

Meaningful Approaches to Algorithms for Decimals

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Emphases

Students extend their methods for operating on whole numbers to operating on decimal numbers. They develop, discuss, and use efficient, accurate, and generalizable methods. They make math drawings, relate the drawing to a written method, and explain the method using place value and properties of operations.

Students eventually become fluent with all operations on decimals without making a drawing.

Mathematical Practices

Students do

about math sense-making
using mathematical structure
to support math drawings
math explaining.

5.NBT Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents **10 times as much** as it represents in the place **to its right** and **1/10** of what it represents in the place **to its left**.

2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

Use **whole-number exponents** to denote **powers of 10**.

3. Read, write, and compare decimals **to thousandths**.

4. Use **place value understanding** to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

7. Add, subtract, multiply, and divide decimals to hundredths, **using concrete models or drawings** and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; **relate the strategy to a written method and explain the reasoning used**.

5.NBT Understand the place value system.

Generalize place value relationships in both directions

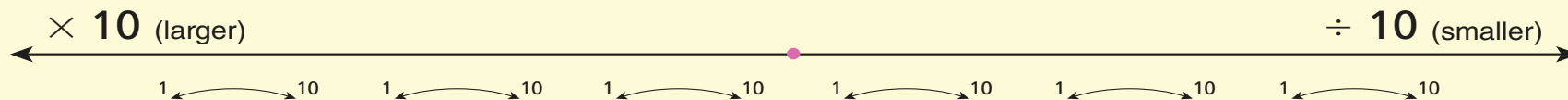
1. Recognize that in a multi-digit number, a digit in one place represents **10 times as much** as it represents in the place **to its right** and **1/10 of** what it represents in the place **to its left**.

Extend from tenths and hundredths in Grade 4 to thousandths

3. Read, write, and compare decimals **to thousandths**.
 - a. Read and write decimals to thousandths using base-ten numerals, number names, and **expanded form**, e.g.,
$$347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000).$$
 - b. Compare two decimals to thousandths **based on meanings of the digits in each place**, using $>$, $=$, and $<$ symbols to record the results of comparisons.

The value of a place is 10 times the value of the place to its right

Place Value



Thousands	Hundreds	Tens	ONES	Tenths	Hundredths	Thousandths
1,000.	100.	10.	1.	0.1	0.01	0.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
\$1,000.00 	\$100.00 	\$10.00 	\$1.00 	\$0.10 	\$0.01 	\$0.001 

G5 Secret Code Cards to the Thousandths



Tenths Card

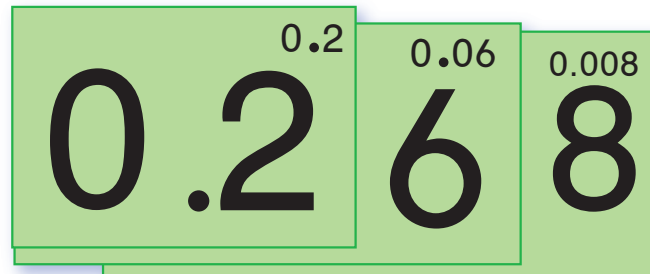


Hundredths Card



Thousandths Card

Ask students to place the cards on top of each other to display the number in standard form.

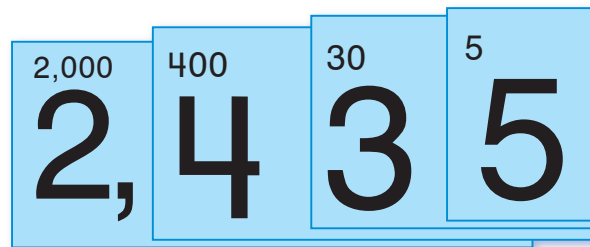


Assembled Cards

G5 Secret Code Cards to the Thousands

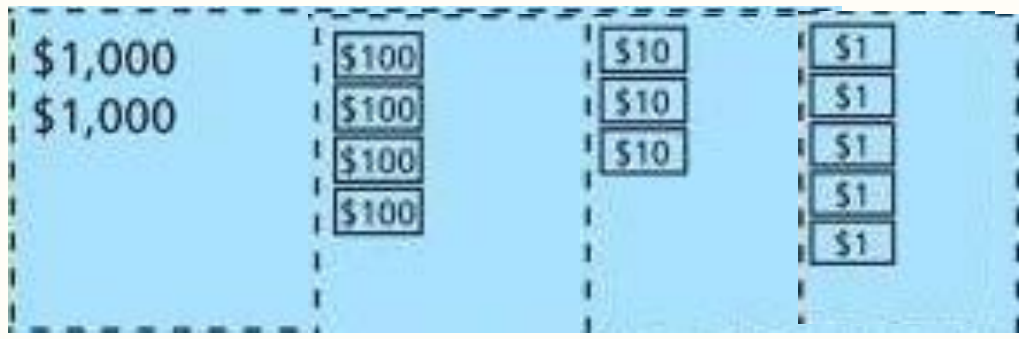
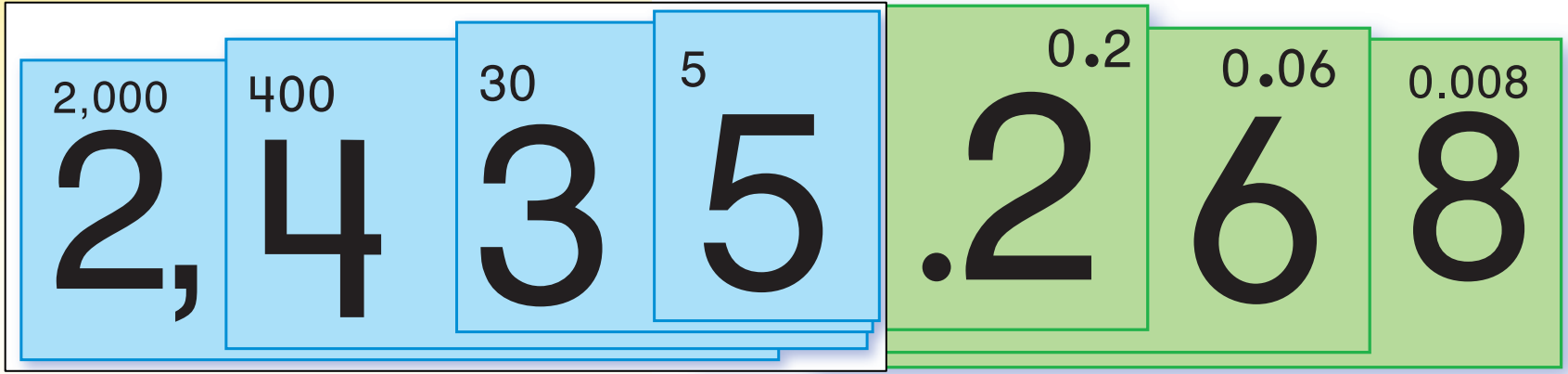


Ask students to place the cards on top of each other to display the number in standard form.



Assembled Cards

G5 Secret Code Cards Show Money Values



G5 5.NBT.7 Adding and Subtracting Decimals

Extend from adding and subtracting whole numbers in Grades K through G4 to adding and subtracting with decimals to hundredths.

7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

To add and subtract multi-digit numbers, you use like units: 2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

So for decimals align like units and add or subtract as with multi-digit numbers.

Align like units and add or subtract as with multi-digit numbers.

You can line up decimals points under each other to put like units under each other. But students need to understand that they do this so **they can add or subtract like units**. They can think:

dollars and dollars, dimes and dimes, pennies and pennies.

$0.37 + 0.9$ rewrite as

D P

0.37

0.9

You can add a decimal point and zeros

after a whole number: $46 + 0.72$

46.00

0.72

G5 Generalizing Composing and Decomposing for Decimals

5.NBT.1 Generalize place value relationships in both directions

1. Recognize that in a multi-digit number, a digit in one place represents **10 times as much** as it represents in the place **to its right** and **1/10 of** what it represents in the place **to its left**.

Because these one-for-ten left-to-right trades work regularly across all positions for whole numbers and for decimals,

composing (make 1 new unit from 10 units to the right) and

decomposing (make 10 new units to the right from 1 unit on the left)

work the same way for adding and subtracting decimals as they did for whole numbers.

Students can extend their G4 general compact methods for adding and for subtracting whole numbers to adding and subtracting decimals.

G5 Avoid the Common Subtraction Error

$$\$6 - 37¢$$

or

$$6 - 0.37$$

\$ D P

6.00

$$\begin{array}{r} 6.00 \\ - 0.37 \\ \hline \end{array}$$

Common
Error

6.00

-0.37

6.37

Draw the
Magnifying Glass

$$\begin{array}{r} \text{9} \\ 5 \text{ } 10 \\ \cancel{6.00} \\ - 0.37 \\ \hline 5.63 \end{array}$$

G5 Multiplying Decimals 5.NBT.2 and 7 and 5.NF.5b

5.NBT Understand the place value system.

2. Explain patterns in the number of zeros of the product when **multiplying a number by powers of 10**, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use **whole-number exponents** to denote powers of 10.

7. Add, subtract, **multiply**, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF.5. Interpret multiplication as scaling (resizing), by:

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); **explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number**; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

G5 The Whole Number Zero Shift Pattern Applies to Decimals


Cost of a Red Phantom Marble

\$. 4 1 2

$\times 1$

$1 \times \$0.412 = \0.412

\times




The diagram shows the cost of one red phantom marble as \$0.412. This is represented by a number line with digits 4, 1, and 2 in the thousandths, hundredths, and tenths places respectively. To the right of the number line are three coins: a quarter, a dime, and two pennies. A dashed box contains the equation $\times 1$. Below it, another dashed box contains the equation $1 \times \$0.412 = \0.412 . A multiplication symbol \times is shown below the equation.

10 Red Phantom Marbles

\$. 4 1 2

$\times 10$

$10 \times \$0.412 = \4.12




The diagram shows the cost of 10 red phantom marbles as \$4.12. This is represented by a number line with digits 4, 1, and 2 in the ones, tenths, and hundredths places respectively. To the right of the number line are four one-dollar bills, one dime, and two pennies. A dashed box contains the equation $\times 10$. Below it, another dashed box contains the equation $10 \times \$0.412 = \4.12 .

100 Red Phantom Marbles

\$. 4 1 2 0

$\times 100$

$100 \times \$0.412 = \41.20




The diagram shows the cost of 100 red phantom marbles as \$41.20. This is represented by a number line with digits 4, 1, and 2 in the tens, ones, and tenths places respectively, and a 0 in the hundredths place. To the right of the number line are four ten-dollar bills, one one-dollar bill, and two dimes. A dashed box contains the equation $\times 100$. Below it, another dashed box contains the equation $100 \times \$0.412 = \41.20 .

1,000 Red Phantom Marbles

\$ 4 1 2 . 0 0

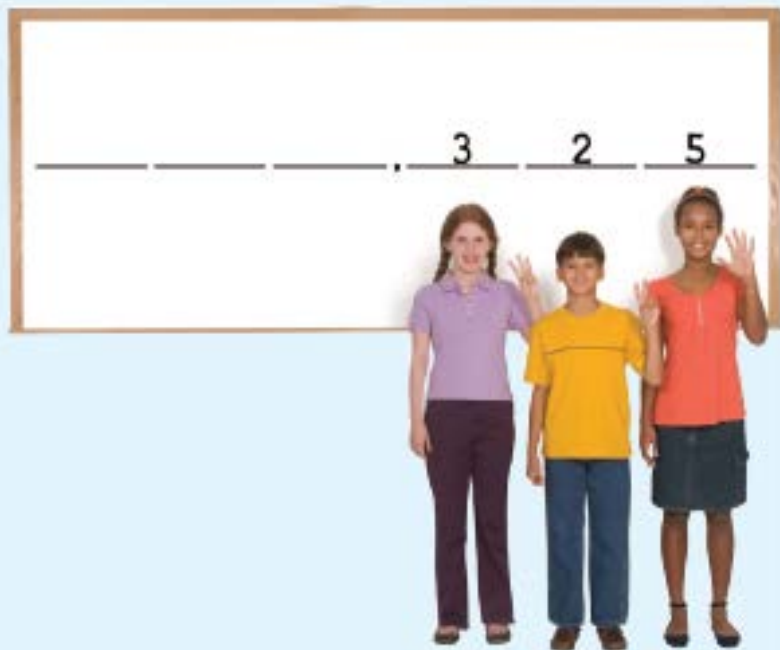
$\times 1,000$

$1,000 \times \$0.412 = \412.00

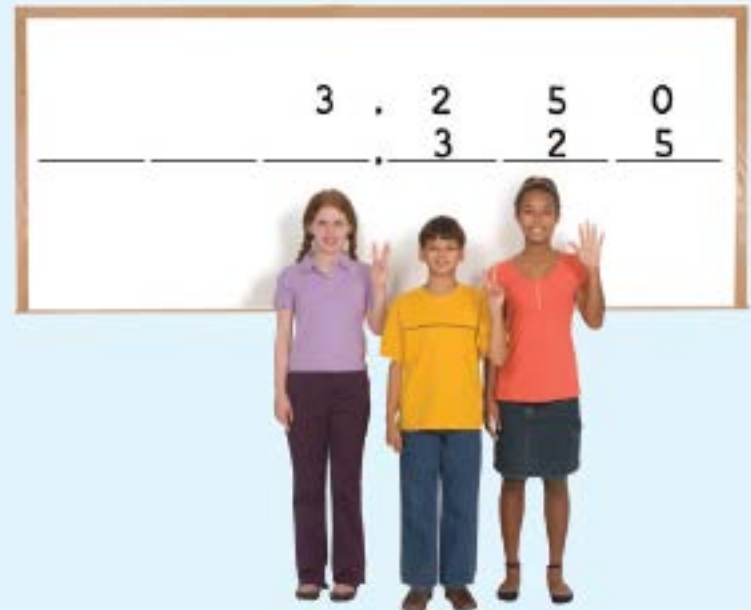


The diagram shows the cost of 1,000 red phantom marbles as \$412.00. This is represented by a number line with digits 4, 1, and 2 in the hundreds, tens, and ones places respectively, and two zeros in the tenths and hundredths places. To the right of the number line are four hundred-dollar bills, one ten-dollar bill, and two one-dollar bills. A dashed box contains the equation $\times 1,000$. Below it, another dashed box contains the equation $1,000 \times \$0.412 = \412.00 .

G5 The Decimal Shift Pattern Explained by Students



Beginning Position



*Problem 1: 325×10
Move 1 place to the left.*

G5 The Decimal Places in Whole Number x Decimal = Decimal Places in the Decimal

$$\begin{array}{r} 0.31 \\ \times \quad 5 \\ \hline 1.55 \end{array}$$

$$\begin{array}{r} 0.31 \\ \times 50 \\ \hline 15.50 \end{array}$$

$$\begin{array}{r} 0.31 \\ \times 500 \\ \hline 155.00 \end{array}$$

The number of decimal places in a whole number x decimal product = the number of decimal places in the decimal.

Because any multidigit number is composed of ones, tens, and hundreds (etc.), this is true for any multidigit number.

So you can multiply such problems as if they were two whole numbers and then count the decimal places and put that many places in the product.

G5 The Shifts for a Decimal Times a Whole Number with Money

Leon's Earnings

\$ 2 1 3 .

$\times 1$




$1 \times \$213 = \213

Save 0.1 Each Month

\$ 2 1 . 3 0

$\times 0.1$



$0.1 \times \$213 = \21.30

Students then explore the shift using 0.01 as a factor.

Save 0.01 Each Month

\$ 2 . 1 3

$\times 0.01$



$0.01 \times \$213 = \2.13

G5 The Decimal Places in Decimal x Whole Number = Decimal Places in the Decimal

The number of decimal places in a decimal x a whole number product = the number of decimal places in the decimal.

$$5 \times 31$$

$$155$$

$$\begin{array}{r} 31 \\ \times \quad 5 \\ \hline 155 \end{array}$$

$$0.5 \times 31$$

$$0.1 \times 5 \times 31$$

$$0.1 \times 155$$

$$15.5$$

$$\begin{array}{r} 31 \\ \times \quad 0.5 \\ \hline 15.5 \end{array}$$

$$0.05 \times 31$$

$$0.01 \times 5 \times 31$$

$$0.01 \times 155$$

$$1.55$$

$$\begin{array}{r} 31 \\ \times \quad 0.05 \\ \hline 1.55 \end{array}$$

Because any multidigit number is composed of ones, tens, and hundreds (etc.), this is true for any multidigit number.

So you can multiply such problems as if they were two whole numbers and then count the decimal places and put that many places in the product.

G5 The Decimal Multiplier Shift Patterns Apply to Decimals

$$1 \times \$41.20 = \$41.20$$

100 Red Phantom Marbles

\$ _____ 4 _____ 1 _____ . _____ 2 _____ 0 _____

The diagram shows a place value chart with columns for tens, ones, tenths, and hundredths. Under the tens column (4), there are four \$10 bills. Under the ones column (1), there is one \$10 bill. Under the tenths column (2), there are two quarters. Under the hundredths column (0), there are no coins.

$$0.1 \times \$41.20 = \$4.12$$

10 Red Phantom Marbles

\$ _____ _____ 4 _____ . _____ 1 _____ 2 _____

The diagram shows a place value chart with columns for tens, ones, tenths, and hundredths. Under the ones column (4), there are four \$1 bills. Under the tenths column (1), there is one dime. Under the hundredths column (2), there are two pennies.

$$0.01 \times \$41.20 = \$0.412$$

Cost of a Red Phantom Marble

\$ _____ _____ _____ . _____ 4 _____ 1 _____ 2 _____

The diagram shows a place value chart with columns for tenths, hundredths, thousandths, and ten-thousandths. Under the hundredths column (4), there are four pennies. Under the thousandths column (1), there is one penny. Under the ten-thousandths column (2), there are two pennies.

G5 The Decimal Places in Decimal x Decimal = Total Decimal Places in the Decimal Factors

The number of decimal places in a decimal x decimal product = the total number of decimal places in the decimal factors.

$$0.2 \times 0.3$$

$$2 \times 0.1 \times 3 \times 0.1$$

$$2 \times 3 \times 0.1 \times 0.1$$

$$6 \times 0.01$$

$$0.06$$

$$0.2 \times 0.03$$

$$2 \times 0.1 \times 3 \times 0.01$$

$$2 \times 3 \times 0.1 \times 0.01$$

$$6 \times 0.001$$

$$0.006$$

$$0.02 \times 0.03$$

$$2 \times 0.01 \times 3 \times 0.01$$

$$2 \times 3 \times 0.01 \times 0.01$$

$$6 \times 0.0001$$

$$0.0006$$

Because any decimal number is composed of tenths and hundredths (etc.), this is true for any decimal number and generalizes to decimals with more places.

So you can multiply decimal x decimal problems as if they had two whole numbers. Then count the decimal places in each factor to find the total number of decimal places in the product.

G5 Relate Decimal Patterns to Fractions

The number of decimal places in a decimal x decimal product
= the total number of decimal places in the decimal factors.

$$0.2 \times 0.3$$

$$2 \times 0.1 \times 3 \times 0.1$$

$$2 \times 3 \times 0.1 \times 0.1$$

$$6 \times 0.01$$

$$0.06$$

$$2/10 \times 3/10$$

$$6/100$$

$$0.2 \times 0.03$$

$$2 \times 0.1 \times 3 \times 0.01$$

$$2 \times 3 \times 0.1 \times 0.01$$

$$6 \times 0.001$$

$$0.006$$

$$2/10 \times 3/100$$

$$6/1000$$

$$0.02 \times 0.03$$

$$2 \times 0.01 \times 3 \times 0.01$$

$$2 \times 3 \times 0.01 \times 0.01$$

$$6 \times 0.0001$$

$$0.0006$$

$$2/100 \times 3/100$$

$$6/10000$$

G5 The Shift Pattern Direction Depends on the Multiplier

Whole Number Multipliers

1. When you multiply by 10, the number gets 10 times as big. The digits shift 1 place(s) to the left.

Decimal Number Multipliers

2. When you multiply by 0.1, the number gets $\frac{1}{10}$ as big. The digits shift 1 place(s) to the right.

5.NF.5. Interpret multiplication as scaling (resizing), by:

b. ..., explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; ...

G5 Shift Patterns and the Number of Decimal Places in the Product

We have explained patterns for the number of decimal places in the answer.

The number of decimal places in a whole number \times a decimal product = the number of decimal places in the decimal.

The number of decimal places in a decimal \times a whole number product = the number of decimal places in the decimal.

The number of decimal places in a decimal \times a decimal product = the total number of decimal places in the decimal factors.

So you can multiply all of these problems as if they had two whole numbers. Then count the decimal places in each factor to find the total number of decimal places in the product.

G5 Practicing All Multiplication Problems

5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm.

Grade 5 practice with **decimal multiplication problems** serves fluency for this whole number standard because you multiply as usual and then count the number of decimal places in each factor to find the total number of decimal places in the answer.

$$\begin{array}{r} 13. \quad 4.6 \\ \times 0.12 \\ \hline 0.552 \end{array}$$

$$\begin{array}{r} 14. \quad 8.5 \\ \times 8.5 \\ \hline 72.25 \end{array}$$

$$\begin{array}{r} 15. \quad 246.1 \\ \times .09 \\ \hline 22.149 \end{array}$$

$$\begin{array}{r} 16. \quad 7.7 \\ \times 3.2 \\ \hline 24.64 \end{array}$$

$$\begin{array}{r} 17. \quad 1.1 \\ \times 0.11 \\ \hline 0.121 \end{array}$$

$$\begin{array}{r} 18. \quad 4.3 \\ \times 2.4 \\ \hline 10.32 \end{array}$$

$$\begin{array}{r} 19. \quad 7.8 \\ \times 0.15 \\ \hline 1.17 \end{array}$$

$$\begin{array}{r} 20. \quad 3.8 \\ \times 4.5 \\ \hline 17.1 \end{array}$$

G5 Dividing a Decimal by a 1-Digit Number

► Divide a Decimal by a One-Digit Number

Three friends set up a lemonade stand and made \$20.25. They will share the money equally. Study the steps below to see how much money each person should get.

When the \$20 is split 3 ways, each person gets \$6. There is \$2 left.

$$\begin{array}{r} 6 \\ 3 \overline{)20.25} \\ \underline{-18} \\ 2 \end{array}$$

We change the \$2 to 20 dimes and add the other 2 dimes. There are 22 dimes.

$$\begin{array}{r} 6. \\ 3 \overline{)20.25} \\ \underline{-18} \\ 2.2 \end{array}$$

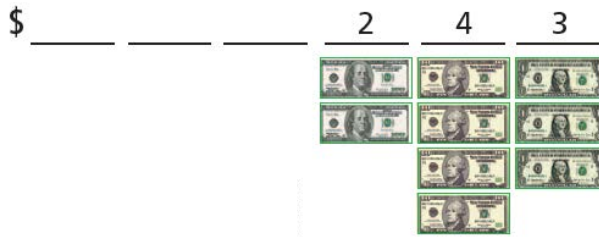
When we split 22 dimes 3 ways, each person gets 7 dimes. There is 1 dime left.

$$\begin{array}{r} 6.7 \\ 3 \overline{)20.25} \\ \underline{-18} \\ 2.2 \\ \underline{-2.1} \\ .1 \end{array}$$

We change the dime to 10 cents and add the other 5 cents. Now we split 15 cents 3 ways.

$$\begin{array}{r} 6.75 \\ 3 \overline{)20.25} \\ \underline{-18} \\ 2.2 \\ \underline{-2.1} \\ .15 \\ \underline{-.15} \end{array}$$

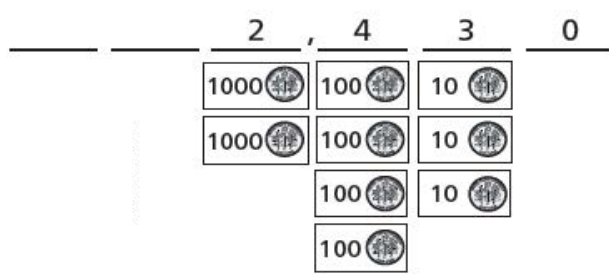
G5 Shift Patterns for Dividing Whole Numbers by Decimals



$$\div 1$$

$$243 \div 1 = 243$$

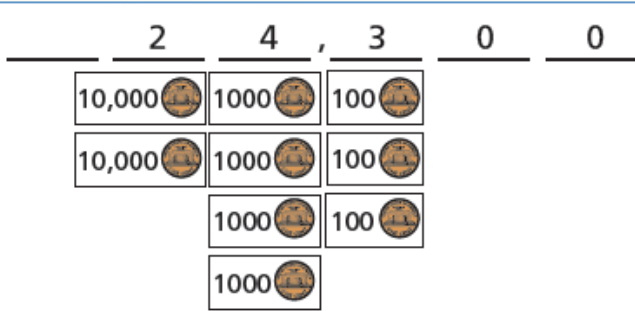
Jordan's Earnings in Dollars



$$\div 0.1$$

$$243 \div 0.1 = 2,430$$

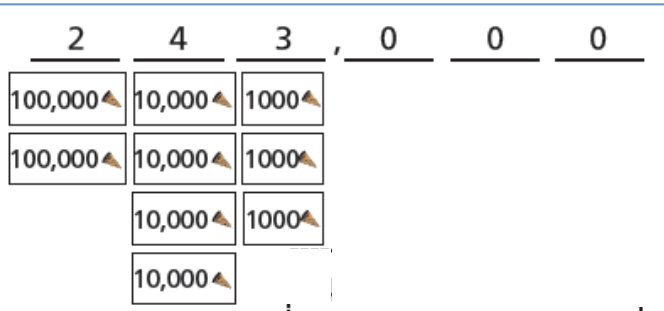
Jordan's Earnings in Dimes



$$\div 0.01$$

$$243 \div 0.01 = 24,300$$

Jordan's Earnings in Pennies



$$\div 0.001$$

$$243 \div 0.001 = 243,000$$

Jordan's Earnings in Tenths of a Cent

G5 Dividing by Decimals

What does it mean to divide 6 by 0.1?

Dividing by 0.1 asks: **How many of the small one-tenths are in 6?**

For **each** of the 6 units, there are **10 one-tenths** (ten 0.1s).

So there are **10 x 6 one-tenths** in 6.

$$6 \div 0.1 = 60$$

The 6 shifts **1 place to the left**. The answer is **10 times as much as 6**.

So $\div 0.1$ is the same as $\times 10$.

$$6 \div 0.1 = 60 \text{ is the same as } 0.1 \times 60 = 6$$

The shifts for division and multiplication are in the opposite directions.

► Change Decimal Divisors to Whole Numbers

You can use the strategy below to change a division problem with a decimal divisor to an equivalent problem with a whole number divisor.

Discuss each step used to find $6 \div 0.2$.

Step 1: Write $6 \div 0.2$ as a fraction.

$$6 \div 0.2 = \frac{6}{0.2}$$

Step 2: Make an equivalent fraction with a whole number divisor by multiplying $\frac{6}{0.2}$ by 1 in the form of $\frac{10}{10}$. Now you can divide 60 by 2.

$$\frac{6}{0.2} \times 1 = \frac{6}{0.2} \times \frac{10}{10} = \frac{60}{2}$$

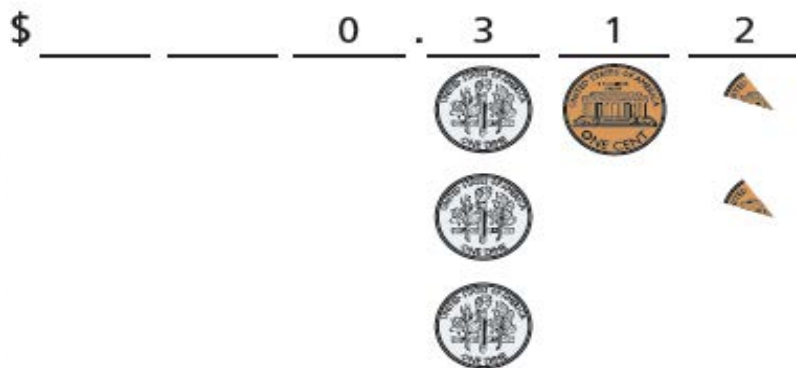
G5 Dividing by Hundredths and Thousandths

$$23. \quad \frac{6}{0.02} = \frac{6}{0.02} \times 1 = \frac{6}{0.02} \times \frac{100}{100} = \frac{600}{2} \longrightarrow \underbrace{0.02} \overline{) \underbrace{6.00}}$$

$$24. \quad \frac{6}{0.002} = \frac{6}{0.002} \times 1 = \frac{6}{0.002} \times \frac{1,000}{1,000} = \frac{6,000}{2} \longrightarrow \underbrace{0.002} \overline{) \underbrace{6.000}}$$

You may have to put one or more 0s at the end of the dividend when you make the divisor a whole number.

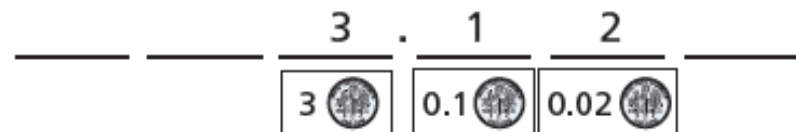
G5 Shift Patterns for Dividing Decimals by Decimals



$$\div 1$$

$$\$0.312 \div 1 = \$0.312$$

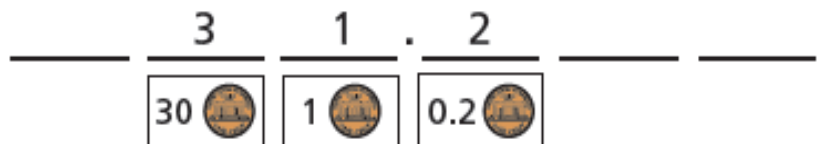
Cost of Cat's Eye Marble in Dollars



$$\div 0.1$$

$$0.312 \div 0.1 = 3.12$$

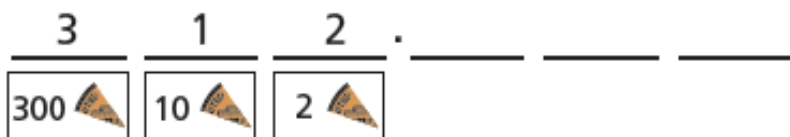
Cost of Cat's Eye Marble in Dimes



$$\div 0.01$$

$$0.312 \div 0.01 = 31.2$$

Cost of Cat's Eye Marble in Pennies



$$\div 0.001$$

$$0.312 \div 0.001 = 312$$

Cost of Cat's Eye Marble in Tenths of a Cent

G5 Dividing a Decimal by Hundredths and Thousandths

15. $\frac{0.06}{0.02} \times 1 = \frac{0.06}{0.02} \times \frac{100}{100} = \frac{0.06 \times 100}{0.02 \times 100} = \frac{6}{2}$

$0.02 \overline{)0.06}$ $\overset{3.}{\phantom{0.02 \overline{)0.06}}}$

16. $\frac{0.06}{0.002} \times 1 = \frac{0.06}{0.002} \times \frac{1,000}{1,000} = \frac{0.06 \times 1,000}{0.002 \times 1,000} = \frac{60}{2}$

$0.002 \overline{)0.060}$ $\overset{30.}{\phantom{0.002 \overline{)0.060}}}$

You may have to put one or more 0s at the end of the dividend when you make the divisor a whole number.

G5 Is it Multiplication or Division? Get Larger or Smaller?

► Multiply or Divide?

Read the problem. Then answer the questions.

1. A turtle walks 0.2 mile in 1 hour. How far can it walk in 0.5 hour?

a. Do you need to multiply or divide to solve? multiply

b. Will the answer be more or less than 0.2 miles? less

c. What is the answer? 0.1 mile

2. Gus ran 3.6 miles. He took a sip of water every 0.9 mile.
How many sips did he take?

a. Do you need to multiply or divide to solve? divide

b. Will the answer be greater or less than 3.6? greater

c. What is the answer? 4 sips

3. Last year 135 cows on Dixie's Dairy Farm had calves.
This year 0.6 times that many cows had calves. How many
cows had calves this year?

a. Do you need to multiply or divide to solve? multiply

b. Will the answer be greater or less than 135? less

c. What is the answer? 81 cows

G6 NS Decimal Standard 6.NS.3

6.NS.3. Fluently add, subtract, multiply, and divide **multi-digit decimals** using the standard algorithm for each operation.

The CCSS do not specify a grade for **fluency with fractions**.

In Grade 5 students do all operations except general division of **fractions**, which is in Grade 6 (6.NS.1).

In Grade 7 all operations with **fractions** are extended to operations with rational numbers in Grade 7.

So in Grade 6 students should overview all operations with **fractions** to relate them to division of **fractions** and to become **fluent** with operations with **fractions** before extending these operations in Grade 7.

In Grade 6 students should also **relate operations with decimals to operations with fractions to understand how the meanings are alike (e.g., you add like units) but the notations create different methods (you find a new equivalent unit in different ways)**. This enriches and deepens student understanding of number systems.









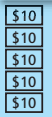




G6 Relating Money Values, Decimals, and Fractions

► Discuss and Summarize

Fill in the blanks and discuss how the parts of each problem are related.

Place Value

$\times 10$ (larger) ← → $\div 10$ (smaller)

Thousands	Hundreds	Tens	ONES	Tenths	Hundredths	Thousandths
1,000.	100.	10.	1.	0.1	0.01	0.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
\$1,000.00 	\$100.00 	\$10.00 	\$1.00 	\$0.10 	\$0.01 	\$0.001 
2,000	300	60	1	0.6	0.03	0.002
2,	3	6	1	.6	3	2
\$1,000 \$1,000						

1 a.

b.

c. $2,361.632 = 2,000 + \underline{300} + \underline{60} + \underline{1} + \underline{0.6} + \underline{0.03} + 0.002$

2 a.

b.

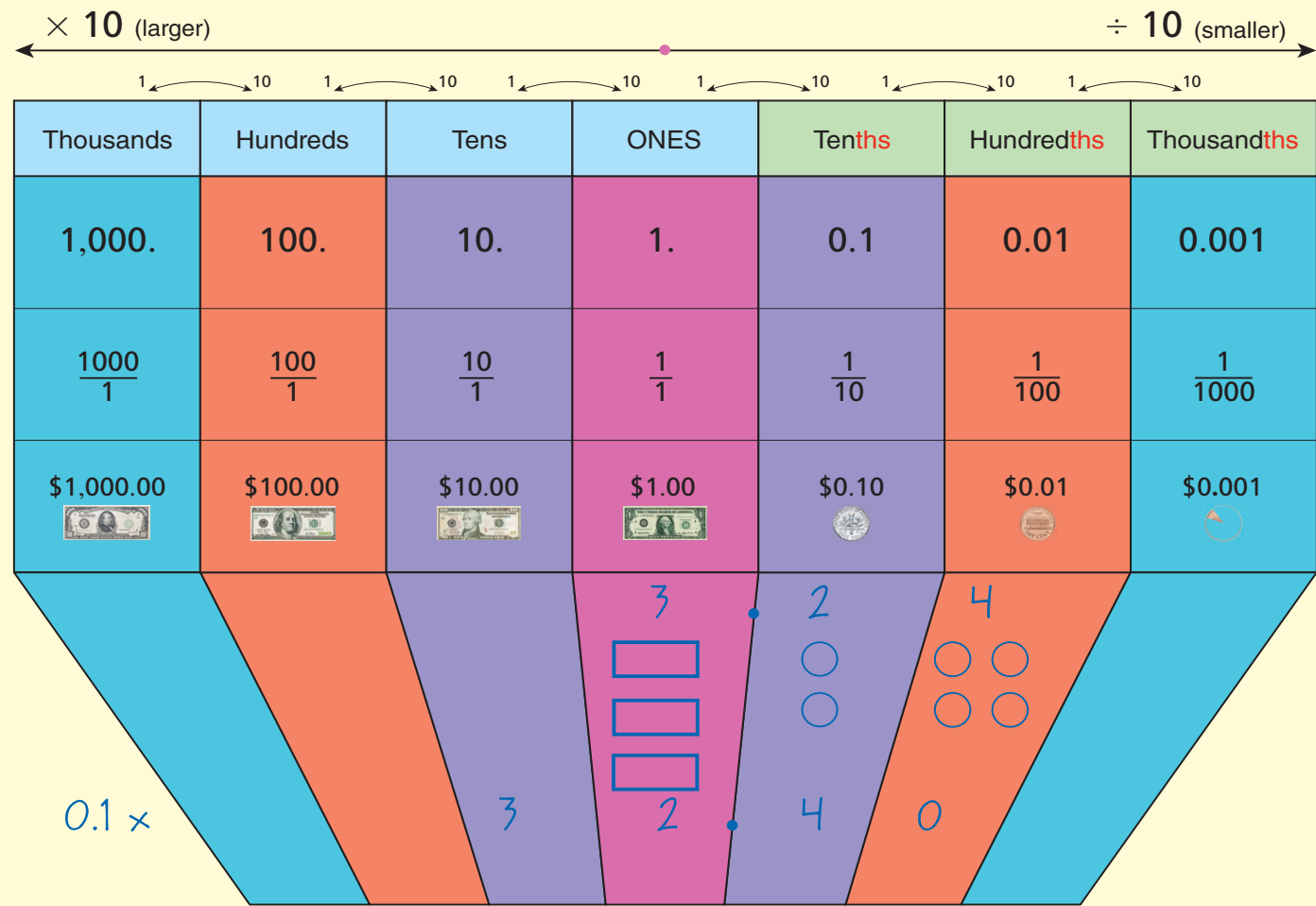
c.

d.

$$\begin{array}{r}
 0 \quad .6 \quad 3 \quad 2 \\
 + \quad \frac{6}{10} + \frac{3}{100} + \frac{2}{1,000} \\
 + \frac{600}{1,000} + \frac{30}{1,000} + \frac{2}{1,000} \\
 \hline
 0 \quad .6 \quad 0 \quad 0 \\
 + 0 \quad .0 \quad 3 \quad 0 \\
 + 0 \quad .0 \quad 0 \quad 2 \\
 \hline
 0 \quad .6 \quad 3 \quad 2
 \end{array}$$

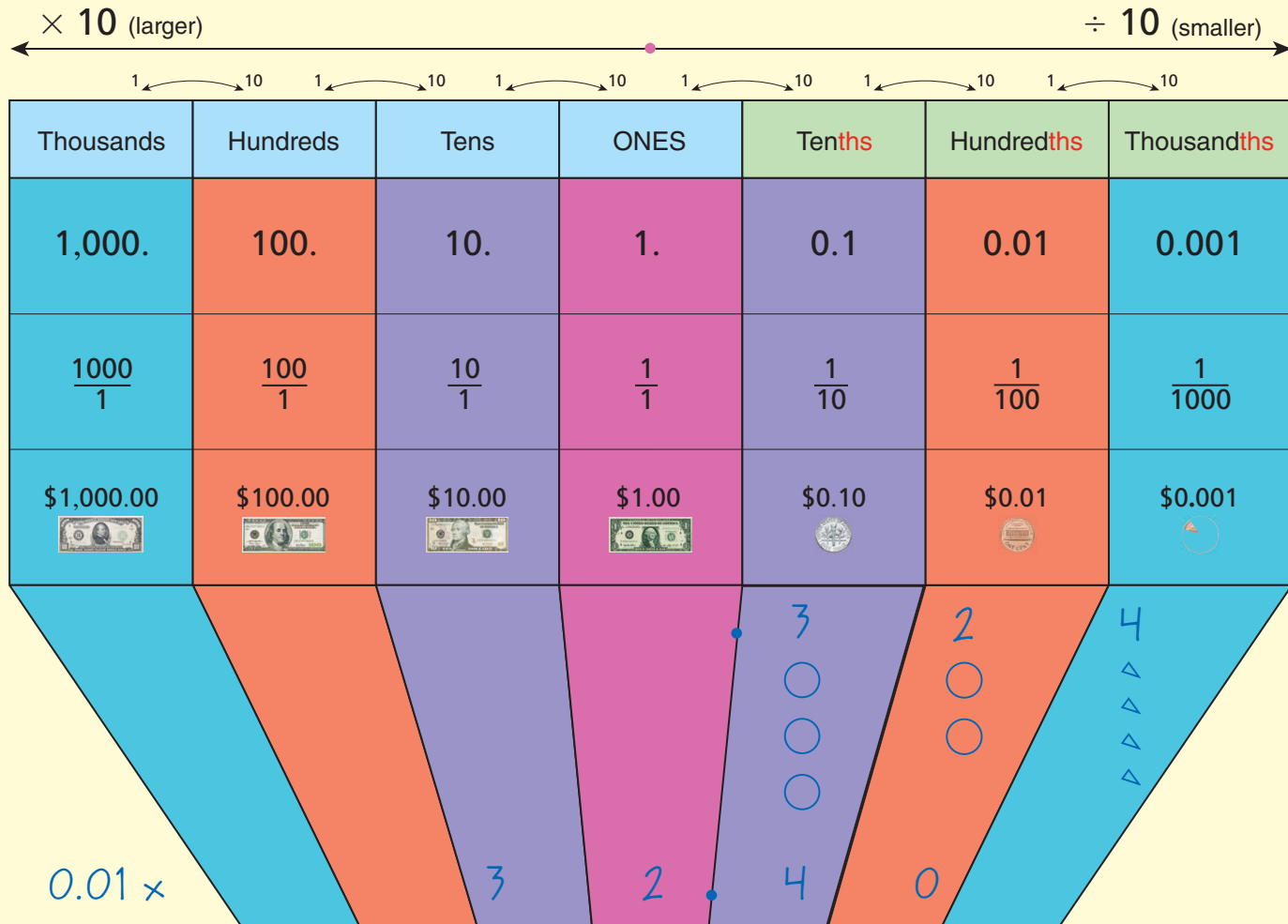
G6 Seeing the Shift to the Right for x 0.1 on the Place Value Poster

Place Value

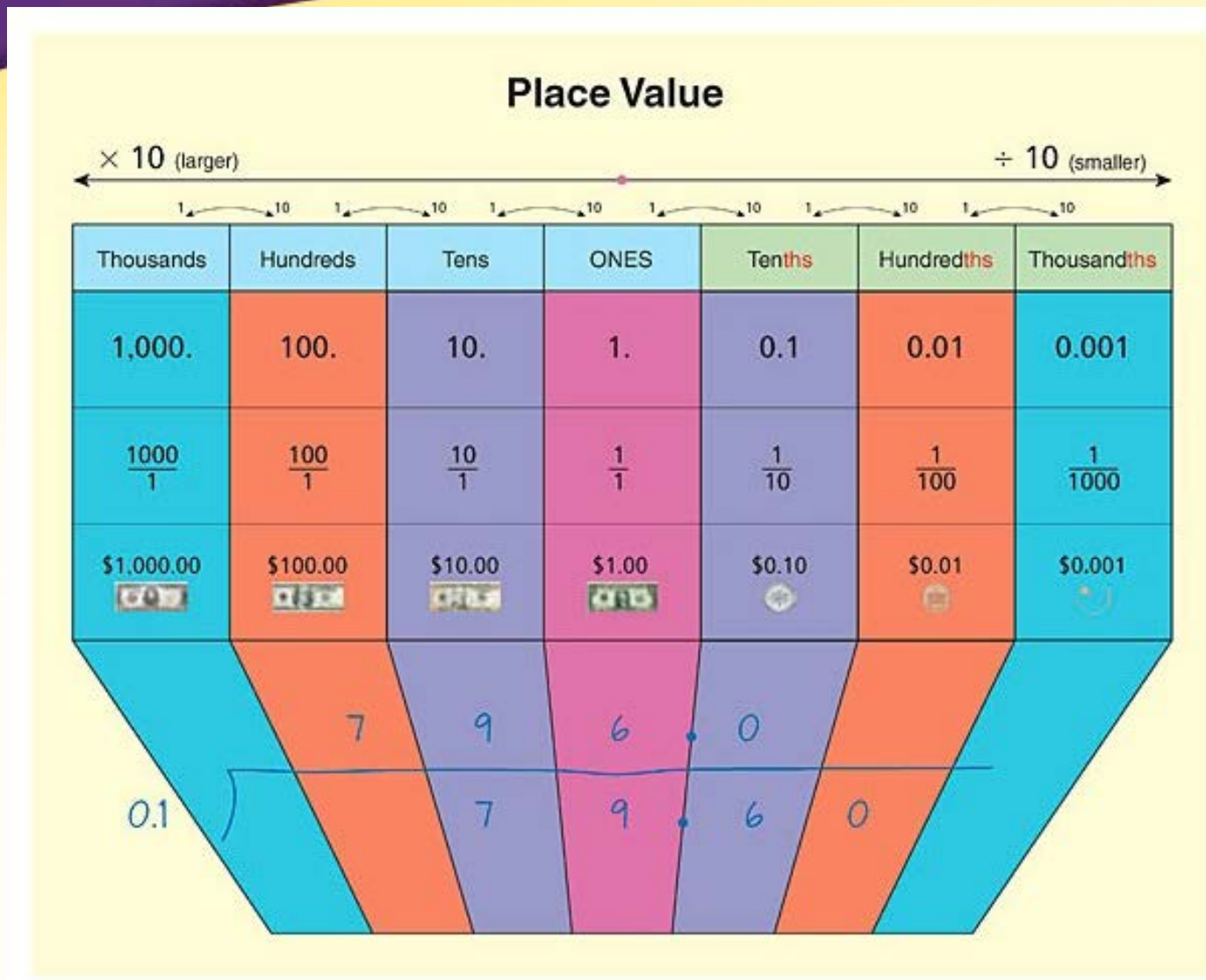


G6 Seeing the Shift to the Right for $\times 0.01$ on the Place Value Poster

Place Value

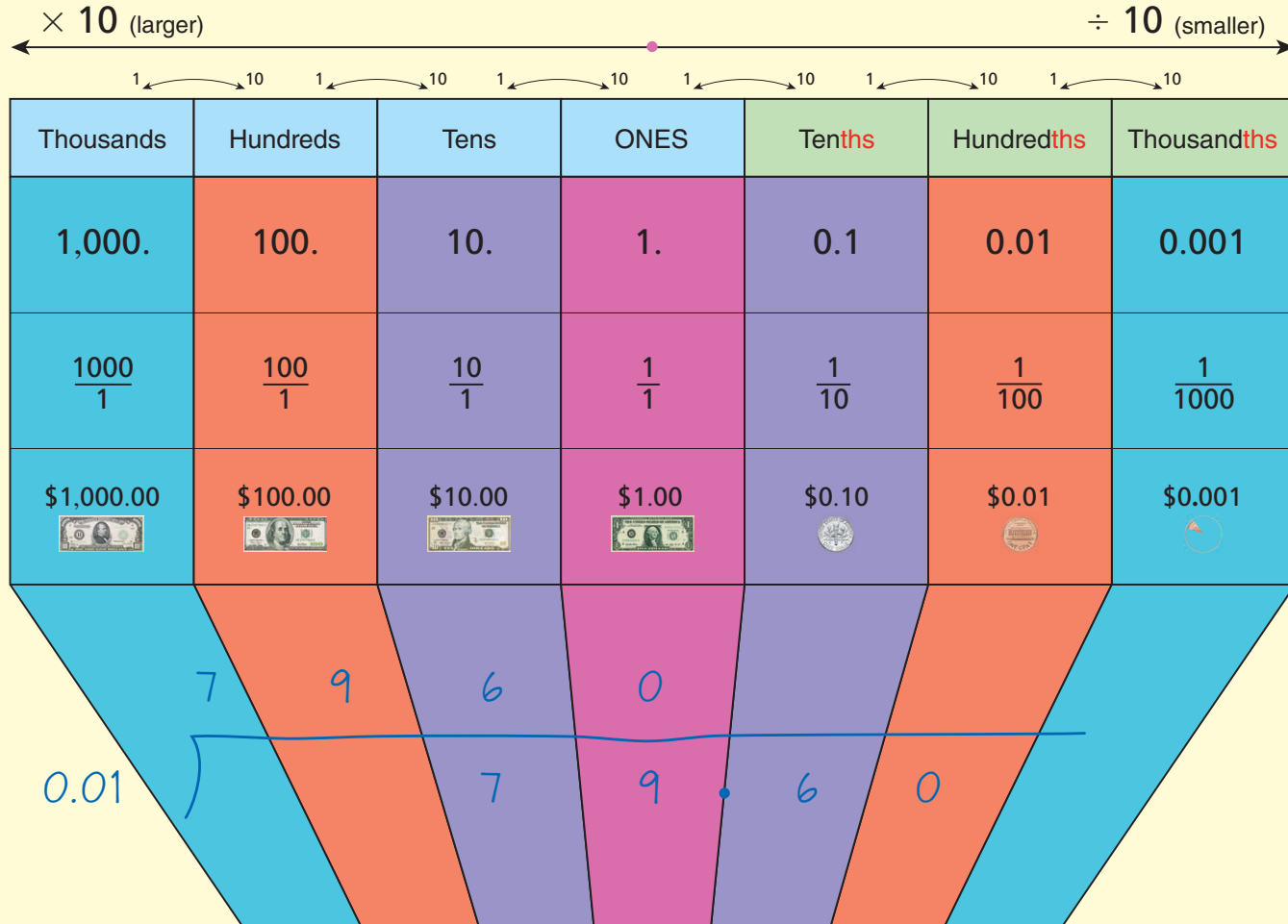


G6 Seeing the Shift to the Left for $\div 0.1$ on the Place Value Poster



G6 Seeing the Shift to the Left for $\div 0.01$ on the Place Value Poster

Place Value



G6 Decimal Shift Directions for Multiplying and Dividing

For decimals less than 1:

Multiplying by 0.1 shifts all digits one place **smaller** (to the **right**) because you are finding **one-tenth of each digit**.

So $\times 0.1$ is the same as $\div 10$.

Dividing by 0.1 shifts all digits one place **larger** (to the **left**) because you are finding how **many of the small one-tenths are in each digit**.

So $\div 0.1$ is the same as $\times 10$.

The shifts for multiplying and dividing by a decimal are **in the opposite direction** of the shifts for multiplying and dividing by a whole number:

Multiplying by a decimal less than 1 makes a smaller number and dividing by a decimal less than 1 makes a larger number.

Dividing by a whole number makes a smaller number and multiplying by a whole number makes a larger number.

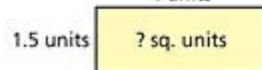
G6 Do Multiplying and Dividing Make Greater or Less?

► Greater, Equal, or Less

1. Explore what happens when you multiply 4 by different numbers.

Case 1: Multiply 4 by a number greater than 1.

$$4 \cdot 1.5 = \underline{6}$$

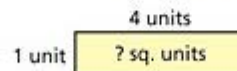


Is the product less than, greater than, or equal to 4?

greater than

Case 2: Multiply 4 by 1.

$$4 \cdot 1 = \underline{4}$$

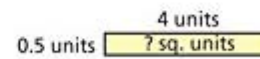


Is the product less than, greater than, or equal to 4?

equal to

Case 3: Multiply 4 by a number less than 1.

$$4 \cdot 0.5 = \underline{2}$$



Is the product less than, greater than, or equal to 4?

less than

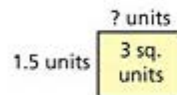
2. Multiply 10 by 1, a number greater than 1, and a number less than 1. Tell how each product compares to 10.

Numbers and products will vary. Students should find that the products are equal to, greater than, and less than 10 respectively.

3. Explore what happens when you divide 3 by different numbers.

Case 1: Divide 3 by a number greater than 1.

$$3 \div 1.5 = \underline{2}$$

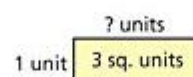


Is the quotient less than, greater than, or equal to 3?

less than

Case 2: Divide 3 by 1.

$$3 \div 1 = \underline{3}$$

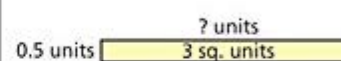


Is the quotient less than, greater than, or equal to 3?

equal to

Case 3: Divide 3 by a number less than 1.

$$3 \div 0.5 = \underline{6}$$



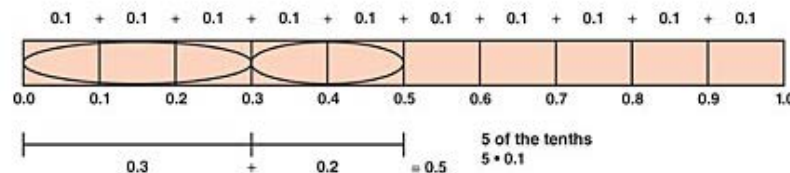
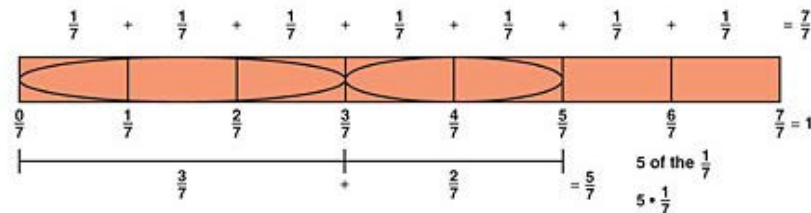
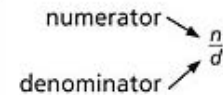
Is the quotient less than, greater than, or equal to 3?

greater than

G6 Adding and Subtracting Unit Fractions and Decimals

► Adding and Subtracting Unit Fractions and Decimals

The **denominator** of a fraction tells the number of equal parts the whole is divided into, and, therefore, the number of unit fractions that are made. The **numerator** tells how many of these unit fractions the fraction represents.



Add.

4. $\frac{3}{9} + \frac{2}{9} = \frac{5}{9}$

5. $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

6. $0.03 + 0.02 = 0.05$

Subtract.

7. $\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$

8. $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$

9. $\frac{5}{9} - \frac{3}{9} = \frac{2}{9}$

10. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

11. $0.5 - 0.3 = 0.2$

12. $0.5 - 0.2 = 0.3$

13. $0.05 - 0.03 = 0.02$

14. $0.05 - 0.02 = 0.03$

To add or subtract fractions with like denominators, add or subtract the numerators.

To add or subtract decimals with like places, add or subtract the numbers in the places.

G6 Finding a Common Denominator for Fractions and Decimals

► Finding a Common Denominator

Example 1 $\frac{4}{5} + \frac{3}{10}$

Jo Anne's Solution

I look at the denominators to see what I need to multiply by.

$$\frac{4}{5} + \frac{3}{10} = \frac{4 \cdot 2}{5 \cdot 2} + \frac{3}{10} = \frac{8}{10} + \frac{3}{10} = \frac{11}{10} = 1\frac{1}{10}$$

Mark's Solution

I see $5 \cdot 2 = 10$.

So, I multiply 4 and 5 by 2.

I find how many tenths.

Now I can add.

$$\frac{4 \cdot 2}{5 \cdot 2} = \frac{8}{10}$$

$$\frac{4 \cdot 2}{5 \cdot 2} = \frac{8}{10}$$

$$\frac{4 \cdot 2}{5 \cdot 2} = \frac{8}{10}$$

$$\begin{aligned} \frac{4}{5} + \frac{3}{10} &= \frac{8}{10} + \frac{3}{10} \\ &= \frac{11}{10} = 1\frac{1}{10} \end{aligned}$$

To compare, add, or subtract two fractions when the denominator of one is a factor of the denominator of the other, change the fraction with the lesser denominator to an equivalent fraction that uses the greater denominator.

Example 2 $0.2 + 0.03$

Shauna's Solution

0.2 is two dimes. That is 20 pennies, so $0.2 = 0.20$.

I add like places.

$$\begin{array}{r} 0.20 \\ + 0.03 \\ \hline 0.23 \end{array}$$

To compare, add, or subtract decimals with different numbers of places, put zeros on the end of the decimal with fewer places to make the number of decimal places in the two numbers equal.

Thank you!

Questions or comments?

Find these slides:

<http://www.math.uga.edu/~sybilla/>

Also at the Mathematics Teaching Community (google it!):

<https://mathematicsteachingcommunity.math.uga.edu>

Tags: standard-algorithms, decimals, numbers-operations-base-ten