Early Childhood Research Quarterly 40 (2017) 150–162

Contents lists available at ScienceDirect

Early Childhood Research Quarterly

journal homepage: www.elsevier.com/locate/ecresq

Review

The research-based balance in early childhood mathematics: A response to Common Core criticisms

Douglas H. Clements[a,*], Karen C. Fuson[b], Julie Sarama[a]

[a] Dept. of Teaching & Learning Sciences, University of Denver, Morgridge College of Education, United States
[b] Professor Emerita, School of Education and Social Policy, Northwestern University, United States

ARTICLE INFO

Keywords:
Mathematics
Standards
Common core
Primary
At-risk students
Learning trajectories

ABSTRACT

We address common criticisms of the Common Core State Standards—Mathematics, evaluating them based on comprehensive reviews of existing documentation and research to better ground future debates and to ameliorate negative effects of possible misconceptions or misinterpretations. The four main criticisms follow. (1) No one who helped develop the standards had any expertise in the education of young children. (2.) The CCSSM dictates scripted curricula and didactic instruction rigidly applied to all children at the same pace. (3.) The standards emphasize academic skills and leave no time for play, exploratory approaches, or social-emotional development. (4.) The standards are too early and therefore developmentally inappropriate for children in the early grades. We conclude that these criticisms are not valid, and that, given the importance of mathematics to academic success in all subjects, all children need and deserve to build a robust knowledge of mathematics in their earliest years and can do so if we use the research knowledge and research-based standards and programs presently available. We summarize and exemplify the research-based balanced approach to teaching based on learning trajectories that can provide guidance for engaging and developmentally appropriate mathematical experiences that have been demonstrated to help all children learn to high standards.

1. Introduction

Snow was falling in Boston and preschool teacher Sarah Gardner's children were coming in slowly, one bus at a time. She had been doing high-quality mathematics all year, but was still amazed at her children's ability to keep track of the situation: The children kept saying, "Now 11 are here and 7 absent. Now 13 are here and 5 absent. Now… ." (Clements & Sarama, 2014; p. 1).

To highlight the importance of high-quality mathematical experiences in the preschool and primary school years (preschool through G2) and to facilitate closing the achievement gap resulting from differences in access to such experiences, the National Research Council (2009) issued a research-based report entitled "Mathematics in early childhood: Paths toward excellence and equity". Its research and recommendations were used in developing the Common Core State Standards—Mathematics. However, blogs, newspapers, and other media, including some documents written by researchers, have criticized the Common Core State Standards Mathematics (CCSSM) as being inappropriate for young children in various ways. In this article, we provide information about the research background of the CCSSM and describe and examine four of the most common criticisms in the light of research.

Although the CCSSM do not include preschool, we include research about preschool at certain points because states and documents about standards (e.g., Scott-Little, Kagan, Reid, & Castillo, 2012) are beginning to apply all or some of the Kindergarten standards to preschool. Furthermore, the preschool years can make a major contribution to closing the gap in opportunity to learn mathematical ideas. Therefore, we want to help educators in preschool early childhood shift their perspectives and embrace the potential of this new knowledge (e.g., as called for by Hachey, 2013; Stipek, 2013) so that all children enter school prepared with foundational mathematical knowledge.

2. The research background of the CCSSM

Preschool mathematics knowledge predicts achievement even into high school (National Mathematics Advisory Panel, 2008; NRC, 2009; Stevenson & Newman, 1986). It also predicts later reading achievement

* Corresponding author at: Morgridge College of Education, Marsico Institute for Early Learning and Literacy, Katherine A. Ruffatto Hall 154, 1999 East Evans Avenue, University of Denver, Denver, CO 80208-1700, United States.
E-mail address: Douglas.Clements@du.edu (D.H. Clements).

http://dx.doi.org/10.1016/j.ecresq.2017.03.005
Received 5 July 2016; Received in revised form 20 February 2017; Accepted 19 March 2017
0885-2006/

as well as early reading skills do (Duncan et al., 2007; see also Farran, Aydogan, Kang, & Lipsey, 2005; Lerkkanen, Rasku-Puttonen, Aunola, & Nurmi, 2005). Early number sense predicts later functional literacy, as measured by an instrument linked to future economic and life outcomes (Geary, Hoard, Nugent, & Bailey, 2013). Thus, mathematical thinking appears to be cognitively foundational (Baroody & Purpura, in press; Clements & Sarama, 2009; Purpura & Reid, 2016; Sarama & Clements, 2009). Given the importance of mathematics itself and to academic success across subjects (Sadler & Tai, 2007), all children need and deserve a robust knowledge of mathematics in their earliest years.

However, opportunities to learn early mathematics are more frequent in some communities and families than in others (Baroody & Purpura, in press; Blevins-Knabe & Musun-Miller, 1996; Ginsburg & Russell, 1981; Griffin et al., 1995; Jordan, Huttenlocher, & Levine, 1992; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Sanachter, Rambaud, Fuller, & Eggers-Pierola, 1995; Saxe, Guberman, & Gearhart, 1987). This opportunity gap can negatively affect children who live in poverty and who are members of linguistic and ethnic minority groups (Brooks-Gunn, Duncan, & Britto, 1999; Campbell & Silver, 1999; Denton & West, 2002; Entwisle & Alexander, 1990; Halle et al., 1997; Mullis et al., 2000; National Research Council, 2001; Natriello, McDill, & Pallas, 1990; Rouse, Brooks-Gunn, & McLanahan, 2005; Secada, 1992; Sylva, Melhuish, Sammons, Siraj-Blatchford, & Taggart, 2005; Thomas & Tagg, 2004), starting in the preschool years (Arnold & Doctoroff, 2003; Chernoff et al., 2007; Denton & West, 2002; Ginsburg & Russell, 1981; Griffin et al., 1995; Jordan et al., 1992; Saxe et al., 1987; Sowder, 1992). Fortunately, high-quality learning experiences result in greater school readiness in kindergarten (Magnuson, Meyers, Rathbun, & West, 2004; National Research Council, 2001; National Research Council and Institute of Medicine, 2000) and help all children to use multiple strategies, with similar accuracy, speed, and adaptive reasoning (Clements & Sarama, 2014; Rouse et al., 2005; Siegler, 1993).

A major goal of the NRC research-based report (2009) was identifying and summarizing research-based foundational and achievable goals for preschool and for grades K, 1, and 2. These goals form learning trajectories across these ages. Learning trajectories (Clements & Sarama, 2014; Sarama & Clements, 2009) show how goals relate to and build on each other and provide ways for mathematics teaching to build related understandings that can help all children move forward. This approach emphasizes the individual learning trajectories each child needs to traverse but provides a cohesive view that permits learning experiences to address groups of children.

The NRC report also found that little mathematics was being taught in pre-school (cf. Piasta, Pelatti, & Miller, 2014) and the early grades and that teaching incidentally through play or integrated with other topics, though sometimes useful, was not sufficient. It concluded that sustained focused teaching and learning time for mathematics is essential. The report summarized research about appropriate teaching-learning practices in early childhood, envisioning an engaging and encouraging climate for children's early encounters, particularly because this develops their confidence in their ability to understand and use mathematics. These positive experiences help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence, which contribute to their future success in and out of school (e.g., Clements, Sarama, & DiBiase, 2004). These developmentally appropriate teaching-learning practices are summarized in Table 1, which appeared in various related forms in books about teaching mathematics in preschool to grade 2 jointly published by the National Council of Teachers of Mathematics and the National Association for the Education of Young Children (see bottom of Table 1). Stipek (2013, p. 434) succinctly summarized this approach as "purposeful instruction that supports the development of deep mathematical understandings and that children enjoy".

The NRC foundational and achievable goals were used in developing the CCSSM for grades K to 2. The major professional organizations concerned with the mathematical education of young children–the National Association for the Education of Young Children, the National Council of Teachers of Mathematics, and the National Council of Supervisors of Mathematics—all endorsed the CCSSM. Thus, these standards and their embedded learning trajectories can guide educators about what foundational mathematics they need to help young children learn.

Table 1
Effective and Developmentally Appropriate Teaching-Learning Practices (Adapted from NCTM, 2010a, 2010b, 2010c, 2011).

A. The teacher expects and supports **children's ability to make meaning and mathematize** the real world by
- providing **settings that connect** mathematical language and symbols to quantities and to actions in the world,
- **leading children's attention** across these crucial aspects to help them make connections, and
- **supporting repeated experiences** that give children time and opportunity to build their ideas, develop understanding, and increase fluency

B. The teacher creates a nurturing and helping **Math Talk Community**
- within which to **elicit thinking** from students, and
- to help students **explain and help** each other explain and solve problems.

C. For each big math topic, the teacher leads the class through a **research-based learning path** based on children's thinking. This allows the teacher to differentiate instruction within whole-class, small group, and center-based activities. This path provides the repetitive experiencing that young children need.

D. For later pre-K and Kindergarten, children need to follow up activities with real 3-dimensional objects by working with math drawings and other written 2-dimensional representations that **support practice and meaning-making with written mathematical symbols**. Children of all ages also need to see and count groups of things in books, that is, they need to experience and understand 3-dimensional things as pictures on a 2-dimensional surface. Working with and on 2-dimensional surfaces, as well as with 3-dimensional objects, supports equity in math literacy because too many children have not had experiences with 2-dimensional representations in their out-of-school environment.

3. Identifying criticisms of and misconceptions about the CCSSM

This background of research and actions of professional groups supporting the CCSSM has been ignored by or is unknown to critics of the standards that have published in blogs, interviews, and position papers in recent years. We respond here because these criticisms are impeding opportunities for young children to learn mathematics and simultaneously develop the competence and positive identity that such opportunities support. We aim to better ground future debates and to ameliorate possible negative effects of invalid criticisms.

3.1. Data sources

To select and organize the criticisms and research relevant to them, we consulted three types of sources. The first two were the research literature (e.g., Tran, Reys, Teuscher, Dingman, & Kasmer, 2016) and Internet sources (see Appendix A for the search procedures and list of blogs and other commentary). We identified research by including published peer-reviewed journal articles from 2000 to 2016 as well as frequently-cited seminal studies conducted before that range. We began by developing a key word search list by brainstorming an initial list of terms to enter when searching for articles. These terms were young children, pre-K, preschool, kindergarten, primary grades, mathematics, math AND < the topic > . The following electronic databases were searched: Medline, PubMed, PsychINFO, PsycArticles, ERIC, Google Scholar, and Applied Social Science Index and Abstract. The search strategy, which aimed to find both studies conducted in the United States and internationally, was limited to the English language. The electronic searches were supplemented by checking the reference lists of included articles, existing systematic reviews and meta-analyses, and hand searching online databases of research. The criteria for the search of social media were determined by the authors and included online commentary and blog posts. The first step taken to initiating a search for articles pertaining to the Common Core State Standards (CCSS) and early childhood education, particularly mathematics, was the development of a key word search list. To compile this list, our research team brainstormed an initial list of key words or phrases; using a Boolean "OR" to connect terms within brackets and a Boolean "AND" between sets of bracketed lists (the first three lists were included in each search; appended to one of the others): [Common Core State Standards; CCSS; CCSSM] [preschool; pre-K; pre- kindergarten; kindergarten; early childhood education; ece]; [mathematics; math] [developmentally appropriate; developmental appropriateness]; [curriculum; and curricula]. To decide whether to include a post; we determined the audience and the aim of the article to see if they aligned with the goals of the search. Next; we determined if the post did any of the following: referenced/analyzed a reputable research study; gave specific emphasis to CCSS's "developmental appropriateness," made claims pertaining to whether the CCSS was "researched-based" or not; and/or included "expert opinions." We searched the following databases: Google Scholar; Google; National Institution of Early Education Research; and The Chronicle of Higher Education; as well as numerous reputable and prevalent news sources (for which we particularly sought out a diverse number of opinion pieces) including The Washington Post; Forbes; Education Week; The Stanford Daily; and National Public Radio (NPR). The electronic searches were supplemented by checking the reference lists of the included posts and hand searching online databases.

3.2. Coding

After compiling all documents, Clements/Sarama and Fuson independently created categories of the criticisms and then re-read through everything to see if each main criticism fit into one of those categories; they all agreed that each main criticism did so fit. We then checked the categories, and they were the same. The goal of this process was to reduce the complexity of the criticism so that the main aspects could be addressed. To broaden the process, we then consulted the third type of source: the main writer of the CCSSM's early childhood sections and the presidents of the National Association of the Education of Young Children (NAEYC), National Council of Teachers of Mathematics (NCTM), and National Council of Supervisors of Mathematics (NCSM) who have been involved in hearing such criticisms. We asked them if the four categories we had identified captured the main criticisms they had encountered. All responses were yes.

4. Misconceptions

Before we address the four main criticisms, we note that the main writer of the CCSSM's early childhood sections and the presidents of NAEYC, NCTM, and NCSM also identified the following three misconceptions, each of which we also had documented. We briefly list these misconceptions and provide brief corrective information.

4.1. Misconception #1: standardized testing is misused in early childhood due to the CCSSM

This criticism of CCSSM is incorrect because the standardized tests developed for CCSSM, namely PARCC and Smarter Balanced, do not start until grade 3. Also, the CCSSM themselves do not address standardized testing.

4.2. Misconception #2: standards began with the CCSSM

Most states such as California, have had standards in kindergarten and grades 1 and 2 for years (Scott-Little, Kagan, Reid, & Castillo, 2011; Scott-Little et al., 2012), and these standards were *not* dissimilar in content to the CCSSM standards, although they varied by state.

4.3. Misconception #3: the CCSSM and CCSS are the same

Many blog or article headlines are critical of the CCSS, but when one reads further, one sees that the objections are to the literacy standards and not to the mathematics standards. This generates the impression that there is more criticism of the CCSSM than there actually is.

5. Four categories of criticism

The following are the four broad categories we identified.

1. No one who helped develop the standards had any expertise in the education of very young children.
2. The CCSSM dictates scripted curricula and didactic instruction rigidly applied to all children at the same pace.
3. The standards emphasize academic skills and leave no time for play, exploratory approaches, or social-emotional development.
4. The standards are too early and therefore developmentally inappropriate for children in the early grades (this also relates to criticisms 2 and 3).

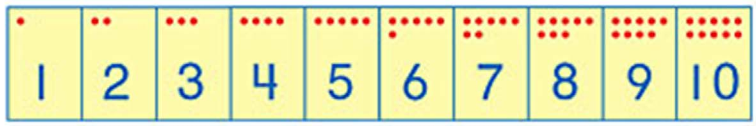
We will now address the four criticisms in turn.

5.1. Is it true that no one who helped develop the standards had any expertise in the education of very young children?

Several authors have made the claim that early childhood educators were not involved in developing the CCSSM (see the Appendix A for the category ECE authors). Two examples are: "Two committees made up of 135 people wrote the standards – and not one of them was a K-3 classroom teacher or early childhood education professional" (DEY, 2014; Ravitch in Strauss, 2014, in Appendix A). These claims are demonstrably false. Members of the Working Group and Feedback Group included public school early and elementary teachers and directors of state PreK-16 and PreK-20 programs (CCSSO/NGA, 2010). Feedback from teachers was encouraged and used in many other ways. For example, K-2 teachers convened by the American Federation of Teachers made substantive contributions to the CCSSM, such as in the connections between numbers and the quantities they name. Experts in early childhood education also suggested that the concept of a tens unit be moved from kindergarten to grade 1. Zimba, one of the lead authors of the CCSSM, summarizes these and other facts that show that this assertion is false (see Appendix A). Further, as described previously, the standards for the early years of the CCSSM were grounded in a NRC (2009) report on early mathematics. The National Research Council advises the nation on scientific issues of national importance. Committees of highly-qualified experts review the available research and make recommendations. The National Association for the Education of Young Children and the National Association of Early Childhood Specialists in State Departments of Education (NAEYC and NAECS-SDE, 2010) issued a joint letter saying that the CCSSM was "fair and age appropriate for Kindergarten through 3rd grade." And finally, we the authors of this paper were involved in helping write the CCSSM, and we have decades of experience teaching researching, developing, and evaluating curricula and working with other teachers to study the teaching and learning of young children preschool to grade 3. In summary, the response to this criticism is clear: Early childhood educators helped develop, refine, review, and endorse the CCSSM.

5.2. Do the CCSSM dictate rigidly paced scripted curricula and didactic instruction?

Some critics assert that the CCSSM requires the use of scripted curricula and didactic instruction, both rigidly applied to all children at the same pace (see the Appendix A under the category curriculum, Number Parade with 5-Groups for 6 to 10



Numbers within 5

A) Put number tiles in order at top; can look at Number Parade.
Put red and blue tiles at bottom (5 red and 5 blue in a baggie; one side is plain and the other side has a white dot).
Point to the number tiles in order and say the number on that tile.

B) Pull down the number tile for the number said by teacher or a student.

C) Show that number of tiles.

D) Have a Math Talk Discussion:

a. Relate the visual quantity 3 to 3 fingers, 3 sounds, and 3 body movements.
b. Practice visual imagery: Close your eyes. Visualize. (See the 3.).
c. Describe different arrangements by color, dot/no dot, spatial relationships (e.g., 3 = 2 + 1)
d. Change your arrangement and discuss why you still have 3.
e. Copy the arrangement of another person.
f. See and describe partners of 3 (decompositions of 3 into 2 numbers) and create new partners that make 3 using attributes listed in c.
g. Graph on a graph mat (2 rows/columns of 10 empty squares) by putting tiles dot side up on the graph mat and say which is more (later say which is less).

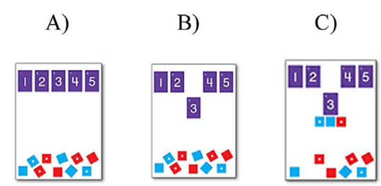


Fig. 1. Counting Mat Activities for Quantity-Word-Symbol and Decomposing Knowledge.

instruction, and tests). But standards do not dictate teaching methods *or* assessments *or* particular policies (Tran et al., 2016; Zimba, 2016). As goals, they do provide guidance—for example, teaching only low-level skills will not meet the CCSSM goals. Experts on early education believe good standards can be positive influences, contradicting fears about inappropriate teaching. Early childhood educator Bredekamp (2004) says: "As long as goals are developed drawing on research and the wisdom of practice, goals can be excellent contributions" (2004, p. 79). See also NAEYC's position (2012).

To those who believe that *any* goal will be inappropriate for some small number of children, we respond *that is precisely why learning trajectories are the developmentally appropriate core*—the main point is to help children meet the goal on the children's pace and way of thinking. The learning trajectories in the CCSSM, drawn from the research-based National Research Council report (2009), help educators know how to move children along in learning.

Two examples of such teaching illustrate the major points in Table 1, including focusing on meaning-making by mathematizing the real world, using a nurturing and helping Math Talk Community, and using research-based learning trajectories. Fig. 1 shows an activity that has been used in hundreds of kindergarten classes at the beginning of the year. Children see the 5-group patterns for numbers 6 through 10 and relate them to the 5-groups on their fingers. They practice counting from 1 to 10 as a student leader points to each number on the Number Parade. Children can just focus on the numerals and dots or raise fingers with the count or jump with each word. Feedback from teachers and observations in classrooms indicate that children enjoy and can do this activity regardless of how familiar they are with counting words, and it is rich enough to engage those who can count much higher. The counting mat activity supports children in developing and using their knowledge of written symbols and their order, counting objects, relating different arrangements of objects for the same number, decomposing a number into its partners (addends), and graphing in two rows or columns. Children engage as they are able and are helped by classmates' displays and responses. Children also explain their own displays and thinking. Many variations are encouraged and discussed. In both activities children build in their own ways coordinated knowledge of the counting words, written number symbols, and real world quantities over the supported repeated experiences (Table 1 third bullet in A).

Fig. 2 shows mathematics drawings made by grade 1 children to represent and solve a more difficult type of "put together" problem—those with one addend unknown. Children earlier in the year had represented and solved the simpler put together problems with unknown totals. At all times children engaged in a Math Talk Community in which children explained their thinking and asked questions of each other. These two classes were in a rural school that had been on state warning for low achievement before they began this approach. Notice the variation across children as they did their own sense-making about the situation and choose their own representations. Supporting children in making mathematics drawings enables the teacher to understand their thinking and help them to debug errors. Five of the six wrong answers in Fig. 2 had correct representations of the problem situation, and so questions by classmates easily lead to corrections. Such drawings and discussion help to advance all children's thinking as they see different ways to represent and solve a problem (e.g., Fuson, Clements, & Sarama, 2015; Hufferd-Ackles, Fuson, & Sherin, 2004; Murata & Fuson, 2006). Most (88%) of these children solved written equations of the form 6 + ? = 9 correctly. Relating such equations to word problem situations and to their own drawings allowed them to make sense of such equations (Fuson & Smith, 2016).

So we see that the CCSSM's rigorous standards do not dictate any particular type of teaching including didactic, rigid teaching. Furthermore, the examples above show how the CCSSM can be met by teaching in the balanced ways summarized in Table 1. Research-based curricula that do result in increased learning and do teach in these ways exist now. Hachey (2013) identifies three such programs: Building Blocks, Number Worlds, and Big Math for Little Kids. Educators can also see the NCTM/NAEYC books about teaching mathematics (Table 1) for preschool, K, 1, and 2 to enact the NRC

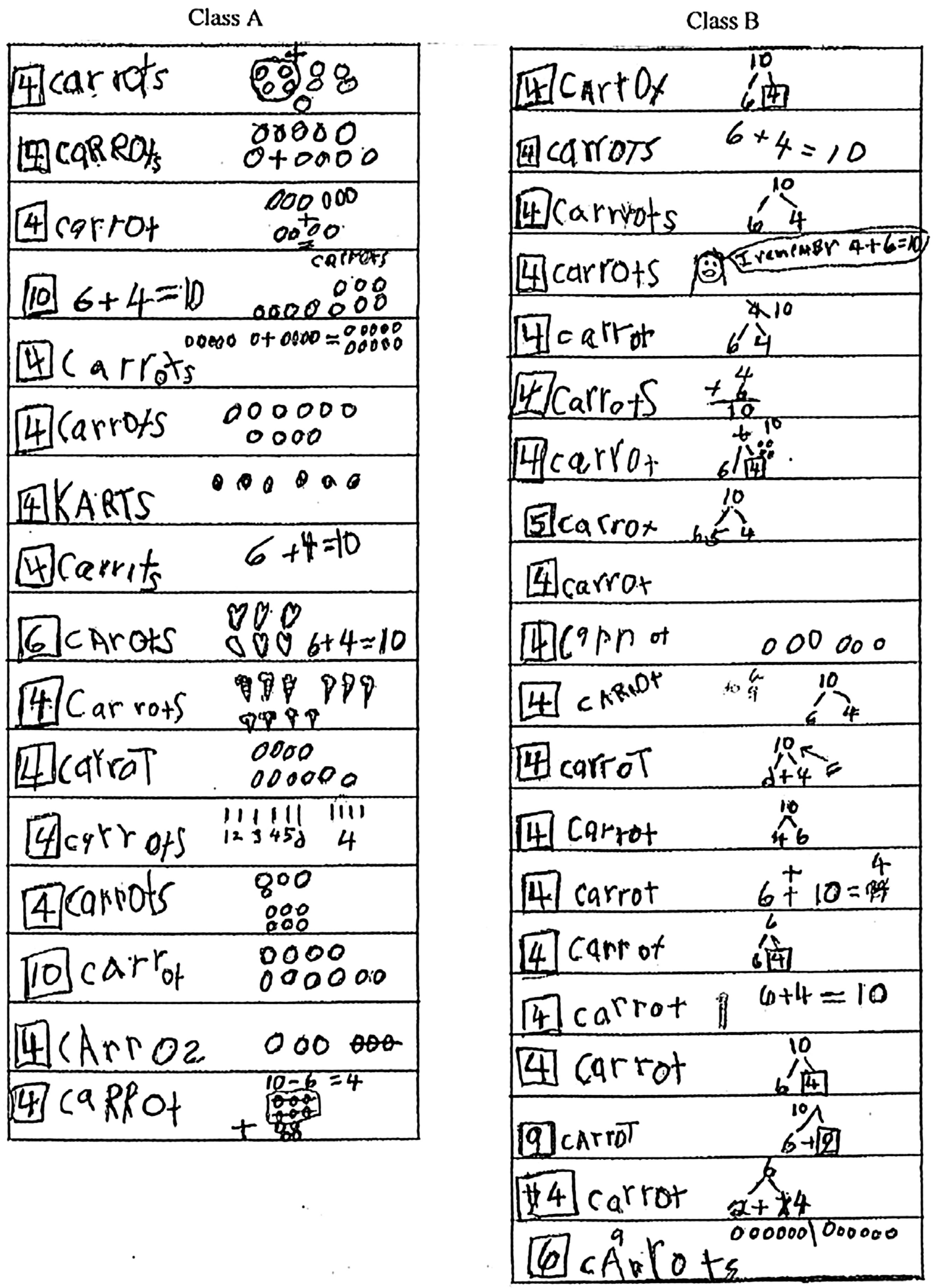


Fig. 2. Grade 1 Math Drawings and Solutions for the Put Together, Addend Unknown Problem "Rosa picked 6 carrots. Her sister picked some too. Together they picked 10 carrots. How many carrots did Rosa's sister pick?".

foundational and achievable goals. Other descriptions of this balanced middle ground that supports all children through their own learning trajectories are available (Clements & Sarama, 2011, 2012, 2014; Fuson, 2009, 2012; Fuson & Murata, 2007; Fuson, Murata et al., 2015; Murata, 2013; Murata & Fuson, 2006).

5.3. Do academic goals eliminate play, exploratory approaches, and social-emotional development?

A third class of criticisms is that academic goals like the CCSSM eliminate play, exploratory approaches, and social-emotional development in mathematics classrooms (see the Appendix A for the play "vs." academics category). Table 1 stemming from the NRC Report (2009) and the two examples we just examined indicate that such assertions are false. Children using a balanced teaching-learning approach are encouraged and enabled to play with the mathematical ideas, explore their own approaches and the approaches of others, and gain valuable social-emotional strengths by engaging in the nurturing and helping Math Talk Community. Table 1 and the NCTM/NAEYC books (2009,2010,2011) can provide support for teaching that has the desired positive character.

We now extend our discussion of this criticism to preschool classrooms because there is considerable research showing that this criticism is also false for preschool children, and it is important for preschool educators to know some of this research so they can understand a balanced approach to teaching in the preschool years. Research shows that high-quality implementations of mathematics curricula in preschool not only increase children's mathematics proficiencies, but also do no harm—and even sometimes positively transfer—to other domains, such as language and self-regulation or executive function (Clements, Sarama, Wolfe, & Spitler, 2013; Farran, Lipsey, & Wilson, 2011; Sarama, Clements, Wolfe, & Spitler, 2012; Sarama, Lange, Clements, & Wolfe, 2012). Further, curricula can successfully combine social-emotional, literacy, language, science and mathematics (Sarama, Brenneman, Clements, Duke, & Hemmeter, 2016)—all the while enhancing, rather than competing with, play-based approaches (Farran et al., 2005).

Mathematics and literacy instruction can also increase the quality of young children's play. An analysis of curricula studied in the Preschool Curriculum Evaluation Research Consortium (PCER) (2008) project showed that children in classrooms with stronger emphasis on literacy or mathematics were more likely to be engaged at a higher-quality socio-dramatic play during free-choice (play) time. Those in classrooms with an emphasis on both literacy and mathematics were more likely to be engaged at a high-quality level than those in classrooms with only one, or no, such emphasis (Aydogan et al., 2005). The new ideas may have energized high-level play activity. The authors state that high-quality instruction in multiple content areas may produce a richer context for learning that appeals to children's diverse interests and fosters interest in a broader range of content. In this richer context, individual children find more opportunities for meaningful engagement in free play. Thus, preventing children from experiencing balanced mathematics teaching may deprive them of the joy and fascination of mathematics (Balfanz, 1999; Clements & Sarama, 2014; Stipek, 2013) but also of higher-quality play resulting from their increased mathematical knowledge.

The NRC report (2009) said that children need intentional and sequenced instruction in mathematics from pre-K through the primary grades. Approaches to early childhood, such as the older view of developmentally appropriate practice (DAP, which especially in misinterpretations, emphasized child-initiated and play activity almost exclusively), have not been shown to increase children's learning (Van Horn, Karlin, Ramey, Aldridge, & Snyder, 2005). Programs based only on an "everyday" or "play" approach to mathematics education frequently show negligible gains (Chien et al., 2010), whereas sequential, intentional mathematical work does (Klein, Starkey, Clements, Sarama, & Iyer, 2008). The characterization of balanced intentional teaching coming from the NRC Report (2009) and summarized in Table 1 applies to preschool classrooms also. Mathematizing children's experiences means to focus children on mathematical aspects of the world. Such experiences enable children to link their lives to important cultural mathematical ideas. Such focused experiences supported by adults in the classroom is especially important for children with limited resources and opportunities to do so outside of school. Adults can encourage children to communicate in their own words, sharing their thinking in balanced mathematics small-group or whole-class activities. Adults can help children connect mathematical language to their own ideas and words (Association of Mathematics Teacher Educators, 2017; National Research Council, 2009) to build meanings for mathematical ideas.

High-quality, explicit, and sequential teaching including mathematics talk should be the core of children's mathematical experiences. However, intentional and sequenced instruction can include activities such as mathematics challenges, puzzles, and games and a focus on mathematizing the play of children (Agodini, Harris, Seftor, Remillard, & Thomas, 2013; Clements & Sarama, 2007, 2013; Griffin, 2004; Griffin et al., 1995; National Research Council, 2009; Senk & Thompson, 2003; Siegler & Booth, 2004; van Oers, 2010). Balanced teaching is interactive, with teachers inviting children to find their own solutions and build on their own ideas, but also guiding them to more sophisticated and successful strategies and sharing solutions among children (e.g., Fuson, Clements et al., 2015). Our experiences with hundreds of teachers using a balanced program in pre-K, Kndg, grade 1, and grade 2 indicate that most teachers are surprised by and enthusiastic about the high levels of learning in their classroom and the quality of thinking that their children express. Some teachers, especially in pre-K and kindergarten, are not willing to try a balanced approach because they believe so strongly that teacher-led activities are intrusive and want to allow only child-initiated free play.

5.4. Are the CCSSM developmentally appropriate?

Perhaps the most common criticism of the CCSSM for young children is that the standards are not developmentally appropriate (Meisels, 2011; RealClearEducation, 2014; Tran et al., 2016; see also Appendix A: DEY, 2014). However, as defined by the developers of the construct, DAP requires both meeting children where they are (knowing children, including their developmental status, unique individualisms, and their social and cultural contexts) and enabling them to reach goals that are both challenging and achievable (NAEYC, 2015). DAP "does not mean making things easier for children. Rather, it means ensuring that goals and experiences are suited to their learning and development and challenging enough to promote their progress and interest. Best practice is based on research and expert knowledge—not on assumptions—of how children learn and develop" (NAEYC, 2015, p. 2). Now let us view several specific claims that the CCSSM are not developmentally appropriate in light of comprehensive reviews of scientific research. We include several examples from Kamii (2015, in Appendix A) because she is the most specific of those identified in the Appendix A and includes the most examples.

5.4.1. Counting

Kamii (2015) in Appendix A criticizes the CCSSM for counting verbally too high at kindergarten. "Counting is social- conventional knowledge, which is teachable, but making kindergartners count to 100 is like making them memorize nonsense syllables. Counting by tens can make even less sense to children who may or may not be able to count five objects correctly" (pp. 8–9). Similarly, high school principal Carol Burris also objects to this standard because "there are also going to be kids that just cannot do it at that point in time" (RealClearEducation, 2014, in Appendix A).

Although *some* kindergartners may not be able to count five objects, it is not true that it is developmentally inappropriate to ask children to learn to count five and more objects correctly at that age—they can do so one or more years *earlier* (see the extensive research summarized in Clements & Sarama, 2014; Fuson, 1988). Teaching this competence has shown to be effective for children as young as 4 years of age (Clements & Sarama, 2014; present a comprehensive review) and can help low-SES 4-year-olds who lacked these and other core competencies not only to learn them, but to spontaneously demonstrate performance characteristics of middle class 6-year-olds on a wide range of tasks (Griffin et al., 1995). For any children who cannot yet count to five, "making" them count to 100 would be inappropriate—but neither the CCSSM, learning trajectories approach, or research-based balanced education would suggest that. One begins for all children with activities like those shown in Fig. 1, building and deepening competence from one to ten before going further.

Further, our number system above ten does not consist of nonsense syllables. Children learn relationships among counting numbers and principles and patterns in the number system. For example, error data indicate that children learn the repetition of the x-ty, x-ty-one, x-ty two, … x-ty nine pattern from 20 through 90 (Baroody, 1987; Fuson, 1992). Children do need repeated experiences to overcome the irregular pattern in the teens and in the first two decade words *twenty* and *thirty* (that would be easier if they were two-ty and three-ty). One way is to have children search for patterns in structured written numerals while

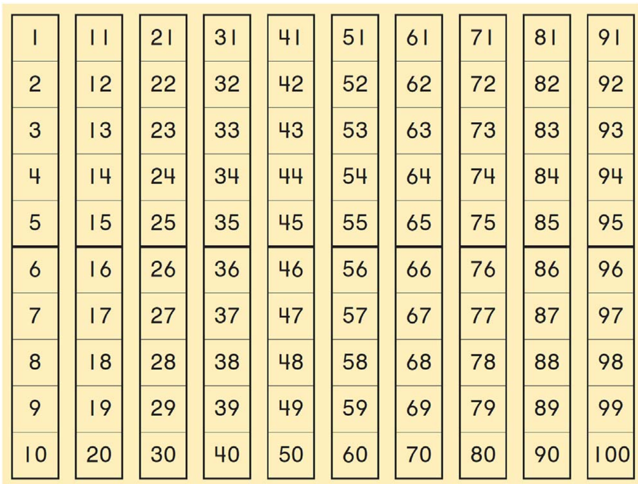


Fig. 3. Seeing the Patterns in the English Words to 100.

they are counting, such as in Fig. 3 (note that groups of ten words are bounded with a rectangle and the vertical arrangement allows children to see the repetition within the twenties, thirties, forties, etc.). Children can discuss these patterns as well as count by ones and later by tens as each number is pointed to by a student leader. Counting by tens helps children solidify the decade word list that creates the x-ty one, x-ty two…patterns. Experiences with and without such visual supports can lead to high levels of children counting to 100 at the end of kindergarten, with the remaining children knowing much of this count sequence (Sarama & Clements, 2009).

Earlier, from the ages of 2–5 years, children learn about the system of number words due to a desire to count larger collections and their own curiosity about the number word system itself (Baroody, 2004; Fuson, 1988, 2004; Griffin, 2004; Steffe, 2004). They play with counting large numbers (Seo & Ginsburg, 2004). Supporting such experiences in preschool can facilitate learning to count to 100 in kindergarten.

Not only researchers, but also kindergarten teachers state that Burris and Kamii are simply incorrect, and they also identify an important equity issue that low expectations can unintentionally exacerbate.

> "Different expectations lead to different achievement levels. It is simply not ok with me that educators like Burris choose what is too easy or too hard for their students. Not only should kindergarteners be able to count to 100 – as noted in the Common Core – but they can. How to get them there is part of the art and science of teaching, and involves making sure that I know what my students know and can move them to where they need to be" (Torney and Trahan, 2014, in Appendix A).

5.4.2. Counting and cardinality

Another counting example criticized by Kamii (2015) in Appendix A is the CCSSM standard: Understand that each successive number name refers to a quantity that is one larger. Kamii claims that Morf's Piagetian research shows that children cannot relate each subsequent number with the +1 operation until third grade. Her description of the study shows that it introduces complexities far beyond that standard. Two glasses held 2 and 15 cubes, respectively. Then 30 cubes were dropped one at a time into the former. The research question was, "When I was dropping one cube after another into this glass, was there a time when the two glasses had *exactly the same number?*" (p. 9, emphasis in original). To respond correctly, children had to say "yes" and state that this was because each additional cube increased the quantity by one. In contrast, in the common preschool activity of adding one to a group of 5 and asking, "How many now?" (Clements & Sarama, 2013; see also Greenes, Ginsburg, & Balfanz, 2004; Klein, Starkey, & Ramirez, 2002), both preschoolers and children with special needs can make that generalization given educational experiences (Baroody, 1999; Clements et al., 2011; Klein & Starkey, 2004). Many of the tasks Kamii uses have similar complexities that involve more knowledge than is in the standard she criticizes.

5.4.3. Wider research-based view of Piaget's theory

These examples reveal a crux of the problem. Both Burris and Kamii cite only the theory of Jean Piaget, and Kamii relies mainly on her own studies from that perspective. But there is a large body of research that has modified Piaget's original research and theories. As one example, Bideaud, Meljac, and Fischer (1992) published a book in France celebrating Piaget's work on number (Piaget & Szeminska, 1941/1952) that included 17 chapters, all of which modify Piaget's theory of number extensively and provide understanding of how much more young children can learn. None of this research is cited in Kamii's paper, nor are the other studies provided throughout this paper and in the NRC report (2009) cited.

Kamii continues to define children's learning and understanding of number and counting only from Piaget's original position as requiring a synthesis of hierarchical inclusion and order. From this perspective counting is ineffectual, with "no connection between the acquired ability to count and the actual operations of which the child is capable" (see also Piaget, Inhelder, & Szeminska, 1960; Piaget & Szeminska, 1941/1952, p. 61). These early examples featured children who could count, but still failed the number conservation task. For example, although at a certain age a child can make a set equal in number to an interviewer's set using matching, when the interviewer spreads her or his objects out, the child may claim that the interviewer now has more. Directly to the point, asking children to count the two sets, according to Piaget, did not help them determine the correct answer. From this view, children do not acquire a notion of quantity and then conserve it; they discover true quantification only when they become capable of conservation. That is, teaching counting was a waste of time before the stage of concrete operations (Piaget & Szeminska, 1941/1952, p. 184).

Recent research has substantially changed this position (for discussions, see Clements, 1984a, 1984b; Wright, Stanger, Stafford, & Martland, 2006). As an example, many preschoolers, when told the cardinal value of each of two sets, can determine whether the items in the sets can be matched (i.e., can be put in one-to-one correspondence), providing evidence that they understand the relation between the number words and quantity and can use number words to reason numerically about one-to-one correspondences that are not perceptually available (Becker, 1989; Sophian, 1988). This provides evidence that even preschoolers have at least initial integration of the cardinality of the collection as a whole and the individual items in the collection. If asked to count, many kindergartners will count and use that information to correctly judge equivalence, and older kindergartners do so spontaneously (see extensive research summaries in Fuson, 1988; Fuson et al., 1983). Many 4-year-olds will similarly use their counting skills effectively if provided a visual display of numbers (numerals and dot patterns), and more do so when conflicting perceptual cues are not present (Michie, 1984). Counting increases when children are given feedback as to the correctness of their previous judgments, showing them that relying on counting was valid, but using length or density was not. Along with evidence on object counting and addition and subtraction (e.g., Frontera, 1994), this indicates that counting can be a meaningful quantifier for children before they reach the Piagetian levels of operational thought about number conservation. In summary, research and practice indicate that the CCSSM related to counting are appropriate as defined by DAP.

5.4.4. Other mathematical topics

Kamii (2015) in Appendix A also criticizes standards on measurement, place value, problem solving, and multidigit computation based on the same combination of unrevised Piagetian theory and tasks more complex than the standards. Considering the large corpi of research that revise Piaget's theory extensively, restricting one's view to the original Piagetian position cannot be justified and is misleading. Kamii's criticisms about these aspects of number and her suggestion that you cannot teach this competence but can encourage it indirectly only—-cleaning up spilled milk and playing Pick-Up Sticks are her examples—ignore considerable research demonstrating the effectiveness of teaching about various aspects of number (National Research Council, 2009).

5.4.5. Summary

The criticisms about developmental appropriateness of the CCSSM ignore comprehensive research reviews and reports from decades of research (see full research summaries in Clements & Sarama, 2014; Fuson, 1988, 1992, 2004; Ginsburg, 1977; National Research Council, 2009; Sarama & Clements, 2009). Moreover, the CCSSM were grounded in a large body of different research reports, such as that summarized in the National Research Council report (2009, see also pp. 91–93 of the CCSSM, for just a "Sample of Works Consulted").

Statements by Kamii and others made in the sources listed in the Appendix A focusing on developmentally appropriate practice mislead teachers and parents because these statements underestimate what children can learn in supportive stimulating environments that enable children to think and to build and use important cultural knowledge. This is potentially damaging to children, especially those with fewer previous opportunities to engage with mathematics, because it denies them opportunities to learn and understand age-appropriate knowledge that their age-mates with such opportunities do learn.

Some might still argue that some of the CCSSM standards are too challenging for some subset of children. But this will be true of any set of standards that pose a worthwhile challenge to our children, and our children deserve that challenge. Based on learning trajectories, teachers should always be working on the challenging-but-achievable levels for their class and for the individuals in it (Hiebert & Grouws, 2007). We cannot allow children starting at lower levels to stay behind others. That would relegate them to a trajectory of failure. Instead, we should work together to help them build up their mathematical foundations. Given this support, research indicates that they can learn (Clements et al., 2011; Frye et al., 2013; National Research Council, 2009)}. It is especially feasible in kindergarten and the primary grades, and even more so in pre-kindergarten, to catch up children who enter those environments with less knowledge than their peers (Clements & Sarama, 2014; Clements et al., 2011; Frye et al., 2013). Then it is important that teachers continue to have grade-level expectations of all children so that children do not fall behind again because of lowered expectations and low-level teaching efforts for children from backgrounds of poverty or from homes where English is not spoken.

So, the concern of developmental inappropriateness results from not knowing the extensive research on what young children can learn and about revisions of the concept of DAP (NAEYC, 2015). This research indicates that what is actually inappropriate is placing age- and stage limitations on what young children are capable of without knowing the research about what they can learn. What is developmentally inappropriate is a preschool in which there is little mathematics (Ginsburg, Klein, & Starkey, 1998; Graham, Nash, & Paul, 1997; Tudge & Doucet, 2004), when successful research-based approaches are available that help children learn so much more (Clements & Sarama, 2011). Also developmentally inappropriate are the many present-day kindergartens and curricula that "teach" most children what they already know (e.g., Carpenter & Moser, 1984; Engel, Claessens, & Finch, 2013; Van den Heuvel-Panhuizen, 1996) or early childhood classrooms that fail to help children who enter with less knowledge to catch up with their peers (Ginsburg et al., 1998).

6. Final words

Understanding balanced teaching and the large amount of research about it can help everyone move from the inaccurate and distorting criticisms discussed above and move on to productive discourse and learning about balanced teaching, especially for children who enter our pre-K or K classrooms behind their peers. Teachers' preparation in all three components of learning trajectories—knowledge of mathematics and mathematics for teaching, of children's mathematical thinking and learning, and of instructional activities and strategies must be increased in quality and amount (Association of Mathematics Teacher Educators, 2017; IOM (Institute of Medicine) and NRC (National Research Council), 2012; National Research Council, 2009). This is why we have emphasized descriptions and examples of balanced teaching and provided references to some valuable resources (the NCTN/NAEYC books). The high middle ground of rich engaging and developmentally appropriate mathematical experiences based on research and on children's learning trajectories is accessible to administrators, teachers, parents, and children if responsible adults move beyond old assumptions and examine new research-based perspectives and programs.

Disclosure

The authors of this paper were involved in helping to write the CCSSM.

Acknowledgements

This research was supported in part by the Institute of Education Sciences, U.S. Department of Education through Grants R305K05157 and R305A110188, the National Science Foundation through Grant DRL-1313695, the Gates Foundation, and the Heising-Simons Foundation. The opinions expressed are those of the authors and do not represent views of these agencies.

Appendix A

Author	Date	Title	Source	Type	Standards	Categories	URL
Block, Austin	2015	*A Clarification of Common Core Misconceptions*	The Stanford Daily	Blog	General	ECE authors Curriculum, instruction, and tests Developmental appropriateness	http://www.stanforddaily.com/2015/03/01/a-clarification-of-common-core-misconceptions/
Brown, Kirsty Clarke	2015	*The Common Core State Standards in Early Childhood*	National Institute of Early Education Research (NIEER)	Blog	ECE math and literacy	ECE authors Developmental appropriateness	https://preschoolmatters.org/2015/04/23/clarity-on-the-common-core-

		Education: Summary				Play "vs" academics Curriculum, instruction, and tests	state-standards-in-early-childhood-education/
DEY	2014	*6 reasons to reject Common Core K-3 standards – and 6 rules to guide policy*	Defending the Early Years	Blog	ECE math and literacy	ECE authors Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	http://www.deyproject.org/dey-archives/september-19th-20168628820
Edelman, Susan	2013	*Playtime's Over, Kindergarteners*	The New York Post	Blog	ECE math and literacy	Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	http://nypost.com/2013/01/27/playtimes-over-kindergartners/
Henderson, Michael B., Peterson, Paul E., West, Martin R.	2014	*No Common Opinion on the Common Core*	Education Next	Research survey	General	(Poll on public opinion, 2–1 in favor of CCSS)	http://educationnext.org/2014-ednext-poll-no-common-opinion-on-the-common-core/
Hess, Frederick M.	2014	*How the Common Core Went Wrong*	National Affairs	Blog/informational	Math and literacy	Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests (Extensive discussion, usually including both sides of issues)	http://www.nationalaffairs.com/publications/detail/how-the-common-core-went-wrong
Jenkins, Rob	2013	*Common Ground on the Common Core*	The Chronicle of Higher Education	Blog	General	ECE authors Curriculum, instruction, and tests	http://chronicle.com/blogs/onhiring/common-ground-on-the-common-core/39937
Joseph, David	2016	*Common Core Standards Losing Clout as New York Students 'Opt-Out'*	The Inquisitr	Blog	ECE math and literacy	Curriculum, instruction, and tests	http://www.inquisitr.com/2981573/common-core-standards-losing-clout-as-new-york-students-opt-out/#utm_source=feedburner&utm_medium=feed&utm_campaign=Feed%3A+TheInquisitrNews+%28The+Inquisitr+%C2%BB+News%29
Kamii	2015	*Selected standards from the Common Core State Standards for Mathematics, grades K-3: My reasons for not supporting them*	Defending the Early Years	Research review/white paper	ECE math	ECE authors Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	Kamii, C. (2015).: Defending the Early Years. http://www.deyproject.org/dey-archives/constance-kamiis-critical-look-at-the-k-3-common-core-state-standards-for-math
Loveless, Tom	2016	*How Well Are American Students Learning? With sections on reading and math in the Common Core Era, tracking and Advanced*	The Brookings Institution	Research review/white paper	General	ECE authors Developmental appropriateness Curriculum, instruction, and tests	http://www.brookings.edu/research/reports2/2016/03/24-brown-center-report-loveless

		Placement (AP), and Principals as Instructional Leaders					
Meisels, S. J.	2011	*Common Core standards pose dilemmas for early childhood*	The Washington Post: The Answer Sheet Blog	Blog	ECE math and literacy	Developmental appropriateness	http://www.washingtonpost.com/blogs/answer-sheet/post/common-core-standards-pose-dilemmas-for-early-childhood/2011/11/28/gIQAPs1X6N_blog.html
NAEYC	2012	*The Common Core State Standards: Caution and Opportunity for Early Childhood Education*	National Association for the Education of Young Children (NAEYC)	Research review/white paper	ECE math and literacy	Developmental appropriateness Curriculum, instruction, and tests	http://www.naeyc.org/files/naeyc/11_CommonCore1_2A_rv2.pdf
NIEER	2015	*Top concerns about Common Core State Standards in early childhood education*	National Institute of Early Education Research (NIEER)	Blog	ECE math and literacy	(Lists concerns about the Common Core in ECE and introduces multiple blogs from experts commenting on this issue, including all categories.)	https://preschoolmatters.org/2015/03/26/top-concerns-about-common-core-state-standards-in-early-childhood-education/
Goonoo, Stephen	2016	*Common Core is Changing how Schools Teach ELA and Math*	eSchoolNews	Blog	General	Curriculum, instruction, and tests	http://www.eschoolnews.com/2016/03/24/common-core-is-changing-how-schools-teach-ela-and-math/
NPR	2014	*Debate: Should Schools Embrace The Common Core?*	NPR	Blog	General	Developmental appropriateness	http://www.npr.org/2014/09/19/347145921/debate-should-schools-embrace-the-common-core%20or%20https://www.washingtonpost.com/news/answer-sheet/wp/2013/10/31/a-ridiculous-common-core-test-for-first-graders/
RealClearEducation.	2014	*Common Core 'goes way, way too far': 5 questions with N.Y. principal Carol Burris*	RealClearEducation	Blog	ECE literacy and math	Developmental appropriateness Curriculum, instruction, and tests	http://www.realcleareducation.com/articles/2014/09/24/common_core_education_principal_carol_burris_1105.html
Schwartz, Katrina	2015	*Does Common Core Ask Too Much of Kindergarten Readers?*	KQED	Blog	ECE literacy	ECE authors Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	http://ww2.kqed.org/mindshift/2015/04/27/does-common-core-ask-too-much-of-kindergarten-readers/
Strauss, Valerie	2013	*A tough critique of Common Core on early childhood education*	Washington Post	Blog	ECE math and literacy	ECE authors Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	https://www.washingtonpost.com/news/answer-sheet/wp/2013/01/29/a-tough-critique-of-common-core-on-early-childhood-education/

						(Also states that CCSS are not based on research and opinions are censored.)	
Strauss, Valerie	2014	*Everything you need to know about Common Core—Ravitch*	Washington Post	Blog	ECE math and literacy	ECE authors	http://www.washingtonpost.com/blogs/answer-sheet/wp/2014/01/18/everything-you-need-to-know-about-common-core-ravitch/
Torney, R., & Trahan, L.	2014	*Common Core classroom perspectives: Teachers respond to N.Y. principal Carol Burris*	RealClearEducation	Blog	ECE math	Developmental appropriateness	http://www.realcleareducation.com/articles/2014/10/09/common_core_classroom_perspectives_teachers_carol_burris_1114.html
Turner, Cory	2014	*A Tale of Two Polls*	NPR	Blog	General	ECE authors (Also discusses polls that differ on public support for CCSS)	http://www.npr.org/sections/ed/2014/08/20/341668003/a-tale-of-two-polls
Walton, Alice G.	2014	*The science of the common core: experts weigh-in on its developmental appropriateness*	Forbes	Blog	General	Developmental appropriateness Curriculum, instruction, and tests	http://www.forbes.com/sites/alicegwalton/2014/10/23/the-science-of-the-common-core-experts-weigh-in-on-its-developmental-appropriateness/#412595e177cc
Zimba, Jason	n.d.	*Developmental Appropriateness*	Student Achievement Partners	Research review	ECE Math	Developmental appropriateness	http://achievethecore.org/content/upload/Developmental%20Appropriateness%20Math.pdf
Zubrzycki, Jaclyn	2011	*Common Core Poses Challenges for Preschools*	Education Week	Blog	ECE math and literacy	Developmental appropriateness Play "vs" academics Curriculum, instruction, and tests	http://www.edweek.org/ew/articles/2011/12/07/13prek_ep.h31.html

References

Agodini, R., Harris, B., Seftor, N., Remillard, J., & Thomas, M. (2013). *After two years, three elementary math curricula outperform a fourth*. Washington, DC: National Center for Education Evaluation and Regional Assistance.

Arnold, D. H., & Doctoroff, G. L. (2003). The early education of socioeconomically disadvantaged children. *Annual Review of Psychology*, *54*, 517–545.

Association of Mathematics Teacher Educators (2017). *AMTE standards for mathematics teacher preparation*. Raleigh, NC: AMTE.

Aydogan, C., Plummer, C., Kang, S. J., Bilbrey, C., Farran, D. C., & Lipsey, M. W. (2005, April). An investigation of prekindergarten curricula: Influences on classroom characteristics and child engagement. *Paper presented at the NAEYC*.

Balfanz, R. (1999). Why do we teach young children so little mathematics? Some historical considerations. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 3–10). Reston, VA: National Council of Teachers of Mathematics.

Baroody, A. J., Purpura, D. J. (in press) Number and operations. In J. Cai (Ed.), *Handbook for research in mathematics education*. Reston, VA: National Council of Teachers of Mathematics (NCTM).

Baroody, A. J. (1987). *Children's mathematical thinking*. New York, NY: Teachers College.

Baroody, A. J. (1999). The development of basic counting, number, and arithmetic knowledge among children classified as mentally handicapped. In L. M. Glidden (Vol. Ed.), *International review of research in mental retardation*. *vol. 22*, (pp. 51–103). New York, NY: Academic Press.

Baroody, A. B. (2004). The developmental bases for early childhood number and operations standards. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 173–219). Mahwah, NJ: Erlbaum.

Becker, J. (1989). Preschoolers' use of number words to denote one-to-one correspondence. *Child Development*, *60*, 1147–1157.

Bideaud, J., Meljac, C., & Fischer, J. P. (1992). *Pathways to number: Children's developing numerical abilities*. Hillsdale, NJ: Lawrence Erlbaum Associates Constance Greenbaum translated from the Fench Edition the Introduction and Chapters 2, 9, 10, 11, 12, 17, and 18, Trans., (First published by Presses Universitaires de Lille).

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting*, *5*, 35–45. http://dx.doi.org/10.1002/(SICI)1099-0917(199603)5:1<35:AID-EDP113>3.0.CO;2-0.

Bredekamp, S. (2004). Standards for preschool and kindergarten mathematics education. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 77–82). Mahwah, NJ: Erlbaum.

Brooks-Gunn, J., Duncan, G. J., & Britto, P. R. (1999). Are socieconomic gradients for children similar to those for adults? In D. P. Keating, & C. Hertzman (Eds.), *Developmental health and the wealth of nations* (pp. 94–124). New York, NY: Guilford.

CCSSO/NGA (2010). *Common core state standards initiative K-12 standards development teams*. from https://www.nga.org/files/live/sites/NGA/files/pdf/2010COMMONCOREK12TEAM.PDF.

Campbell, P. F., & Silver, E. A. (1999). *Teaching and learning mathematics in poor communities*. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, *15*, 179–202.

Chernoff, J. J., Flanagan, K. D., McPhee, C., & Park, J. (2007). *Preschool: First findings from* *the third follow-up of the early childhood longitudinal study, birth cohort (ECLS-B) (NCES 2008–025)*. Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education.

Chien, N. C., Howes, C., Burchinal, M. R., Pianta, R. C., Ritchie, S., Bryant, D. M., ... Barbarin, O. A. (2010). Children's classroom engagement and school readiness gains in prekindergarten. *Child Development*, *81*(5), 1534–1549. http://dx.doi.org/10.1111/j.1467-8624.2010.01490.x.

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the *Building Blocks* project. *Journal for Research in Mathematics Education*, *38*(2), 136–163.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York, NY: Routledge.

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science*, *333*(6045), 968–970. http://dx.doi.org/10.1126/science.1204537.

Clements, D. H., & Sarama, J. (2012). Learning and teaching early and elementary mathematics. In J. S. Carlson, & J. R. Levine (Eds.), *Instructional strategies for improving student learning: Focus on early mathematics and reading* (pp. 107–162). Charlotte, NC: Information Age Publishing vol. 3 of Psychological perspectives on contemporary educational issues.

Clements, D. H., & Sarama, J. (2013). *Building blocks*, *vols. 1 and 2*. Columbus, OH: McGraw-Hill Education.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach*(2nd ed.). New York, NY: Routledge.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum.

Clements, D. H., Sarama, J., Spitler, M. E., Lange, A. A., & Wolfe, C. B. (2011). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education*, *42*(2), 127–166.

Clements, D. H., Sarama, J., Wolfe, C. B., & Spitler, M. E. (2013). Longitudinal evaluation of a scale-up model for teaching mathematics with trajectories and technologies: Persistence of effects in the third year. *American Educational Research Journal*, *50*(4), 812–850. http://dx.doi.org/10.3102/0002831212469270.

Clements, D. H. (1984a). Foundations of number and logic: Seriation, classification, and number conservation from a Piagetian perspective. *Psychological Documents (Ms. No. 2607)*, *14*(4).

Clements, D. H. (1984b). Training effects on the development and generalization of Piagetian logical operations and knowledge of number. *Journal of Educational Psychology*, *76*, 766–776.

Denton, K., & West, J. (2002). *Children's reading and mathematics achievement in kindergarten and first grade*. U.S. Department of Education, National Center for Education Statistics. website: http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2002125.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. http://dx.doi.org/10.1037/0012-1649.43.6.1428.

Engel, M., Claessens, A., & Finch, M. A. (2013). Teaching students what they already know? The (mis)alignment between mathematics instructional content and student knowledge in kindergarten. *Educational Evaluation and Policy Analysis*, *35*(2), 157–178. http://dx.doi.org/10.3102/0162373712461850.

Entwisle, D. R., & Alexander, K. L. (1990). Beginning school math competence: Minority and majority comparisons. *Child Development*, *61*, 454–471.

Farran, D. C., Aydogan, C., Kang, S. J., & Lipsey, M. (2005). Preschool classroom environments and the quantity and quality of children's literacy and language behaviors. In D. Dickinson, & S. Neuman (Eds.), *Handbook of early literacy research* (pp. 257–268). New York, NY: Guilford.

Farran, D. C., Lipsey, M. W., & Wilson, S. J. (2011, November). Curriculum and pedagogy: Effective math instruction and curricula. *Paper presented at the Early Childhood Math Conference*.

Frontera, M. (1994). On the initial learning of mathematics: Does schooling really help? In J. E. H. Van Luit (Ed.), *Research on learning and instruction of mathematics in kindergarten and primary school* (pp. 42–59). Doetinchem, Netherlands: Graviant.

Frye, D., Baroody, A. J., Burchinal, M. R., Carver, S., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children: A practice guide*. Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education.

Fuson, K. C., & Murata, A. (2007). Integrating NRC principles and the NCTM process standards to form a class learning path model that individualizes within whole-class activities. *NCSM Journal of Mathematics Education Leadership*, *10*(1), 72–91.

Fuson, K. C., & Smith, S. (2016, April). Children living in poverty can solve CCSS OA word problems. *Paper presented at the Annual Conference of the National Council of Teachers of Mathematics*.

Fuson, K. C., Secada, W. G., & Hall, J. W. (1983). Matching, counting, and conservation of number equivalence. *Child Development*, *54*, 91–97.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York, NY: Springer-Verlag.

Fuson, K. C. (1992). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. Putman, & R. A. Hattrup (Eds.), *Handbook of research on mathematics teaching and learning* (pp. 53–187). Mahwah, NJ: Erlbaum.

Fuson, K. C. (2004). Pre-K to grade 2 goals and standards: Achieving 21 st century mastery for all. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 105–148). Mahwah, NJ: Erlbaum.

Fuson, K. C. (2009). Avoiding misinterpretations of Piaget and Vygotsky: Mathematical teaching without learning, learning without teaching, or helpful learning-path teaching? *Cognitive Development*, *24*, 343–361. http://dx.doi.org/10.1016/j.cogdev.2009.09.009.

Fuson, K. C. (2012). The common core mathematics standards as supports for learning and teaching early and elementary school. In J. S. Carlson, & J. R. Levine (Eds.), *Instructional strategies for improving student learning: Focus on early mathematics and reading* (pp. 177–186). Charlotte, NC: Information Age Publishing Vol. 3 of Psychological perspectives on contemporary educational issues.

Fuson, K. C., Clements, D. H., & Sarama, J. (2015). Making early math education work for all children. *Phi Delta Kappan*, 63–68.

Fuson, K. C., Murata, A., & Abrahamson, D. (2015). Using learning path research to balance mathematics education: Teaching/learning for understanding and fluency. In R. C. Kadosh, & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1036–1054). Oxford, England: Oxford University Press.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *Public Library of Science*, *8*(1), e54651.

Ginsburg, H. P., & Russell, R. L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development*, *46*(6) Serial No. 193.

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In W. Damon, I. E. Sigel, & K. A. Renninger (Eds.), *Handbook of child psychology. Volume 4: Child psychology in practice* (pp. 401–476). New York, NY: John Wiley & Sons.

Ginsburg, H. P. (1977). *Children's arithmetic*. Austin, TX: Pro-ed.

Graham, T. A., Nash, C., & Paul, K. (1997). Young children's exposure to mathematics: The child care context. *Early Childhood Education Journal*, *25*(1), 31–38.

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly*, *19*, 159–166.

Griffin, S., Case, R., & Capodilupo, A. (1995). Teaching for understanding: The importance of the Central Conceptual Structures in the elementary mathematics curriculum. In A. McKeough, J. Lupart, & A. Marini (Eds.), *Teaching for transfer: Fostering generalization in learning* (pp. 121–151). Mahwah, NJ: Erlbaum.

Griffin, S. (2004). Number Worlds: A research-based mathematics program for young children. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 325–342). Mahwah, NJ: Erlbaum.

Hachey, A. C. (2013). The early childhood mathematics education revolution. *Early Education and Development*, *24*(4), 419–430. http://dx.doi.org/10.1080/10409289.2012.756223.

Halle, T. G., Kurtz-Costes, B., & Mahoney, J. L. (1997). Family influences on school achievement in low-income: African American children. *Journal of Educational Psychology*, *89*, 527–537.

Hiebert, J. C., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Vol. Ed.), *Second handbook of research on mathematics teaching and learning. vol. 1*, (pp. 371–404). New York, NY: Information Age Publishing.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk community. *Journal for Research in Mathematics Education*, *35*(2), 81–116.

IOM (Institute of Medicine), & NRC (National Research Council) (2012). *The early childhood care and education workforce: Challenges and opportunities: A workshop report* Washington, DC: The National Academies Press.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, *28*, 644–653.

Klein, A., & Starkey, P. (2004). Fostering preschool children's mathematical development: Findings from the Berkeley math readiness project. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 343–360). Mahwah, NJ: Erlbaum.

Klein, A., Starkey, P., & Ramirez, A. B. (2002). *Pre-K mathematics curriculum*. Glenview, IL: Scott Foresman.

Klein, A., Starkey, P., Clements, D. H., Sarama, J., & Iyer, R. (2008). Effects of a pre-kindergarten mathematics intervention: A randomized experiment. *Journal of Research on Educational Effectiveness*, *1*, 155–178.

Lerkkanen, M.-K., Rasku-Puttonen, H., Aunola, K., & Nurmi, J.-E. (2005). Mathematical performance predicts progress in reading comprehension among 7-year-olds. *European Journal of Psychology of Education*, *20*(2), 121–137.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*(5), 1309–1319. http://dx.doi.org/10.1037/a0019671.

Magnuson, K. A., Meyers, M. K., Rathbun, A., & West, J. (2004). Inequality in preschool education and school readiness. *American Educational Research Journal*, *41*, 115–157.

Meisels, S. J. (2011, November 29). Common Core standards pose dilemmas for early childhood. The Washington Post: The Answer Sheet Blog. Retrieved from http://www.washingtonpost.com/blogs/answer-sheet/post/common-core-standards-pose-dilemmas-for-early-childhood/2011/11/28/gIQAPs1X6N_blog.html =.

Michie, S. (1984). Number understanding in preschool children. *British Journal of Educational Psychology*, *54*, 245–253.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., ... Smith, T. A. (2000). *TIMSS 1999 international mathematics report* Boston: The International Study Center, Boston College, Lynch School of Education.

Murata, A., & Fuson, K. C. (2006). Teaching as assisting individual constructive paths within an interdependent class learning zone: Japanese first graders learning to add using 10. *Journal for Research in Mathematics Education*, *37*, 421–456.

Murata, A. (2013). Diversity and high academic expectations without tracking: Inclusively responsive instruction. *Journal of the Learning Sciences*, *22*(2), 312–335.

http://dx.doi.org/10.1080/10508406.2012.682188.
NAEYC, & NAECS-SDE (2010). *Common core standards initiative related to kindergarten through third grade*. Retrieved from National Association for the Education of Young Children website: http://www.naeyc.org/files/naeyc/file/policy/NAEYC-NAECS-SDE-Core-Standards-Statement.pdf.
NAEYC (2012). *The common core state standards: Caution and opportunity for early childhood education*. Washington, DC: National Association for the Education of Young Children (NAEYC).
NAEYC (2015). *Developmentally appropriate practice and the common core state standards: framing the issues [Research brief]*. Retrieved from https://www.naeyc.org/files/naeyc/15_Developmentally Appropriate Practice and the Common Core State Standards.pdf.
NCTM (2010a). *Focus in grade 1: Teaching with the curriculum focal points reston*. VA: National Council of Teachers of Mathematics.
NCTM (2010b). *Focus in kindergarten: Teaching with the curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics.
NCTM (2010c). *Focus in prekindergarten: Teaching with the curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics.
NCTM (2011). *Focus in grade 2: Teaching with the curriculum focal points*. Reston, VA: National Council of Teachers of Mathematics.
National Research Council (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: National Academy Press.
National Mathematics Advisory Panel (2008). *Foundations for success: the final report of the National Mathematics Advisory Panel* Washington, DC: U.S. Department of Education, Office of Planning, Evaluation and Policy Development.
National Research Council and Institute of Medicine (2000). *From neurons to neighborhoods: The science of early childhood development*. Washington, DC: National Academy Press.
National Research Council (2001). *Eager to learn: Educating our preschoolers*. Washington, DC: National Academy Press.
Natriello, G., McDill, E. L., & Pallas, A. M. (1990). *Schooling disadvantaged children: Racing against catastrophe*. New York, NY: Teachers College Press.
Piaget, J., & Szeminska, A. (1941/1952). *The child's conception of number*. London: Routledge and Kegan Paul.
Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry*. London: Routledge and Kegan Paul.
Piasta, S. B., Pelatti, C. Y., & Miller, H. L. (2014). Mathematics and science learning opportunities in preschool classrooms. *Early Education and Development*, *25*(4), 445–468.
Preschool Curriculum Evaluation Research Consortium (2008). *Effects of preschool curriculum programs on school readiness (NCER 2008–2009)*. Retrieved from Government Printing Office website: http://ncer.ed.gov/.
Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly*, *36*, 259–268. http://dx.doi.org/10.1016/j.ecresq.2015.12.020.
RealClearEducation (2014, September 24). Common Core 'goes way, way too far': 5 questions with N.Y. principal Carol Burris. *RealClearEducation*. Retrieved from http://www.realcleareducation.com/articles/2014/09/24/common_core_education_principal_carol_burris_1105.html
Rouse, C., Brooks-Gunn, J., & McLanahan, S. (2005). Introducing the issue. *The Future of Children*, *15*, 5–14.
Sadler, P. M., & Tai, R. H. (2007). The two high-school pillars supporting college science. *Science*, *317*, 457–458.
Sanachter, J. E., Rambaud, M. F., Fuller, B., & Eggers-Pierola, C. (1995). What is appropriate practice at home and in child care?: Low-income mothers' views on preparing their children for school. *Early Childhood Research Quarterly*, *10*, 451–473.
Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.
Sarama, J., Brenneman, K., Clements, D. H., Duke, N. K., & Hemmeter, M. L. (2016). *Connect4Learning: The pre-K curriculum*. Lewisville, NC: Connect4Learning.
Sarama, J., Clements, D. H., Wolfe, C. B., & Spitler, M. E. (2012). Longitudinal evaluation of a scale-up model for teaching mathematics with trajectories and technologies. *Journal of Research on Educational Effectiveness*, *5*(2), 105–135. http://dx.doi.org/10.1080/19345747.2011.627980.
Sarama, J., Lange, A., Clements, D. H., & Wolfe, C. B. (2012). The impacts of an early mathematics curriculum on emerging literacy and language. *Early Childhood Research Quarterly*, *27*, 489–502. http://dx.doi.org/10.1016/j.ecresq.2011.12.002.
Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processesn early number development. *Monographs of the Society for Research in Child Development*, *52*(2) Serial #216.
Scott-Little, C., Kagan, S. L., Reid, J. L., & Castillo, E. (2011). *Early mathematics standards in the United States: Understanding their content*. Los Altos, CA: Heising-Simons Foundation.
Scott-Little, C., Kagan, S. L., Reid, J. L., & Castillo, E. (2012). *Early mathematics standards in the United States: The quest for alignment*. Los Altos, CA: Heising-Simons Foundation.
Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York, NY: Macmillan.
Senk, S. L., & Thompson, D. R. (2003). *Standards-based school mathematics curricula. What are they? What do students learn?* Mahwah, NJ: Erlbaum.
Seo, K.-H., & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91–104). Mahwah, NJ: Erlbaum.
Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*, 428–444. http://dx.doi.org/10.1111/j.1467-8624.2004.00684.x.
Siegler, R. S. (1993). Adaptive and non-adaptive characteristics of low income children's strategy use. In L. A. Penner, G. M. Batsche, H. M. Knoff, & D. L. Nelson (Eds.), *Contributions of psychology to science and mathematics education* (pp. 341–366). Washington, DC: American Psychological Association.
Sophian, C. (1988). Early developments in children's understanding of number: Inferences about numerosity and one-to-one correspondence. *Child Development*, *59*, 1397–1414.
Sowder, J. T. (1992). Making sense of numbers in school mathematics. In G. Leinhardt, R. Putman, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching*. Mahwah, NJ: Erlbaum.
Steffe, L. P. (2004). *PSSM* From a constructivist perspective. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 221–251). Mahwah, NJ: Erlbaum.
Stevenson, H. W., & Newman, R. S. (1986). Long-term prediction of achievement and attitudes in mathematics and reading. *Child Development*, *57*, 646–659.
Stipek, D. (2013). Mathematics in early childhood education: Revolution or evolution? *Early Education and Development*, *24*(4), 431–435. http://dx.doi.org/10.1080/10409289.2013.777285.
Sylva, K., Melhuish, E., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2005). *The effective provision of pre-school education [EPPE] project: A longitudinal study funded by the DfEE (1997–2003)*. London: DCSF/Institute of Education, University of London.
Thomas, G., & Tagg, A. (2004). *An evaluation of the early numeracy project 2003*. Wellington, NZ: Ministry of Education.
Tran, D., Reys, B. J., Teuscher, D., Dingman, S., & Kasmer, L. (2016). Analysis of curriculum standards: An important research area. *Journal for Research in Mathematics Education*, *47*(2), 118–133.
Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly*, *19*, 21–39.
Van Horn, M. L., Karlin, E. O., Ramey, S. L., Aldridge, J., & Snyder, S. W. (2005). Effects of developmentally appropriate practices on children's development: A review of research and discussion of methodological and analytic issues. *Elementary School Journal*, *105*, 325–351.
Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht, The Netherlands: Freudenthal Institute, Utrecht University.
van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics*, *74*(1), 23–37. http://dx.doi.org/10.1007/s10649-009-9225-x.
Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). *Teaching number in the classroom with 4–8 year olds*. London: Paul Chapman/Russell Sage.
Zimba, J. (2016). The Common Core and the potential for mathematicians to improve the teaching of school mathematics. *Notices of the American Mathematical Society*, *63*(02), 154. http://dx.doi.org/10.1090/noti1327.