) and "Tell me two numbers that are bigger than three and smaller than seven" (cardinal). The number word pairs given in the questions always had three words between them to provide an opportunity for different strategies of response as described above. Steffe, Spikes, and Hirstein (Note 16) used such pairs for Between questions for that reason; we adapted their between questions to our two-phrase forms. In each condition of this study half of the questions involved pairs ten and below and half used pairs between ten and twenty. These pairs were used in the same random order for all subjects. For each condition the phrases within each question were ordered so that half of the questions had the number word pairs in ascending and half in descending order (e.g., "Tell me two numbers that are smaller than nine and bigger than five"). Half of the subjects began with each order. Grade, Word Context (sequence or cardinal), and Order of First Pair were between-subjects variables and Size of Number Word was a within-subjects variable.

Preliminary analyses indicated no main effect and no interaction with Order of First Pair on the number of correct responses, so further analyses collapsed over this variable. A 2 (Grade) by 2 (Word Context) by 2 (Size of Number Word) repeated measure analysis of variance on correct responses revealed significant main effects of Grade [$F(1, 44) = 14.73, p < .001$], Word Context [$F(1, 44) = 5.01, p < .03$], and Size of Number Word [$F(1, 44) = 12.66, p < .001$], and a significant Grade by Word Context interaction [$F(1, 44) = 16.62, p < .001$]. The percentage of correct performance is given in Table 2.11. The kindergarten and first grade children performed equally well on the sequence questions (78% and 76%), but on the cardinal questions the kindergarten children did much worse than did the first graders (37% vs. 88%). In three of the four Grade by Word Context cells the children did slightly (about 10%) better on the pairs below ten than on those above ten.

The two responses to each question were classified according to their location within the word sequence with respect to the given word pair (examples will be given for the pair "five and nine"). Three major types of response patterns depending upon the sequence were identified. The first type consisted of a two-part strategy in which a single response was given to each of the two questions asked. These responses used the And Then relation on the smaller number and the And Then Before relation on the larger number. The classification of the responses in the Cardinal condition as And Then and And Then Before responses was based on the earlier reported evidence of children's performance in sequence and cardinal conditions. Supporting evidence that this strategy in fact did consist of answering each of two questions separately was that many children paused between their two responses and some also then asked for a verification of one of the questions ("Was that 'after five'?") or subvocalized the question to themselves (e.g., lip movement for "comes after five" and then vocalization of "six"). The other two strategies used on the between questions required the integration of the two responses together into the sequence. The first consisted of simple count up or count down strategies (responses were "six, seven" or "eight, seven"). In the second, the first word given was the "middle" spatial response discussed earlier (giving the word exactly in between the question pair); then the next word up or down from that middle word was given (responses were "seven, eight" or "seven, six"). Note that both of these sequence types had both forward and backward counterparts. Over all the condi-

Table 2.11 Percentage of Correct Responses to Two-Phase "Between" Questions

	Kindergarten		First grade	
	Single digits	Teens	Single digits	Teens
Sequence condition	85	70	76	76
Cardinal condition	42	32	93	83

tions and over all subjects, the responses of these two types going up outnumbered the down responses by a ratio of five to one.

Table 2.12 contains the percentage of uses of these strategies by age and by number word question asked. There is very little difference in the strategies used by the two age levels in the sequence conditions (i.e., with the Comes After and Comes Before questions) but considerable difference between the two age levels in the strategies used in the cardinal condition (with the Bigger Than and Smaller Than questions). More first grade than kindergarten children used or tried to use the two-part strategy in the cardinal condition (59% vs. 27%) and a higher proportion of those who used it were able to use it successfully (75% vs. 23%). Earlier findings had indicated that kindergarten children could successfully use the And Then sequence relation in a cardinal context but that they were experiencing difficulty in using the And Then Before relations. The correct two-part responses given by the kindergarten children were consistent with this earlier finding: 70% were words immediately following the given word (And Then responses) and only 30% were And Then Before responses. By the first grade, correct And Then and And Then Before responses were evenly balanced (49% and 51%). In addition to this difference in the use of the two-part strategy, the kindergarten children seemed to have particular difficulty with the directionality of the words in the cardinal conditions; many of their incorrect two-part strategy responses were in the wrong direction (e.g., smaller than rather than larger than the word). They did not show the same directional difficulty in the sequence condition. There was also a considerable difference between the sequence and the cardinal condition in the kindergarteners' percentage of errors that were close errors (within two words of the given number word pair): 69% of the errors in the sequence condition were of these close errors while only 43% of the errors in the cardinal condition were close. A final difference between the two age groups was that the first grade children were also somewhat more advanced than the kindergarten children in their use of the integrated sequence strategies in the cardinal condition (38% vs. 17%).

The order in which the number word pair was presented in the very first question seemed to influence the answer strategy used over all the questions by the first

Table 2.12 Percentage of Strategy Use by Grade and Question Type on Two-Phrase "Between" Questions

	Two-part strategy			Integrated sequence strategies			
	Both correct	One correct	Total	Count up/ down two words	Middle and then up/ down	Total	Other
Sequence							
Kinderg.	28	17	45	36	6	43	13
First	28	20	48	26	13	39	14
Cardinal							
Kinderg.	6	21	27	10	7	17	55
First	44	15	59	23	15	38	4

graders. In the sequence condition, if the words were given in their sequence order (e.g., "Give two words that come after five and come before nine"), children were more likely to use the integrated sequence strategies than the two-part strategy (25 vs. 4 occurrences). If the words were given in reverse sequence order (e.g., "Give two words that come before nine and come after five"), the two-part strategy was more likely to be used than an integrated sequence strategy (23 vs. 12 occurrences). In the cardinal conditions, the children responded to the order in which they heard the number pair in the opposite way, presumably because they responded first to the last number word they heard. If the words were given in nonsequence order, they would respond to the last question first (by giving the word that was bigger than five: "six") and then would continue with the word following six ("seven"), that is, they used an integrated sequence strategy rather than the two-part strategy by 28 to 9 occurrences. If the words were given in their sequence order, a number smaller than nine would first be given and then a response to the second part of the question (bigger than five) would be given; that is, the two-part responses outnumbered the integrated sequence responses 33 to 8. This finding would seem to indicate that children, in fact, possess both the integrated sequence and the two-part strategies, and they employ the one that seems best to fit their initial view of a task. Task variables seem to present particularly tricky problems here, and such variables may be responsible for underestimating the extent to which children have in fact coordinated sequence and cardinal meanings and can use one type of meaning to respond to a question in the opposite context.

This study, of course, was only a very preliminary step toward understanding the development of the sequence and the cardinal Between relations. Studies which compare the two-part terminology used here with the use of the word "between" obviously need to be done. The end of the year kindergarteners in the present study seemed to have much greater difficulty with the cardinal Greater Than/Less Than questions than with the sequence Comes After/Comes Before questions. These cardinal difficulties may have been exacerbated by the juxtaposition of two Greater Than/Less Than questions in the opposite direction when no concept of "between" was present to impose constraints on these directions. The sequence relations may have been simpler because the sequence itself may have imposed some sense of "betweeness" on the two relational statements.

Use of Counting Up and Down Skills in Addition and Subtraction. At the breakable chain level the ability to count up from "*a*" to "*b*" may be used in addition situations such as "8 + ? = 13." Such users will begin counting up at eight and will stop at thirteen. However, children at this level fail to keep track of how many words they have counted up, and so they cannot give any accurate answer at the end of this procedure. Steffe, Richards, and von Glasersfeld (Notes 7 and 17) reported such failures by some children. Such performances occurred in the studies that we report in the next section. The breakable chain level also seems to occur in other cultures. New Guinea Oksapmin children and adults unfamiliar with economic transactions also use counting up from "*a*" to "*b*" without keeping track on their body parts counting system (see Saxe, Chapter 5 of this volume). They say and point to the body parts from "*a*" to "*b*" (e.g., from elbow to ear), but fail to count or match these parts with any other set, and so they also fail to produce any answer. What is required for all of these problems in any culture is that the words of the sequence be taken as units that represent the missing addend and that some means of assessing the numerosity of these units be used. Both of these occur at the next level, the numerable chain level.

Numerable Chain Level

Forward and Backward Sequence Skills. At the numerable chain level, the number words in the sequence can be taken as distinct units, and the numerosity of word segments (words contiguous in the sequence: seven, eight, nine, ten, eleven, twelve) can be ascertained. At this level, the number words are not just produced—they can also be counted or matched to a set of items of known numerosity (e.g., five fingers). Two new forward sequence skills exist at this level: "count up a specified number '*n*' from '*a*' " and "count up from '*a*' to '*b*' to find the number of words from '*a*' to '*b*.' " Parallel skills for the backward sequence become evident some time later: "count down '*n*' from '*b*' " and "count down from '*b*' to '*a*' to find the number of words from '*b*' to '*a*.' "

Counting up from "*a*" to "*b*" and counting down from "*b*" to "*a*" while keeping track of how many words are counted up or down require one to remember the word to which one is counting up (or down) while keeping track in some way of the number of words being produced. Counting up or down by "*n*" requires one to remember the number of words that one is counting up or down while also keeping track of the number of words that one has already produced. Both types of skills require both the memory of a number while one is counting up or down and some method of keeping track of how many one is counting up or down. We have done research on three of the four sequence skills and on keeping-track methods. The first study compared performance on counting up and down by "*n*" and the second compared the two counting-up skills using larger second numbers than were used in the first study. Each of these studies and the work on keeping-track methods will be discussed in turn.

Our own research and the research of Steffe, Richards, and von Glaserfeld (Note 17) has indicated a considerable delay between the ability to count up or down with small numbers (one, two, three, and perhaps four) and with larger numbers. Counting up or down two or three seems able to be done with methods that are used relatively early and do not generalize to larger numbers. For a particular chain to be at the numberable chain level, we therefore require performances on one of the word sequence skills with "*a*" and "*b*" differing by five or more.

Counting Up or Down by "*n*". We investigated the approximate age of acquisition of the skills of counting up and counting back by "*n*" and also explored the effects on this skill of the size of "*n*" and of the word being counted from. Initial piloting in an urban school with a heterogeneous population indicated that many 5-year-olds had considerable trouble counting up with second addends of five or more, and some of them could not even produce a backward counting sequence. Therefore, a sample of 32 randomly selected 6-year-olds (half aged 6 years to 6 years 5 months and half, 6 years 6 months to 6 years 11 months) from this school was given matched counting up and counting down problems. Three sizes of number words counted up or down were used (two, five, and eight), and three starting points in the word sequence were employed (three, seven, and fourteen). Instructions of the form, "Start counting with '*a*' and count up (or back) '*n*' more numbers," were provided, and repeated demonstrations were given if necessasy. Experimenters recorded any "keeping-track" behavior exhibited, and after the final problem they asked children who had displayed no observable strategies how they had known when to stop counting. The order of the counting-up/counting-down sets of problems was counterbalanced. Sex was balanced within each age by order cell. Two scoring systems were used to evaluate the responses. The strict system gave a point only for a correct answer; this system thus identified problems for which a correct keeping-track method had been selected and been used properly. The lenient scoring system gave credit for any answer that was within one number of being correct.

Most of the children could do some, but not all, of the problems. Four children performed perfectly on the counting up tasks, while none did perfectly on the counting down tasks (although one child overcounted by one on one problem and had the rest correct). Only two of the 32 subjects could not do any of the counting up tasks correctly, while five did not get any of the counting down problems correct. A 2 (Age: young sixes and old sixes) by 2 (Direction: counting up and counting down) by 3 (Size of number word counted up or down: two, five, and eight) analysis of variance was done on the strict and on the lenient criterion scores. For the strict criterion scores, the main effects of Direction [$F(1, 30) = 14.73, p < .01$] and Size [$F(2, 60) = 42.74, p < .01$] were significant. None of the interactions nor the main effect of Age attained significance. Counting up was significantly easier than counting down; performance was 51% correct for counting up and 33% correct for counting down. Many children were still having difficulty producing a backward word sequence, and some still had to produce it piece by piece from the forward sequence. With respect to the size of number counted up or down, children did much better when they had to count up or count down two than when they had to count up or count down five or eight, and they did somewhat better for five than for eight. A Newman-Keuls test on the means for each size indicated that each of these size differences was significant. The percentages of correct responses for counting up two, five, and eight were 79%, 45%, and 30%, respectively, and for counting down by these amounts were 68%, 18%, and 13%, respectively.

The strict criterion scores assessed correct procedures correctly carried out. The lenient criterion scores (correct score +1 or −1) gave credit to children who were using a basically correct procedure but who made some minor error. A 2 (Age) by 2 (Direction) by 3 (Size) analysis of variance on the lenient criterion scores revealed several more subtle effects. As with the strict criterion scores, the main effects of Direction [$F(1, 30) = 21.57, p < .001$] and Size [$F(2, 60) = 28.57, p < .001$] were significant. Counting up was still significantly easier than counting down (74% vs. 52% correct). A Newman-Keuls test on the means for each size of number word counted up or down indicated that the difference between counting up or down by two and by five and the difference between counting up or down five and eight were each significant. A significant Size by Direction interaction [$F(2, 60) = 6.83$, $p < .005$] indicated that the difference in performance between counting up and counting down was very small for two (80% vs. 74%), but was larger for five (83% vs. 47%) and for eight (59% vs. 34%). A significant Size by Age interaction [$F(2, 60) = 6.83, p < .005$] indicated that the difference in performance between counting up and counting down was very small for two (80% vs. 74%), but was larger for five (83% vs. 47%) and for eight (59% vs. 34%). A significant Size by Age interaction [$F(2, 60) = 5.18$, $p < .01$] indicated that the difference in the performance of younger (6-6½) versus older (6½-7) children on the problems involving counting up or down eight was much greater than was the age difference for counting up or down five or two (35% vs. 57% correct for eight, 63% vs. 68% for five, and 79% vs. 75% for two). A Direction by Age interaction that approached significance [$F(1, 30) = 3.03, p < .10$] revealed a tendency for the younger children to perform the same as the older children on counting up (74% correct vs. 74%) but to do much more poorly on counting down (44% vs. 60% correct). These results on this narrow age range (6-6½ vs. 6½-7) seem to indicate that the ability to count down develops fairly rapidly from early to late in the seventh year and that the young 6-year-olds are working on, but have not yet mastered, ways to keep accurate track of the number words. The findings of significant Direction and Size of Number Word effects with both criterion scores emphasizes the relative difficulty of counting down as compared with counting up and of keeping track of eight versus five versus two number words counted up or down.

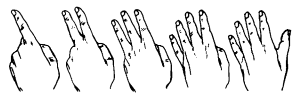
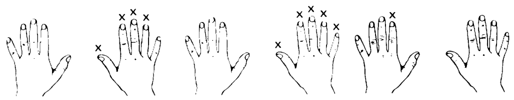
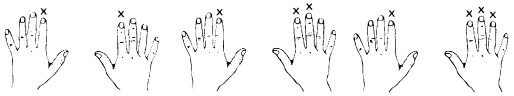
There seemed to be no difference in the accuracy of counting up or counting down as a function of the magnitude of the first number word (three, seven, or fourteen). By age 6, these children seemed to be about equally proficient at counting up from fourteen as from three. However, this result may be partially a result of a lack of complete counterbalancing of these items; the larger number words tended to come somewhat later, and so a practice effect may have been operating. In addition, only the number of errors and not the speed of response was recorded. Different measures might have indicated differences due to location in the number word sequence. The effect of the place in the number word sequence where the counting up/counting down occurs needs further study.

Forward Sequence Skills. Twenty first graders attending a school whose population was computer selected to reflect the racial and economic composition of the city of Chicago participated in this end-of-the-year study. Because the difference between "*a*" and "*b*" was larger than in the previous study, young 7-year-olds were selected for the sample (age range 7 years 1 month to 7 years 5 months, mean 7 years, 3.4 months). The "count up '*n*' from '*a*' " questions were of the same form as the last study: "Start counting with '*a*' and count up '*n*' more number words." The question employed for the number of words from "*a*" to "*b*" skill was of the form, "Count up from '*a*' to '*b*' and tell me how many number words you counted up." The difference between "*a*" and "*b*" was either medium (six and seven) or large (thirteen and fourteen), and "*a*" ranged from two to twelve ("*b*" was always less than twenty). The questions for each skill were blocked, and the order in which the blocks were given was counterbalanced. As in the earlier counting up "*n*" study, some children (in this study, about half) counted the starting word as one of the "*n*" words and thus produced an answer one word before (one less than) the correct answer. This occurred even more frequently for the "count up from '*a*' to '*b*' " questions (in 70% of the sample) and was probably exacerbated by the form of the question used here. In future studies a question form more directly parallel to the other question should be tried (i.e., one that begins, "Start counting with '*a*' "). Overall, performance on these two skills was roughly the same. If both exact answers and those subject to the "one less than" error noted above are pooled and if responses obtained by the use of number facts are also included, 73% of the "Count up from '*a*' to '*b*' " medium responses were correct, and 65% of those for the "Count up '*n*' from '*a*' " were correct. Number facts were only used in response to the latter questions—7% of the time. Performance for the large numbers "*n*" (thirteen, fourteen) was much poorer, 15% and 20%, respectively. This difference resulted chiefly from the fact that most subjects used their fingers to keep track of the number of words they were producing, and they could not figure out how to use their fingers for numbers which exceeded their own ten fingers. Though overall performance for these two skills was generally at about the same level, for individual children it was not always so. Five of the subjects performed better on the "Count up from '*a*' to '*b*' " tasks than on the "Count up '*n*' from '*a*' " task, and eight performed better on the latter than on the former. Seven children performed equally well on both tasks, but three of these reflected ceiling effects and one, a floor effect. This finding of individual children showing superiority in one or the other of these skills should be explored in the future with tasks that have small differences between "*a*" and "*b*" (as well as larger ones) to ensure that subjects understand each type of task.

Procedures for Keeping Track of *n*. All of the skills at this level require that one keep track of the number of words uttered in a given counting up or counting down production. Fuson (in press) developed a classification of the keeping-track methods observed both with word sequences and addition situations in her studies and those of others (e.g., Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, in press; Moser, Note 12; Steffe, Spikes, & Hirstein, Note 16; Steffe, Richards & von Glaserfeld, Note 17). This classification appears in Fuson (in press) and is presented in Table 2.13. The first type of keeping track of the second addend (counting entities) is used in addition situations and requires only a word sequence at the breakable chain level. Objects, not words, are used for each addend, and it is objects that are counted. In the last two major types of keeping track methods ("matching the count" and "counting the count"), the word sequence must be at the numerable chain level, for words now form the addends which are matched with other types of countable units (e.g., fingers or "beats" in an auditory pattern) or are counted to assess the numerosity of the second addend.

The use of these various keeping-track methods has not been studied very systematically. The "counting real entities" method is the first one to develop; it

Table 2.13 Keeping-Track Methods

	Example: 8 + 5 = 13
COUNTING ENTITIES	
Real	"E* N TN EL TV TH"
Represented	"E N TN EL TV TH"
MATCHING THE COUNT	
Match count to estimate	"E N TN EL TV"
Match count to fingers	"E N TN EL TV TH" 
Match count to auditory pattern	"E N-TN-EL TV-TH"
COUNTING THE COUNT	
Auditory count of fingers (Chisenbop)	 "Eight" "One" "Two"
(X means that finger is pressed down on the table)	 "Three" "Four" "Five" Fingers say thirteen.
Auditory count of visual-symbolic (number line)	1 2 3 4 5 6 7 8 9 10 11 12 13 — "ONE (9) TWO (10) THREE (11) FOUR (12) FIVE" (13)
Auditory count of auditory (double counting)	"EIGHT. NINE IS ONE, TEN IS TWO, ELEVEN IS THREE, TWELVE IS FOUR, THIRTEEN IS FIVE."
Visual count of visual (slide rule)	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 / 0 1 2 3 4 5 6 7 8 9 — 8 + 5 = 13

* Abbreviations represent auditory counting words:

E	N	TN	EL	TV	TH
(8,	9,	10,	11,	12,	13)

may involve the use of real entities already present or the use of readily available entities such as fingers. After that, for "n" four or greater, some children seem to use the "counting represented entities" (counting a mental representation of entities which may or may not be in a figural pattern), while others use the "match count to estimate" method, the "match count to fingers" method (successively producing a finger with each word until a given number of fingers has been produced), or the "match count to auditory pattern" method (producing words in a rhythmic pattern). Three of these are fairly accurate, but the "match count to estimate" method is not. It entails no visible means of keeping track and seems to consist of the production of additional words until "about enough" of these have been produced. It thus is probably only a breakable chain level production: a child may simply be counting up some approximate number of words with no well-defined notion of each word as a separate unit. The "match count to auditory" pattern is the method that was seemingly used by most children for $n = 2$ in our counting-up/counting-down study; the sound of the next two words seemed to be sufficient to stop further production. All of the "counting the count" methods need to be learned in school, with the possible exception of the "auditory count of auditory words," which has been observed by all of the above researchers in a few children when there was no evidence that this method had been taught in school. For more details concerning these methods, see Fuson (in press).

Uses of Sequence Skills in Addition and Subtraction. The word sequence skills at the numerable chain level permit tremendous advances to be made in the solution procedures available for addition and subtraction problems (see Table 2.8). A child can now solve problems like "8 + 6 = ?" by counting up six words from "eight," problems like "8 + ? = 14" by counting up from eight to fourteen or by counting down from fourteen to eight while keeping track of how many words have been produced, problems like "14 - 6 = ?" by counting up from six to fourteen or counting down from fourteen to six while keeping track, problems like "? + 6 = 14" by trial-and-error counting up six from arbitrary numbers or by counting down six from fourteen while keeping track. Some first and many second grade children have been observed to use all of these word sequence solution procedures (Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, in press; Houlihan & Ginsburg, 1981; Moser, Note 12; Secada, Note 18; Steffe, Richards, & von Glaserfeld, Note 17), with the particular solution procedure used dependent upon the sizes of the numbers involved and, for verbal story problem versions, upon the situation portrayed in the story. These solution procedures all involve beginning the counting up or down with one of the addends rather than with one. They are called "counting on" and "counting back." For discussions of the additional concepts involved in these procedures see Fuson (in press), Steffe, Thompson, and Richards (in press), Briars and Larkin (Note 19), Davydov and Andronov (Note 20), and Steffe, Richards, and von Glaserfeld (Note 17).

The discrepancy between the ability to count up with small and with large numbers has its counterpart in the use of counting up in addition situations. When the second addend is one or two and objects clearly portray a counting-up addition situation (i.e., the number of objects is known and then one object is added), even some 3-year-olds can use counting up one word to find the total number of objects. In the word sequence acquisition studies reported in the earlier sections, children ascertained the number of blocks in a row and then one or two more blocks were added to the end of the row. When only one block was added, on at least one trial 3 out of 12 3-year-olds uttered the word that had been the number of the row of blocks on the last trial and then its immediate successor. They said, for example, "Eleven, twelve. There are twelve blocks now." Six out of 12 5-year-olds, for at least 60% of their trials, counted on from their previous response when one or two blocks were added, and two more 5-year-olds counted on for at least 40% of their trials. Five months later 6 of those 12 children counted on for at least 90% of their trials and four more did so for at least 60% of their trials. However, when "*n*" is five or greater, we have seen that most 5-year-olds and many 6-year-olds do not even possess the numerable chain level word sequence skills, let alone being able to apply them in addition and subtraction situations.

More Complex Sequence Skills. Counting up or down can be done by tens and by ones. These more advanced counting up and counting down skills permit the solution of two-place addition and subtraction problems. For example, 54 + 37 could be solved by counting up from 54 three more tens and then seven more ones: "Fifty-four. Sixty-four, seventy-four, eighty-four. Eighty-five, eighty-six, eighty-seven, eighty-eight, eighty-nine, ninety, ninety-one." Or this problem could be solved by counting up three decades from fifty and then counting up the ones: "Fifty. Sixty, seventy, eighty. Eighty-seven, eighty-eight, eighty-nine, ninety, ninety-one." The extent to which such counting up or down could be used to measure or to facilitate understanding of our base ten system of numeration or of the usual addition and subtraction computational procedures might be examined in future research.

Counting up or down repeatedly by the same number (e.g., counting up by eight: "Eight, sixteen, twenty-four, thirty-two, forty, forty-eight, fifty-six, sixty-four, seventy-two") will yield the multiplication or division sequence (the "facts") for that number. Such sequences might be used in at least four ways. First, they might be studied for patterns which could facilitate the remembering of facts. Second, the lists for the larger numbers (say, six through nine) could be memorized as a first step in remembering the multiplication facts; the various lists would serve to organize all of the separate multiplication facts. Then factors that went with each product would need to be learned. Something like this happens now with the fives: The list "five, ten, fifteen, twenty, . . ." serves to circumscribe the "fives facts" and then one needs only to sort out a few particulars. Third, learning that one could generate multiplication and division answers by such counting up and counting down might add to a child's understanding of multiplication and division. Fourth, such generation procedures might be used in more limited ways–in the production of one fact from another. For example, 3 X 6 might be found from 2 X 6 by counting up six from twelve. Houlihan and Ginsburg (1981) reported the use of such counting up from known facts by second graders in addition and subtraction problems.

Bidirectional Chain Level

The sequences below the numerable chain level are all strongly unidirectional. Each word is a vector—an entity with direction. The forward or backward recitation context in which each sequence is produced strongly influences the production. We have seen this directional influence earlier in forward intrusions when backward sequences are beginning to be produced. In our studies we have also observed backward intrusions in forward tasks when the forward tasks followed a backward condition. That is, a child seems to set a particular recitation context and then has some difficulty shifting out of it. A sequence at the bidirectional chain level possesses two attributes that distinguish it from other levels: (a) strongly automatized forward and backward sequences that contain no directional intrusions, and (b) the ability to change directions rapidly and flexibly. At the moment the developmental relationship between the bidirectional chain and the numerable chain is not known. The bidirectional chain level may develop independently of the numerable chain level, or it may follow the latter. If these levels develop independently, some children will be at the bidirectional level without being at the numerable level and vice versa.

Steffe, Richards, and von Glaserfeld (Note 17) discussed two uses of bidirectional word sequences: bidirectional counting and reversible counting. In bidirectional counting a child can indicate the counting number of a particular object in a row by counting backwards from a given counting word. For Steffe and co-workers, this bidirectional counting indicates that a child has connected the forward and backward counting actions and knows that they will result in the same counting word for that object. In reversible counting, a child makes a conceptual abstraction and can use backward counting from a known number in a row of objects to determine the numerosity of a group of those objects hidden under a cloth.

The bidirectional level ability to change word production direction rapidly and flexibly enables a child to select the most efficient direction to use to solve a particular problem. It also can lead to an understanding of the inverse relationship between addition and subtraction through either one of two routes: through relating forward and backward counting of the same set of objects or through relating counting up and counting down sequence skills. With respect to the former, children as young as 3 and 4 evidently understand in an intuitive way that "putting together" and "taking away" are inverse operations in the sense that, if the number of objects in a set has been altered, a child will, by "taking away" or "putting together," attempt to recover the original set (Brush, 1978; Gelman & Gallistel, 1978; Starkey & Gelman, in press; Blevins, Mace, Cooper, & Leitner, Note 21). However, these operations are not quantified at this point; children will do the correct replacement operation but will not use the correct amount. A bidirectional chain used in counting objects would seem to be one way to lead to such quantification of the inverse operation. The relating of the forward and backward counting up/counting back sequence skills in order to understand the inverse nature of the addition and subtraction operations may occur in several situations: in verbal problems, in object situations, and in symbolic situations (e.g., $8 + 5 = 13$ is related to $13 - 5 = 8$ and to $13 - 8 = 5$), and thus these may differ considerably. Most present models of addition and subtraction problem solving place understanding of this inverse relationship at the highest level (Riley, Greeno & Heller, in press; Briars & Larkin, Note 19; Nesher, Note 22; Steffe, Richards, & von Glaserfeld, Note 17). Future research may uncover ways in which the number word sequence at the bidirectional level contributes to the understanding of this inverse relationship.

Conclusion

The sequence of counting words is one of the most important tools of early mathematics learning. Its acquisition is a structured process, with children showing consistent individual patterns before the full conventional sequence is learned. After acquiring initial segments of the conventional number word sequence, there is a period of elaboration during which various sequence skills are acquired and relations between words in the sequence are established. The sequence is first used as a problem-solving tool in the act of counting objects and then later the counting words themselves become the objects that are counted. This elaborated, flexible, and easily produced sequence can then become a representational tool that is used in sophisticated counting procedures. In this chapter we have provided an outline of the acquisition and elaboration of the number word sequence. Further work is required for fuller and more detailed understanding of many parts of this developmental learning process.

Our preliminary efforts at examining sequence number words have consisted largely of isolated studies of certain aspects of these changes. Such intensive and isolated efforts are needed in the future, but they need to be complemented by research that involves performance by the same child across many tasks and across longer periods of time. The developmental sequence proposed in this paper is a description of levels, of static states. To date there has been little focus upon the processes by which a child moves from level to level. It is hoped that future work will be able to move from attempting to verify performance at certain levels to explicating the transitions between levels. We also wish to reiterate our caveat at the beginning of the paper about our use of the word "levels." These levels surely are "messier" than Table 2.8 implies. However, they do seem to be useful conceptual distinctions which can facilitate our consideration of changes in children's acquisition and elaboration of the sequence of number words.

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