Connecting Geometric Measurement, Other Measures, and Data Use Standards to Each Other and to Other CCSS Domains

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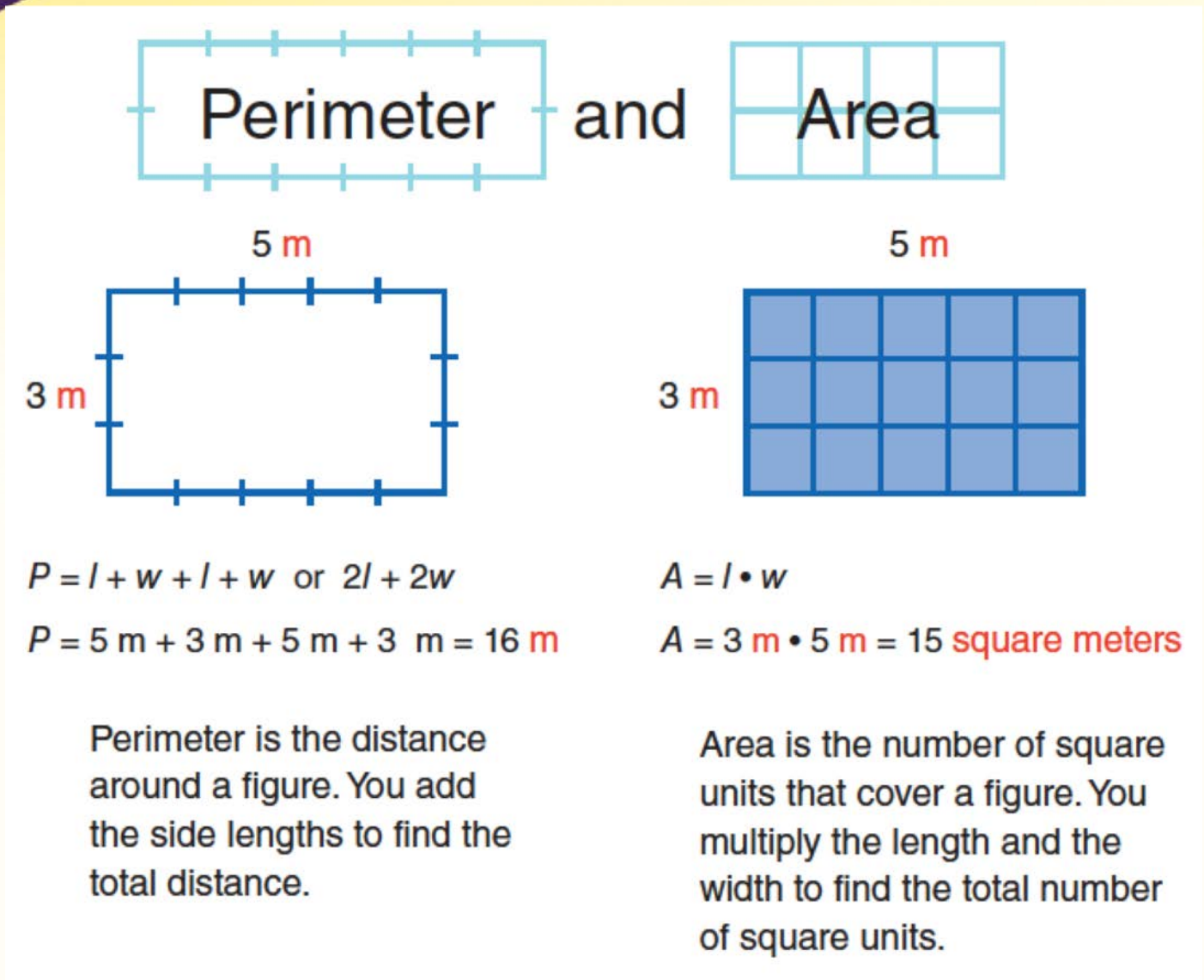
For more details about all CCSS domains including Measurement and Data, please see the 18 hours of audio-visual Teaching Progressions I have made. You can find links to these and to papers and other presentations at karenfusonmath.com

This presentation is also posted there.

Units in the Major Parts of MD: Measurement and Data

	K	1	2	3	4	5
	MD Measurement and Data: K to 5					
	Geometric Measurement: K to 6 Use length to make area and volume units					
Length	(Describe attributes)	(Length)	Length	Area	Angles	Volume [G6 geometry: surface area and area of triangles, special quadrilaterals, and polygons
	Other Measures: K to 5					
Various	(Describe attributes)	(Time)	Time Money	Time Liq volume Mass	Larger to smaller units x	Convert units both ways x ÷
Base ten				Metric liq vol, mass are multiples of ten		
	Represent and interpret data: K to 5					
			Line plots	½ ¼	½ ¼ 1/8	Use fraction operations
Things	Classify into categories, count	Up to 3 categories compare	Picture & bar graphs all problems	Picture & bar graphs scale multiple unit 1- and 2-step compare		

Grade 4 Poster Shows Perimeter and Area Formulas and the Length and Square Units



Differentiate the length units in perimeter and the square units in area.

For area, check that the side lengths have the same units so you can make the square units.

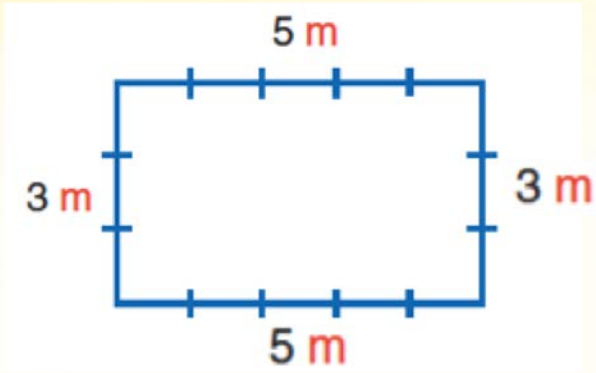
Students draw length or area units in rectangles.

Label rectangles for perimeter on all 4 sides to show what must be added. Then use any of 3 strategies:

3 + 5 + 3 + 5

(3 + 5) x 2

2 x 5 + 2 x 3



Students must see all 4 sides for any perimeter problem.

7 cm

?

Area = 28 sq cm

4 cm

21.

7 cm

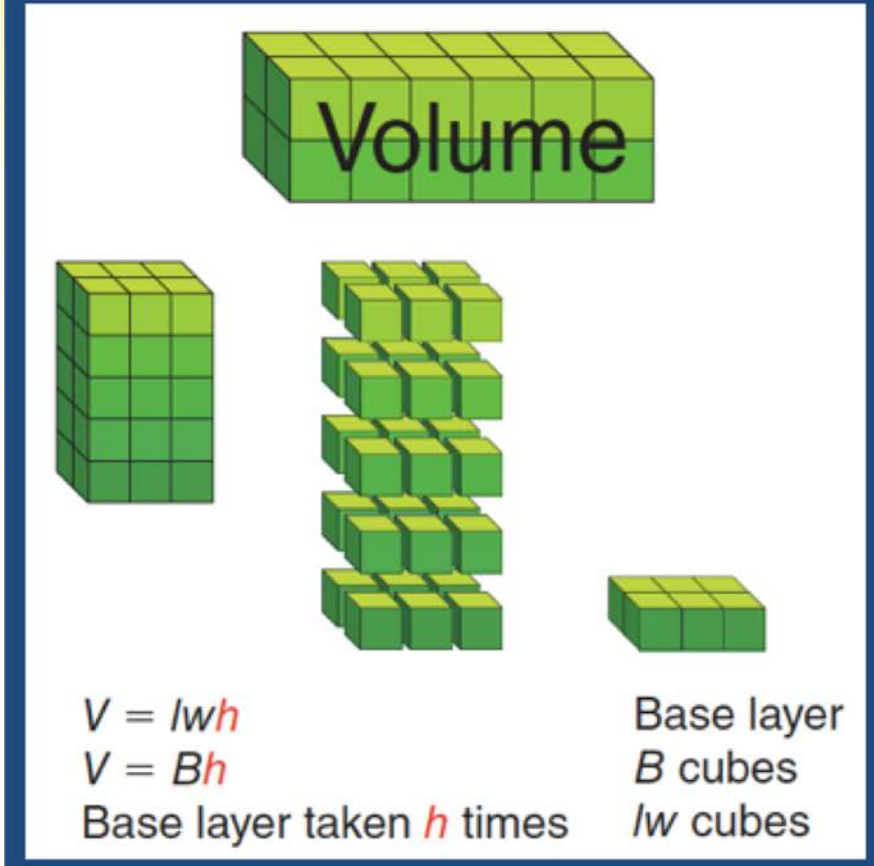
?

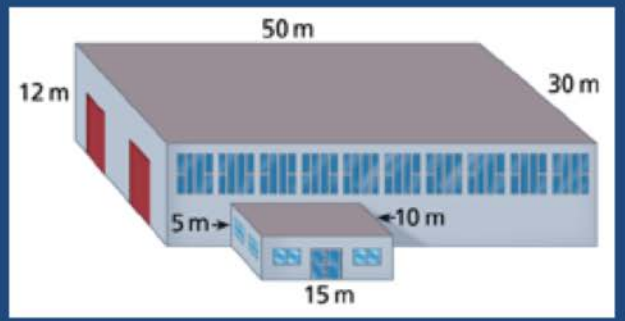
Perimeter = 28 cm

7 cm

Length x length x length to get cubic units

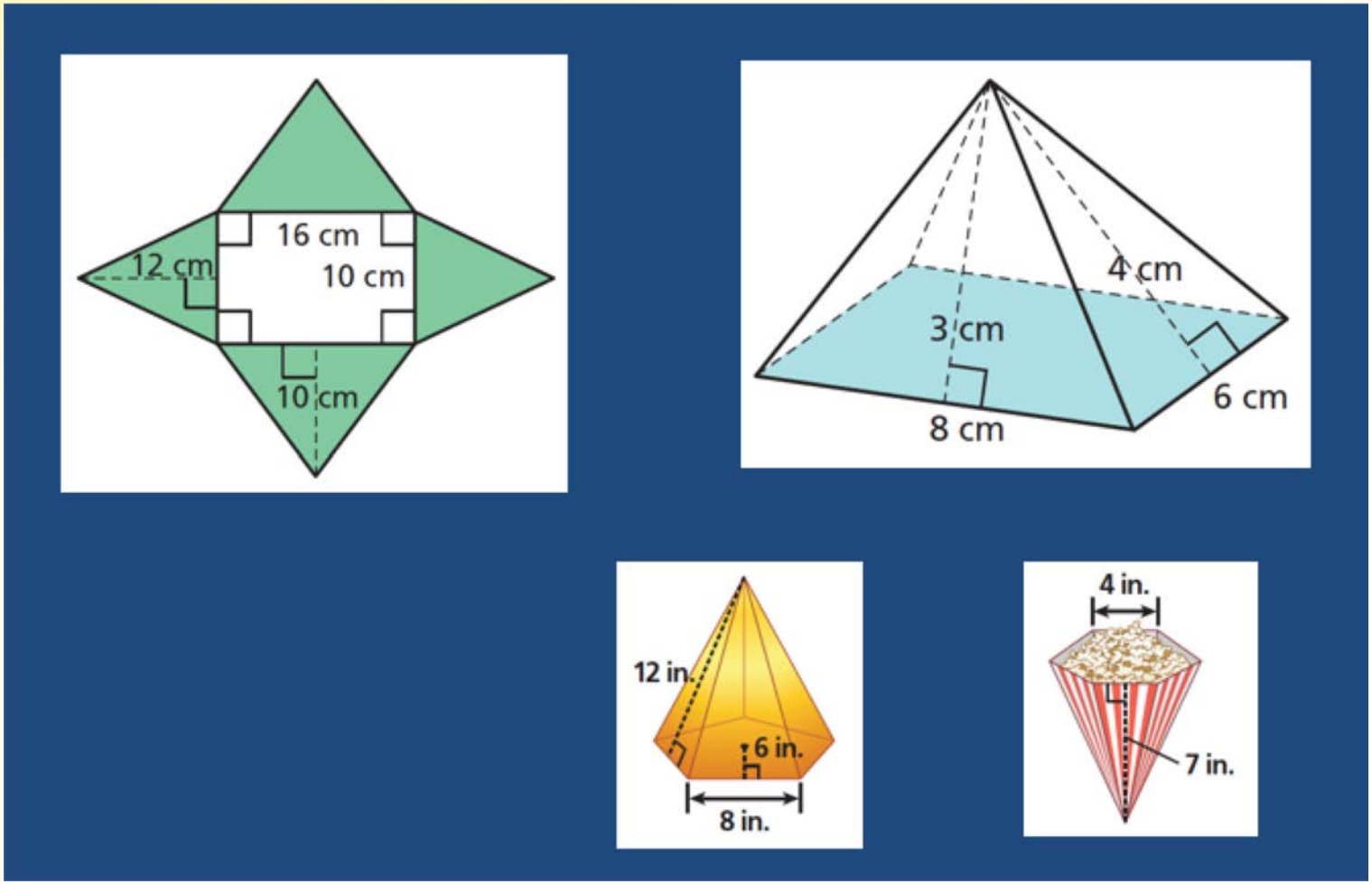
Volume Grades 5 and 6





Seeing square units on different shapes

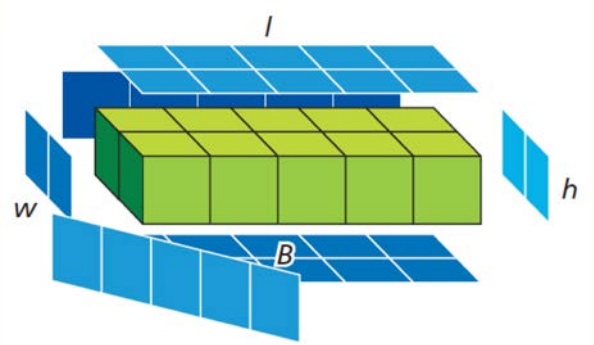
Surface Area Grade 6



G6: Students Differentiate Surface Area and Volume of Prisms

Students see and identify the kinds of units used to measure surface area and volume.

- They see the **square units** that make the surface area and review that they write the answer as unit^2.
- They see the **cubic units** that make the volume and review that they write the answer as unit^3.

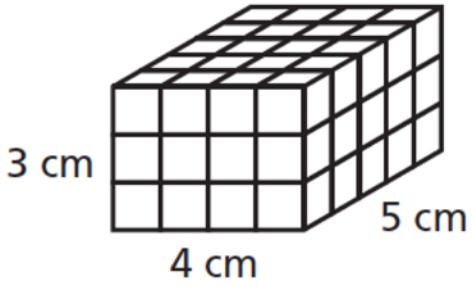


11. What is the surface area and volume of the prism you made?

$SA =$ 34 cm^2 2x5 + 2x2 + 2x2x5 cm^2

$V =$ 10 cm^3 2x5x1 cm^3

16.



$SA =$ 94 cm^2 2(3x4 + 3x5 + 4x5) cm^2

$V =$ 60 cm^3 3x4x5 cm^3

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Length Tools

Conceptual Tools in Measurement, Data, and Fractions

Length Tools

Geometric measurement: Rulers, tape measures

Other measures: Measurement scales for liquid volume and mass

Data: Bar graph scales and line plots

Fractions: Number-line diagrams

Problem solving and ratio: Double number-line diagrams

Coordinate graph scales

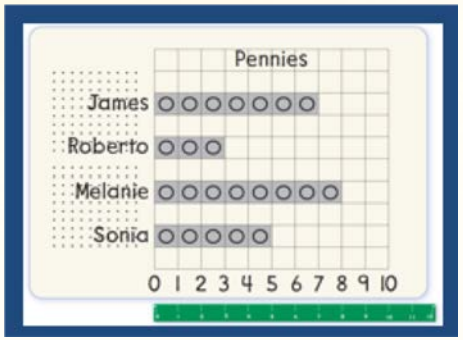
Conceptual Tools in Measurement, Data, and Fractions

Length Tools

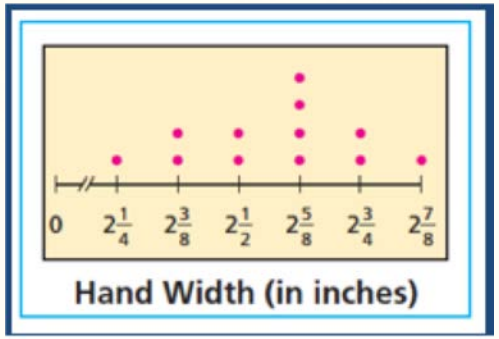
Rulers

Measurement scales for liquid volume and mass

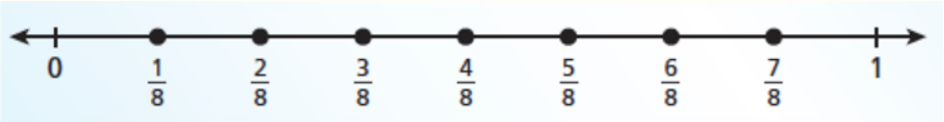
Bar graph scales



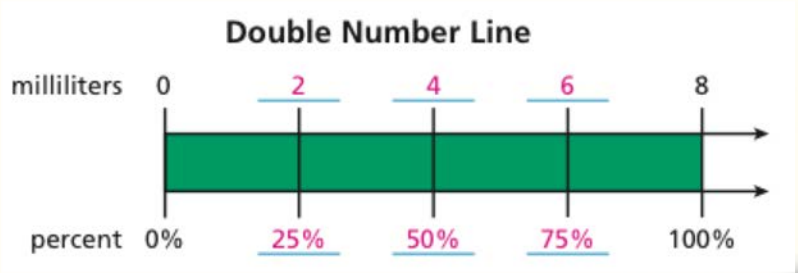
Line plots



Number-line diagrams



Double number-line diagrams



2.MD.1: Measure Lengths in Standard Units Using Tools.

Length tools are visually difficult.

Children are wired to see things, so they see the marks on rulers.

Numbers by the marks draw the eye even more to marks.

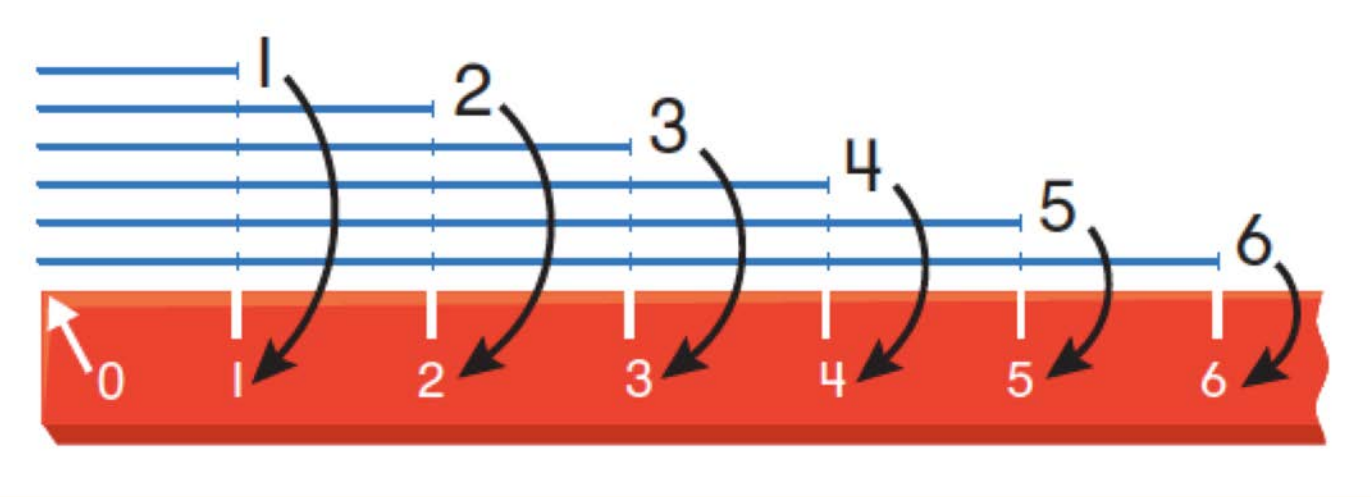
All length tools share this problem.

G2: The Ruler as a Stack of Length Units

Composing length units to make a ruler.

Draw length units one by one marking the ends.

Move a finger along each unit as count the units.



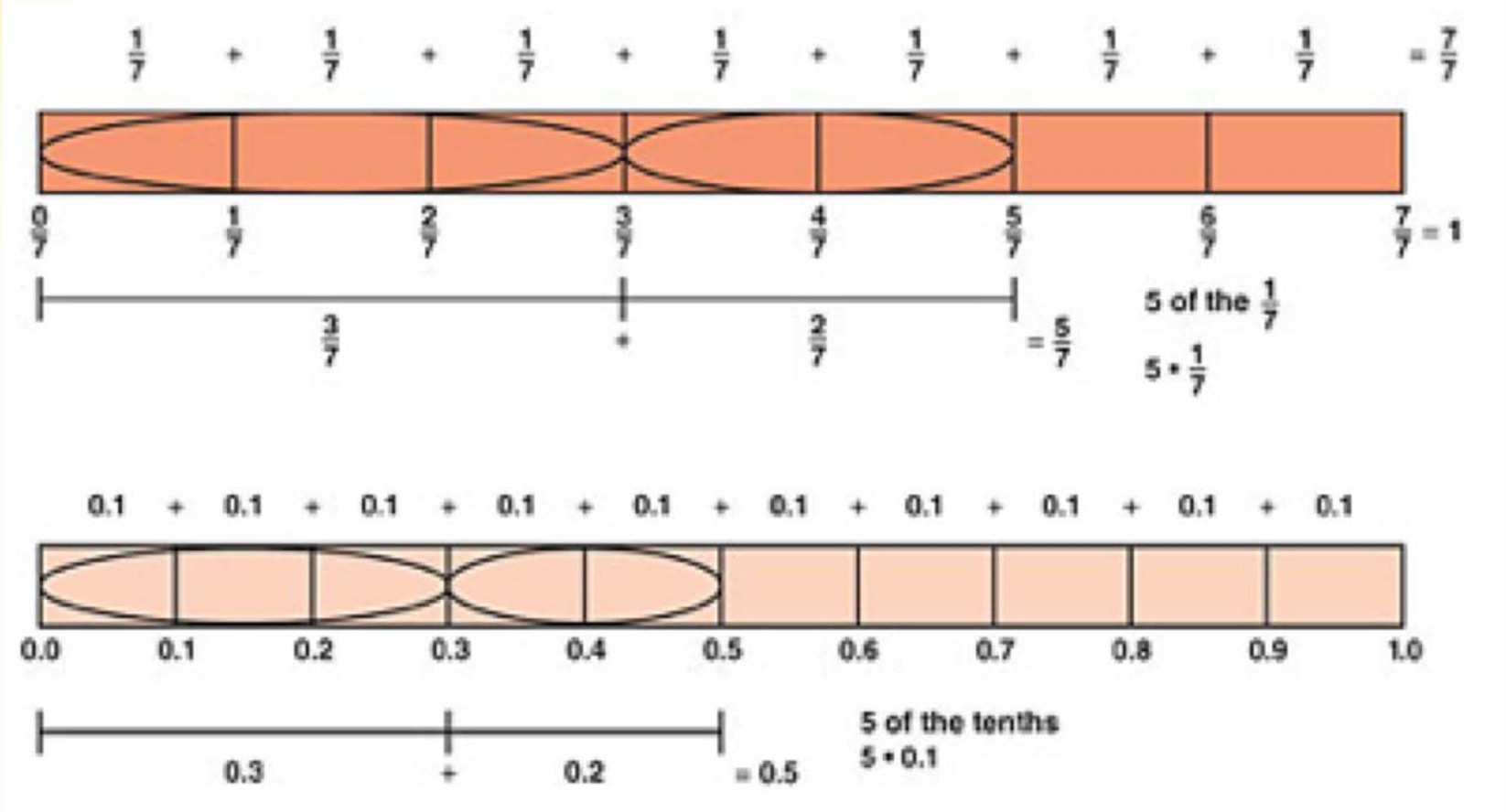
After drawing all of the smaller length totals close to each other, imagine sliding them all on top of each other. All of the lengths are now embedded within the ruler.

2.MD.1: How to Help Students See and Count the Length Units

See and feel the length units.

- Draw length units one by one marking the ends.
- Move a finger along each unit as count the units.
- Color alternating unit lengths to see the lengths.
- Imagine a unit walker has shoes of the unit size and is walking heel to toe to make those units.
- Show length units by holding fingers apart that much (see the invisible length).
- Work with partner lengths.

See lengths by fraction bar labelling and by encircling



G6 Seeing Division as Finding the Unknown Factor in an Equal Groups Situation

2. The mugs at a restaurant hold $\frac{2}{3}$ cup of hot chocolate. The restaurant has $\frac{8}{15}$ cup hot chocolate left in its pot. How many servings of $\frac{2}{3}$ cup are in the pot?

$\frac{4}{5}$ serving

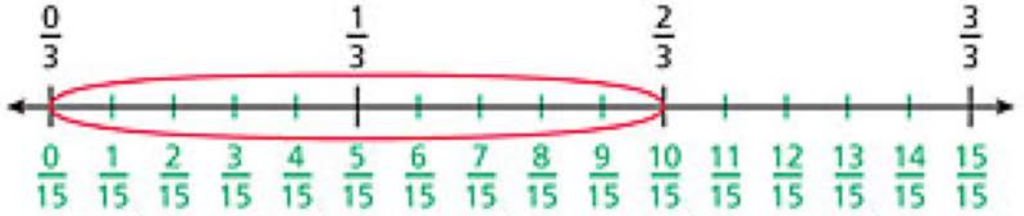
Step 1 Write an equation.

$\frac{?}{?} \cdot \frac{2}{3} = \frac{8}{15}$

Step 2 Look at the denominators.

Divide each $\frac{1}{3}$ into 5 equal parts to make fifteenths.

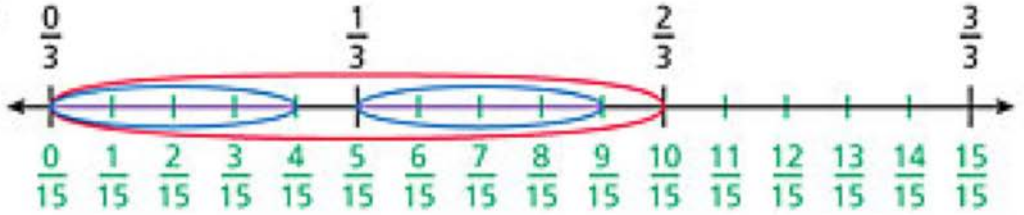
$\frac{?}{5} \cdot \frac{2}{3} = \frac{8}{15}$



Step 3 Look at the numerators.

Take 4 fifteenths from each of the 2 thirds to make $\frac{8}{15}$.

$\frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15}$



Length models do not show place value well.

And they limit computation methods to more difficult methods.

2.MD.6: Lengths, Sums, and Differences on a Number Line Diagram

This standard is often misunderstood:

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

This must be an accurate number line diagram within 100. Students cannot make these diagrams because each point is less than 2 mm apart.

An "open number line" is NOT a number line diagram because the scale is not accurate.

This standard does NOT say that students must use the number line diagram to add and subtract. Students only need to represent lengths, sums, and differences on a number line diagram.

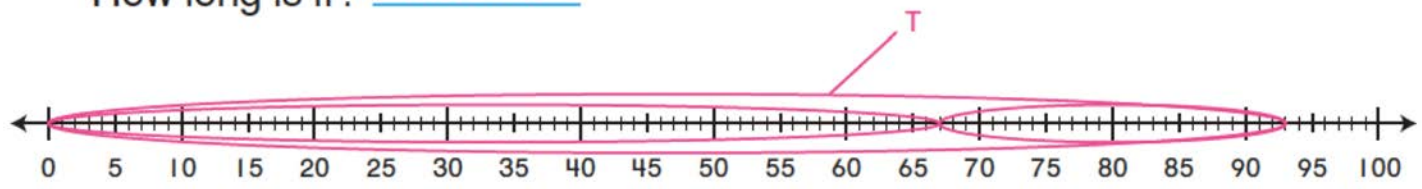
2.MD.6: Lengths, Sums, and Differences on a Number Line Diagram

Students in a late unit represent whole-number lengths, sums, and differences within 100 on number line diagrams on the student activity book page.

They do this because they cannot draw such diagrams accurately.

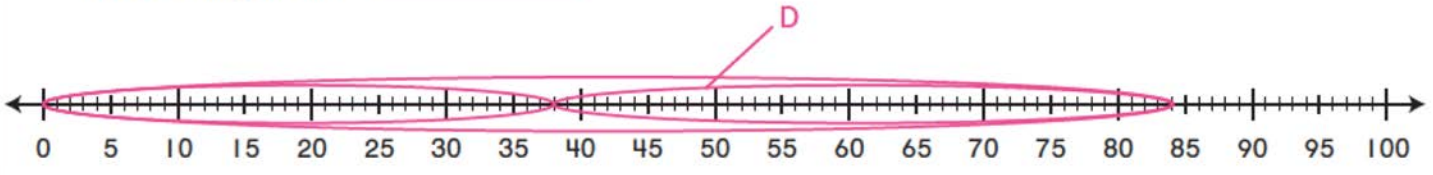
16. Loop 67. Add 26 to it. Loop the total *T*.

How long is it? 93 units



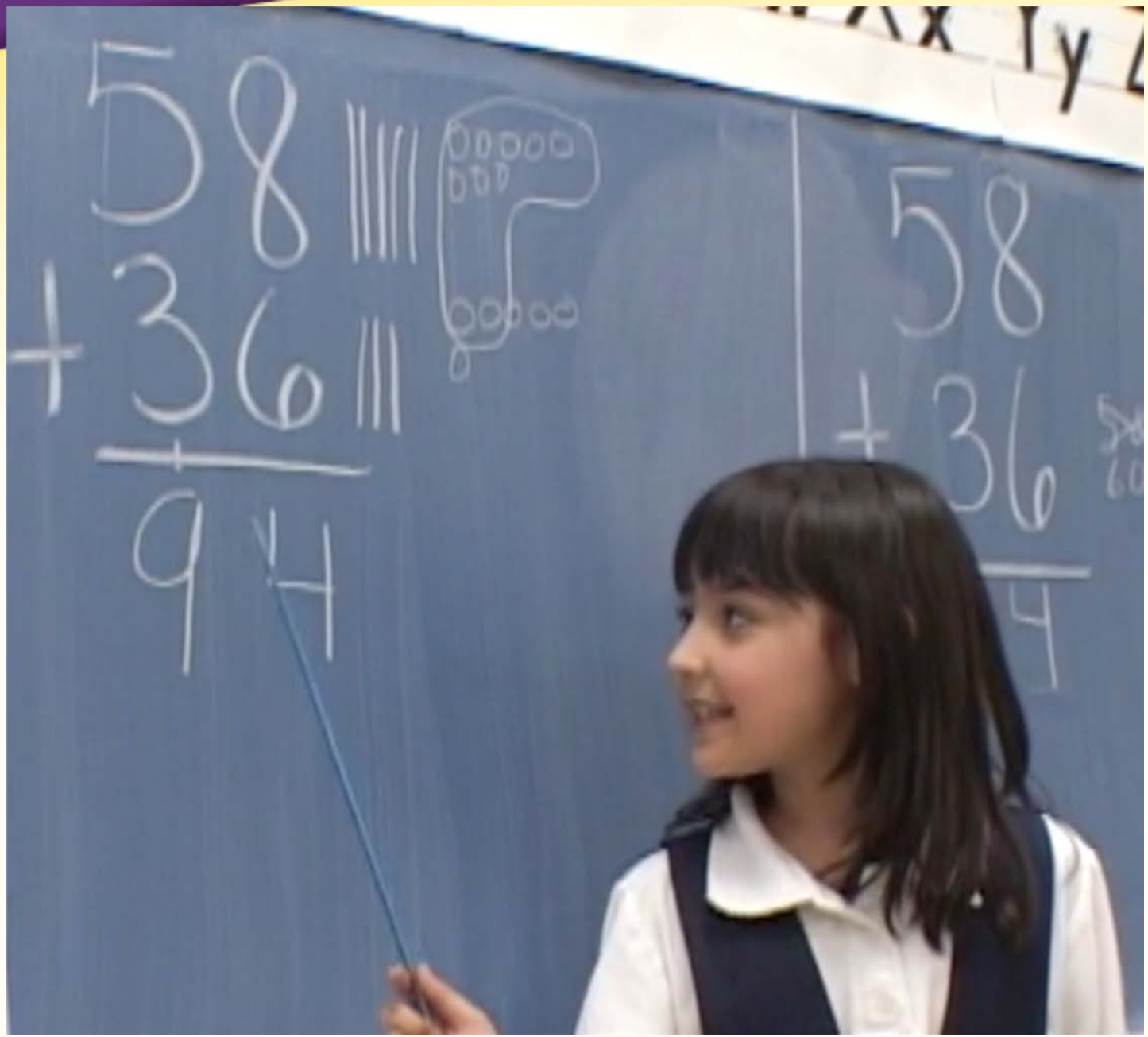
15. Loop 38 and 84. Loop the difference *D*.

How long is it? 46 units



Addends are inside the sums, and subtracting is finding an unknown addend.

G1: Use 10-sticks and ones-circles to show the addends



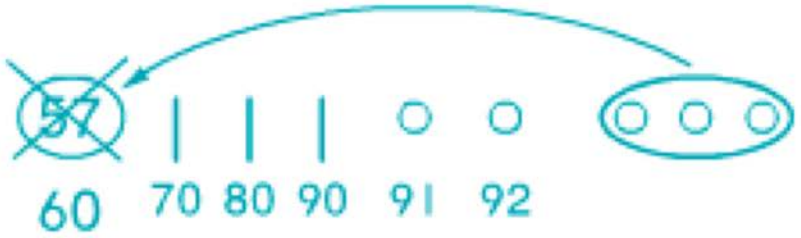
Count On and Add On Methods Do Not Generalize Easily

In Grade 2 count on and add on methods for 3-digit numbers are not emphasized because they

- are difficult for many children and
- do not generalize easily to larger numbers.

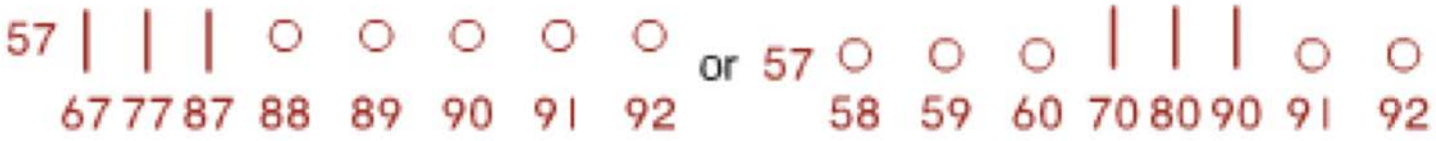
It is easier to use tens and ones drawings than a number line diagram for keeping track of the adding on for 2-digit and for 3-digit problems.

Make a Ten from One Number **57 + 35**



"35 gives 3 to 57 to make 60. 60 and 32 is 92."

Counting On By Tens



For more about what is ”the standard algorithm” and the best methods for adding and subtracting

see Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 14 (2),*14-30.

and Fuson, 2018, about limitations of length models for place value understanding and adding/subtracting

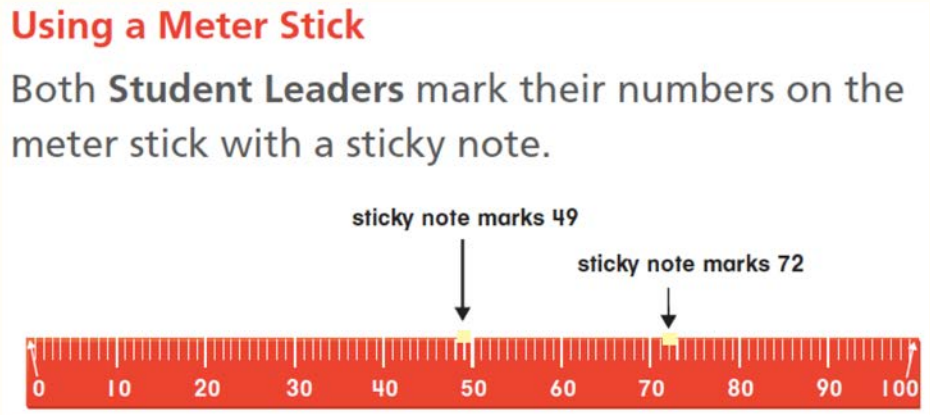
at karenfusonmath.com

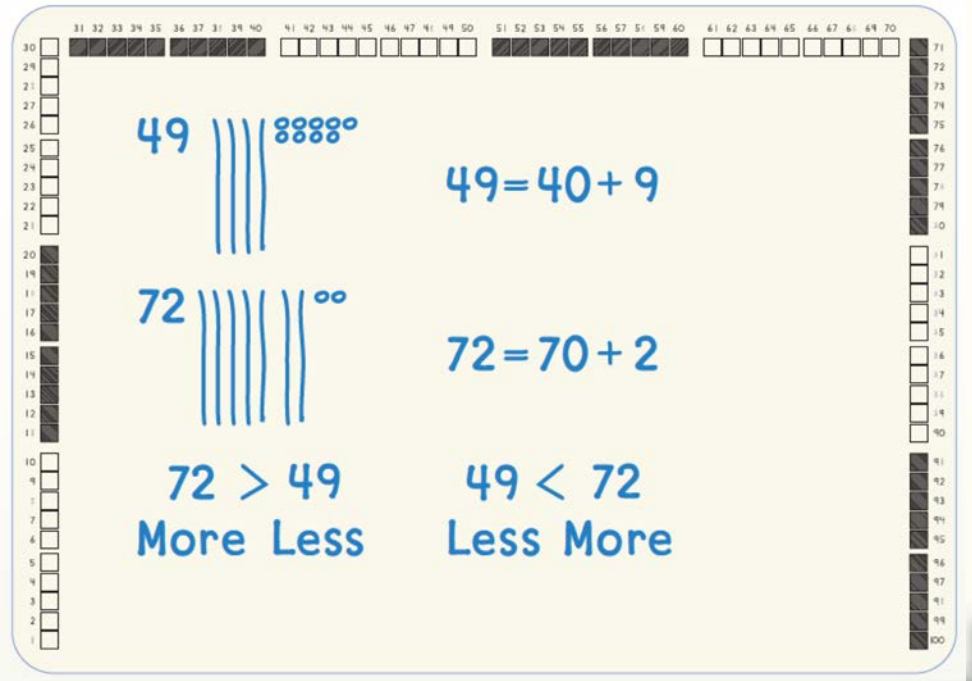
2.MD.6: Lengths on a Number Line Diagram as a Meter Stick

A meter stick showing 100 centimeters is a number line diagram.

Unit 2 Daily Routines show and compare two numbers by children

- putting sticky notes on a meter stick to show the two lengths,
- showing the numbers in ten-sticks and ones and as decades and ones,
- flashing ten fingers and then ones to show each number,
- saying a full comparing sentence in both ways.

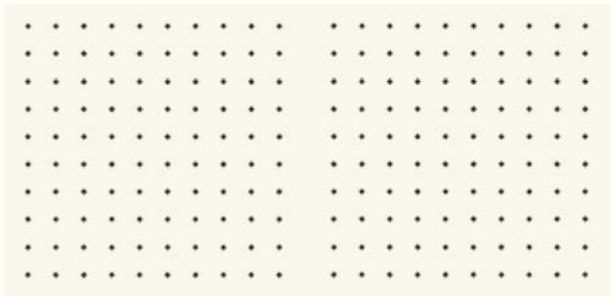


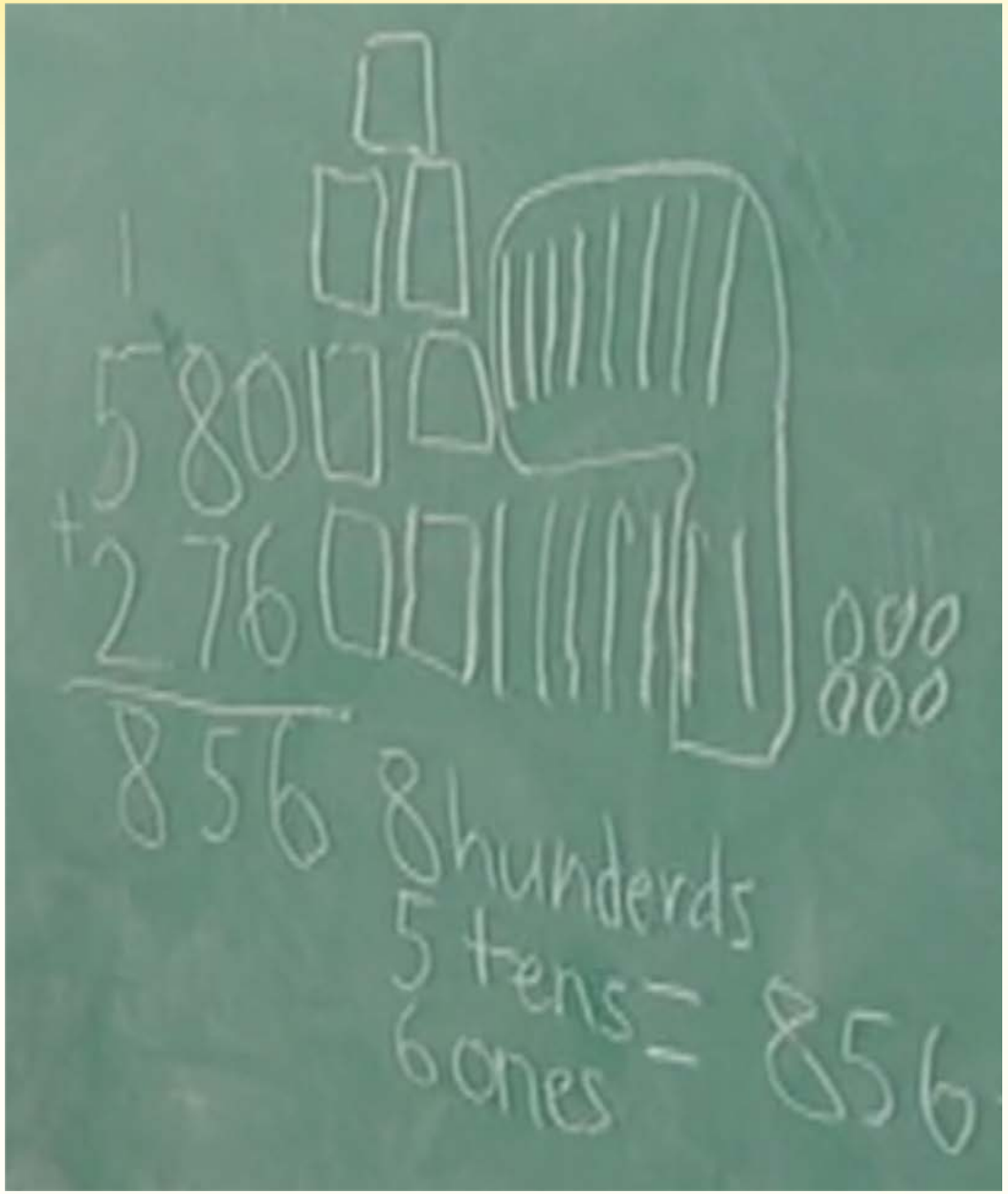


Can relate and see place values with 100-boxes,quick-tens, and quick-ones

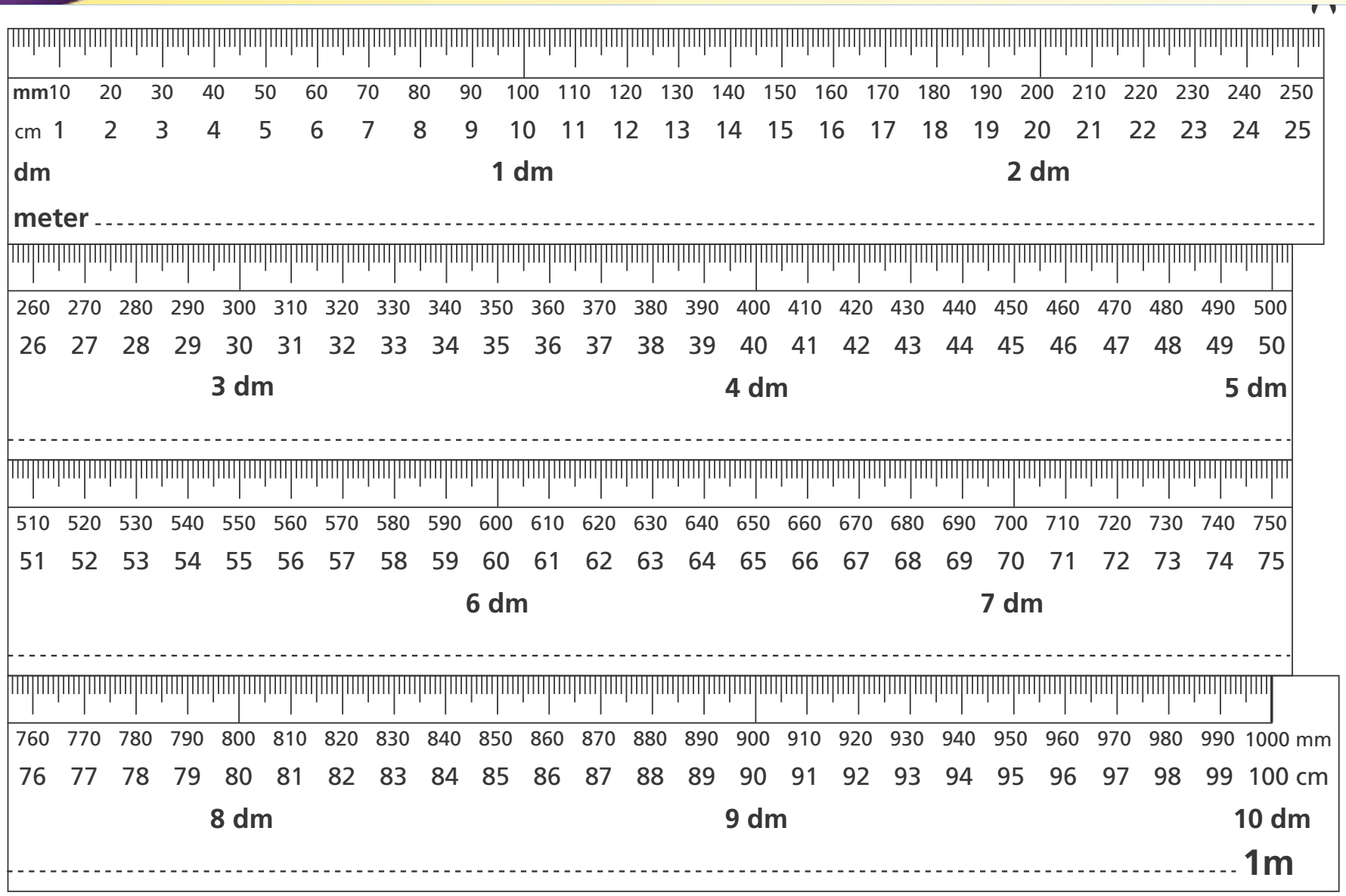
Children draw vertical 10-sticks on columns to see the ten hiding in a quick-ten

and see 100 as ten tens and as 100 dots.





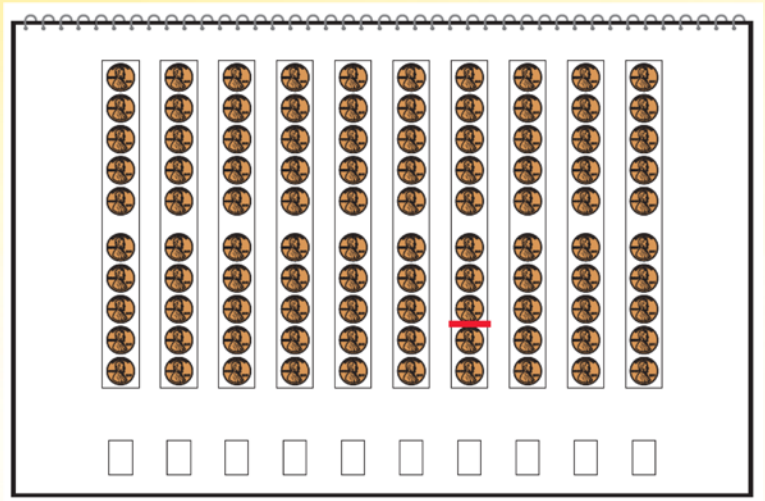
G5 Using Metric Length to Understand Decimals

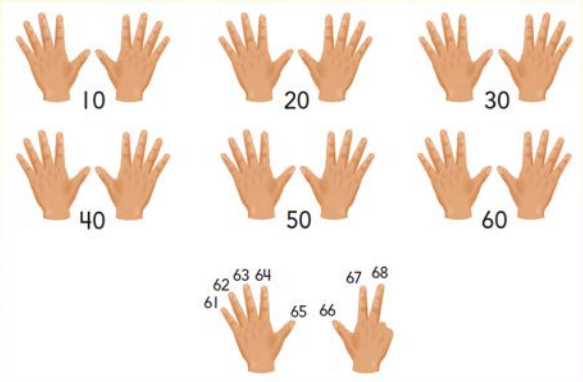


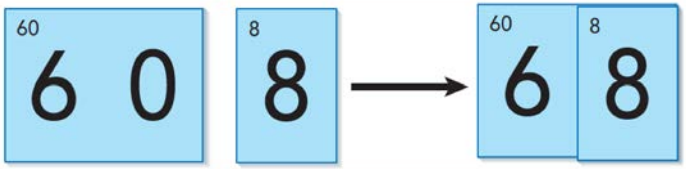
Using money to build place value knowledge and computation for decimal numbers

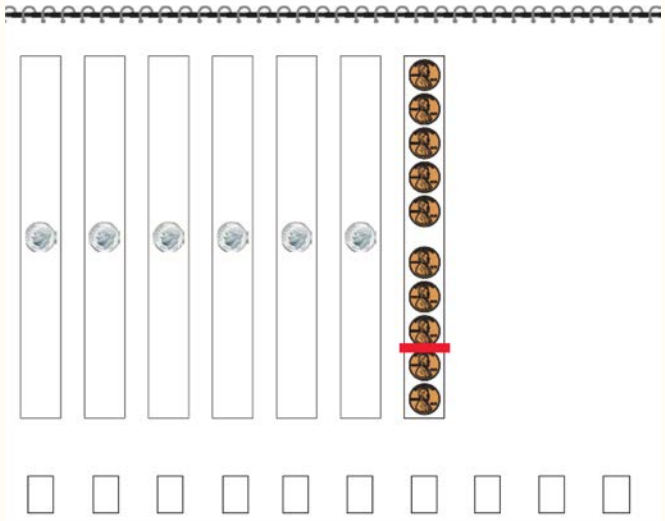
G2: Counting on the Money Flip Chart is Done First by Columns of Ten Pennies and Then by Dimes

Students flash ten fingers as they count by tens on the penny side of the strips on the Money Flip Chart.







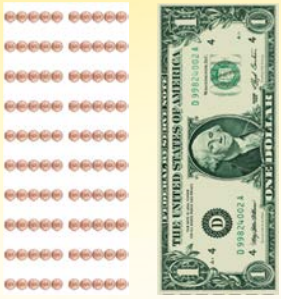


Later students flash ten fingers as they count by tens on the dime side of the strips on the Money Flip Chart.

Both kinds of counting mean that they must shift from counting by tens to counting by ones.

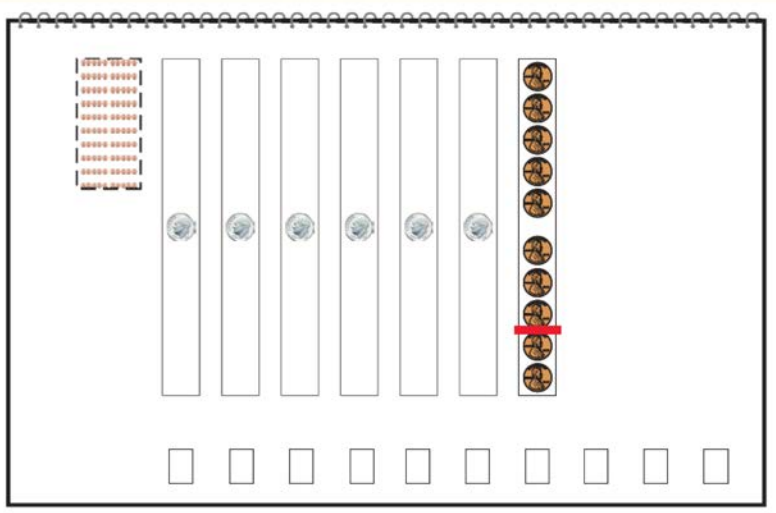
G2: Students Learn to Count from 100 to 200 Using Dime Strips and a Dollar Showing 100 Pennies

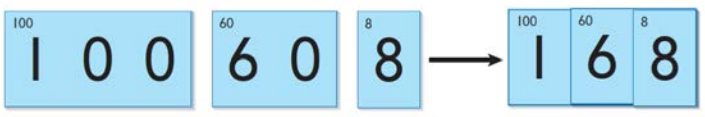
Students see the back of one dollar as ten rows of pennies to remind them that one dollar is equal to 100 pennies.

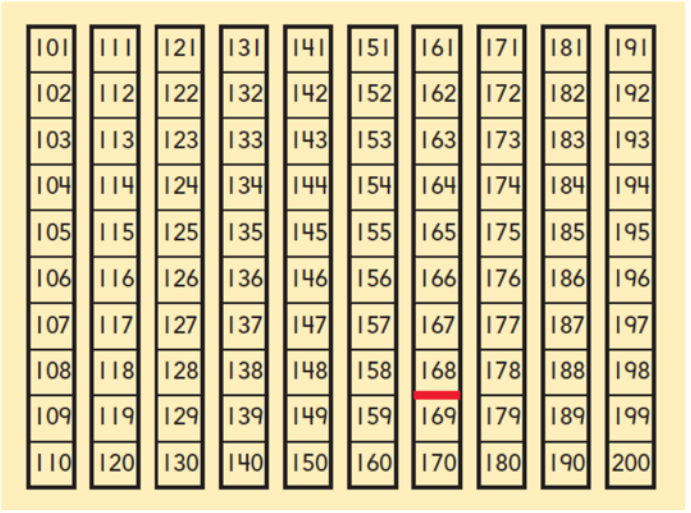


This dollar is put to the left of the Money Flip Chart, and students count from 100 to 200 by tens and by ones.

Students do the same counting by tens and by ones on the 101 to 200 Poster.





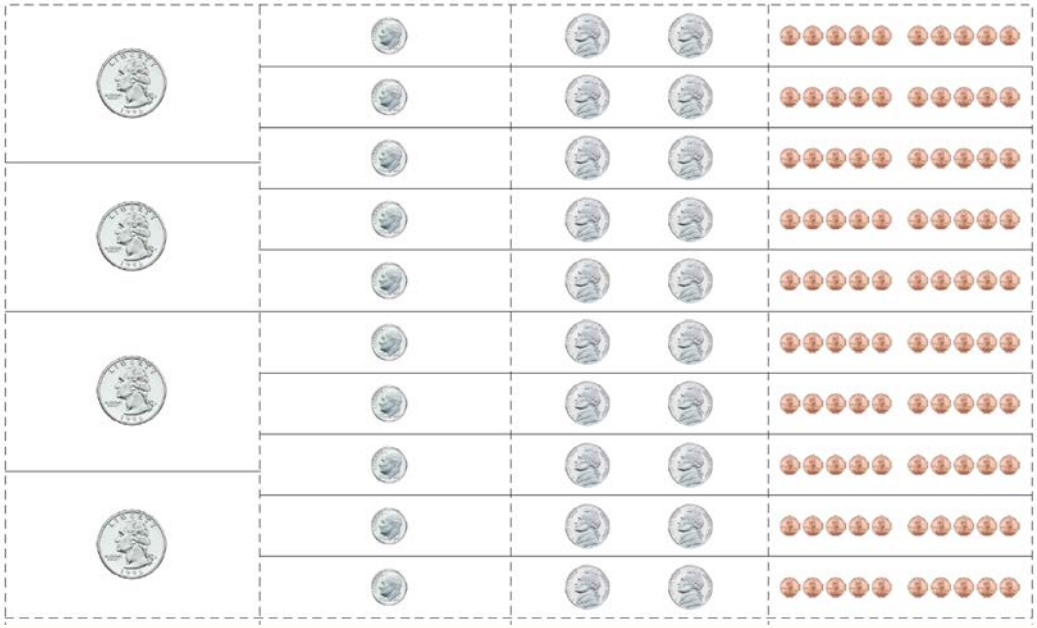


G2: Students See That 4 Quarters, 10 Dimes, 20 Nickels, and 100 Pennies Make One Dollar

Students are introduced to quarters by showing 4 quarters on the back of a dollar bill.

Students discuss what they know about quarters and see that 4 of them equal 1 dollar.

They also see 10 dimes, 20 nickels, and 100 pennies on the back of a dollar so they can relate these coins and see other coins that make a quarter.



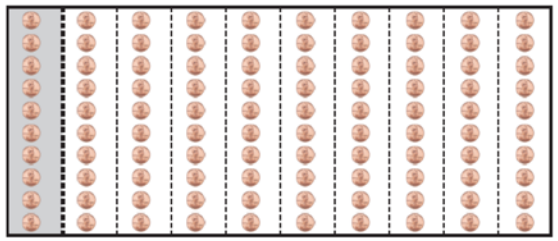
G4: Students Develop Fraction and Decimal Concepts Using Money

A penny is a hundredth of a dollar.
A dime (ten pennies) is one tenth of a dollar.

1 penny $= \frac{1}{100} = 0.01$

$\frac{10}{100}$ 10 of 100 equal parts

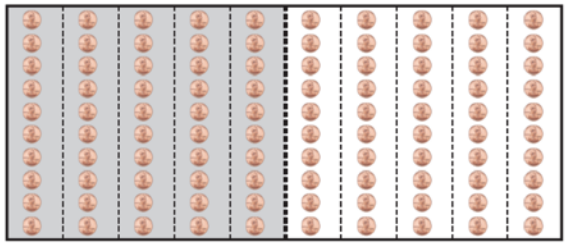
$\frac{1}{10}$ 1 of 10 equal parts



0.1

0.10

$\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$



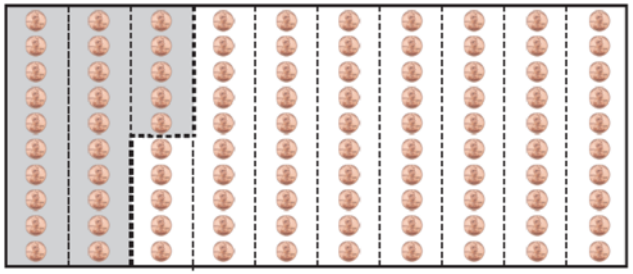
$0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 0.5$

$0.10 + 0.10 + 0.10 + 0.10 + 0.10 = 0.50$

A quarter is 25 hundredths of a dollar and also ¼ of a dollar.
Three quarters are 75 hundredths of a dollar.

$\frac{10}{100} + \frac{10}{100} + \frac{5}{100} = \frac{25}{100}$

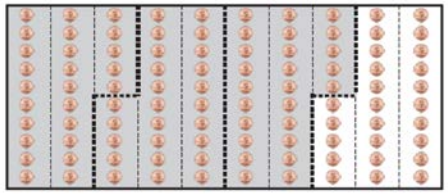
$\frac{1}{10} + \frac{1}{10} + \frac{5}{100} = \frac{25}{100}$



$0.1 + 0.1 + 0.05 = 0.25$

$0.10 + 0.10 + 0.05 = 0.25$

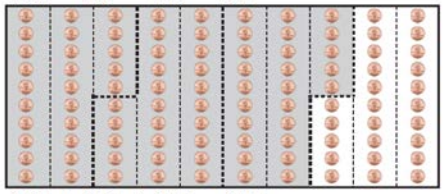
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ 3 of 4 equal parts



$0.25 + 0.25 + 0.25 = 0.75$

$\frac{25}{100} + \frac{25}{100} + \frac{25}{100} = \frac{75}{100}$

$\frac{25}{100} + \frac{25}{100} + \frac{25}{100} = \frac{75}{100}$

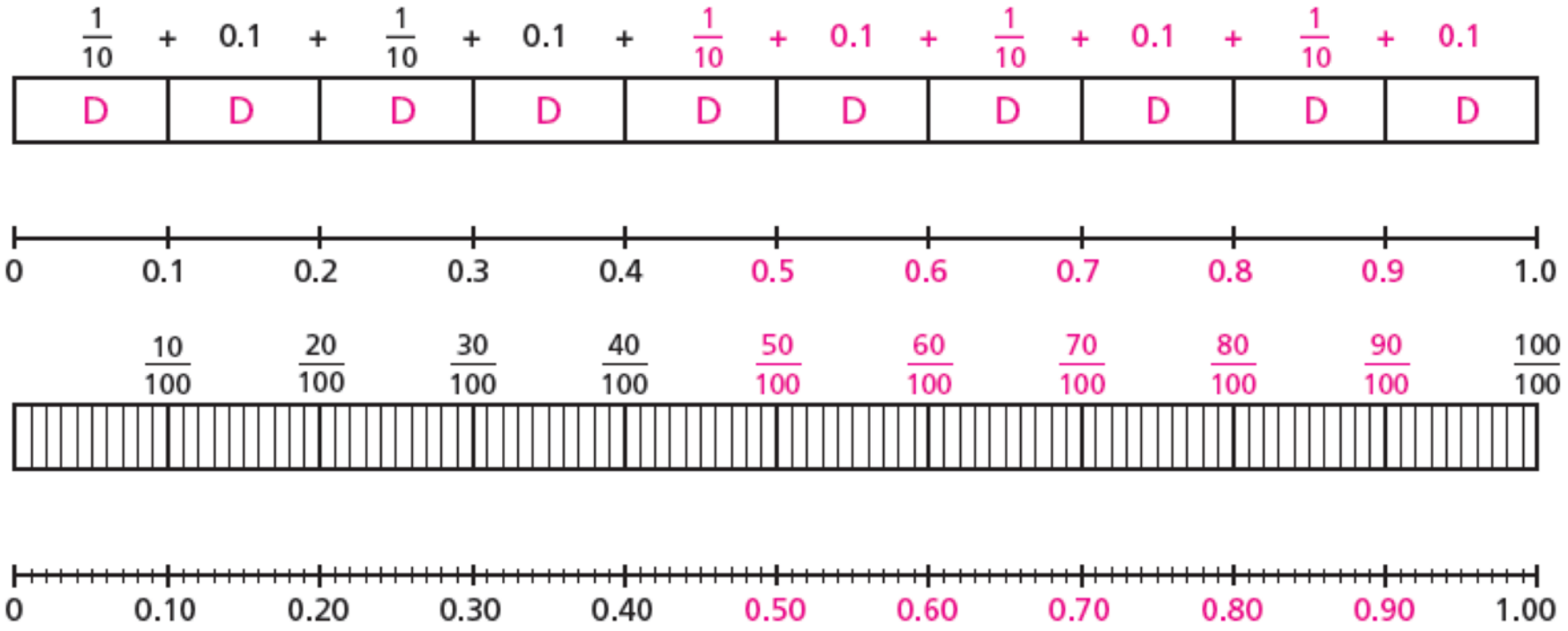


$0.25 + 0.25 + 0.25 = 0.75$

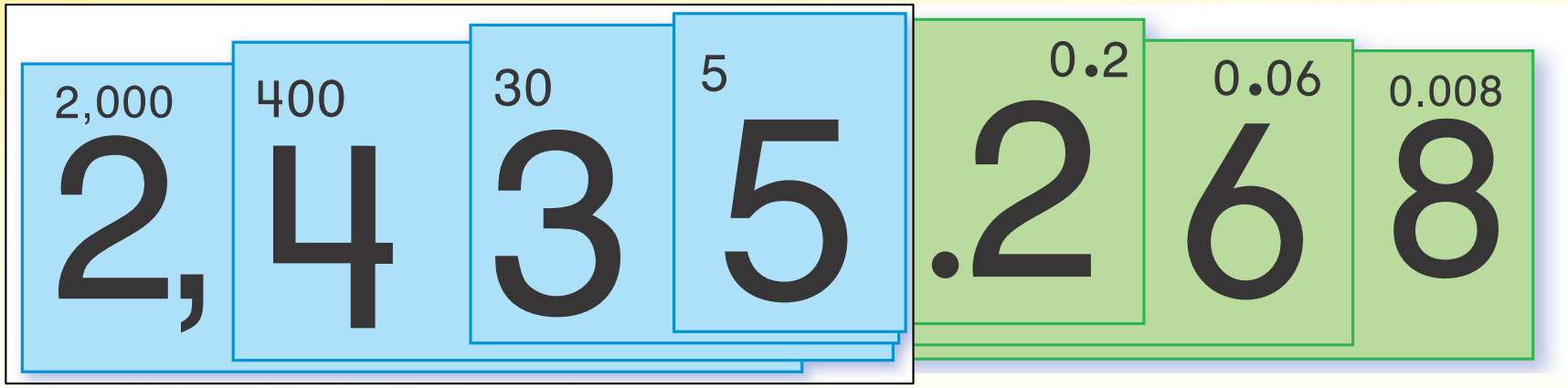
G4 Length Models for Decimals and Fractions

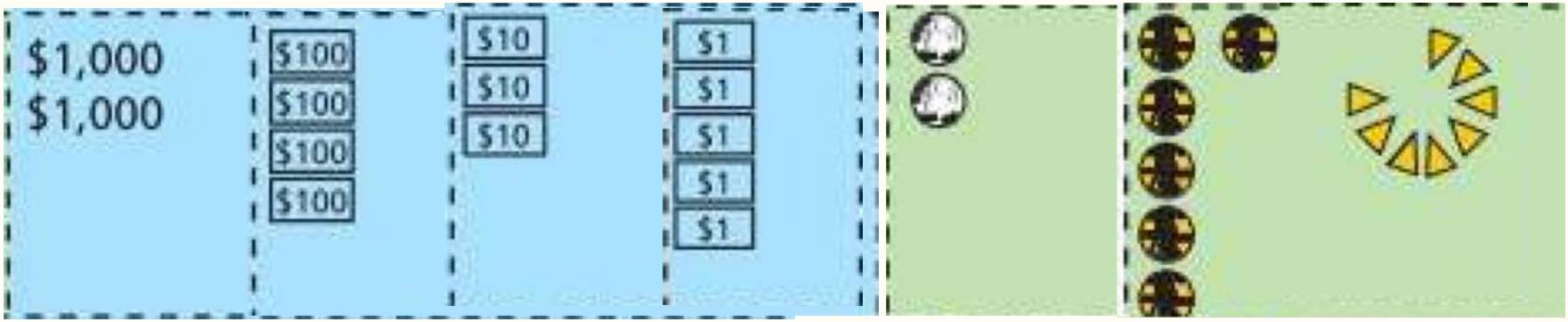
► Understand Tenths and Hundredths

Answer the questions about the bars and number lines below.

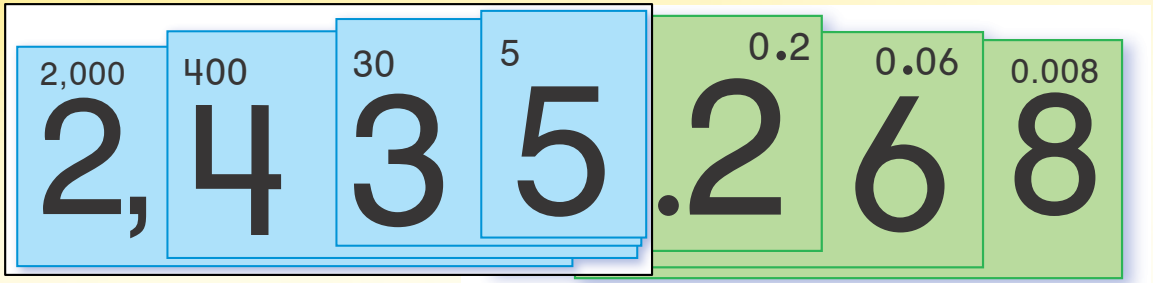


G5 Secret Code Cards Show Money Values

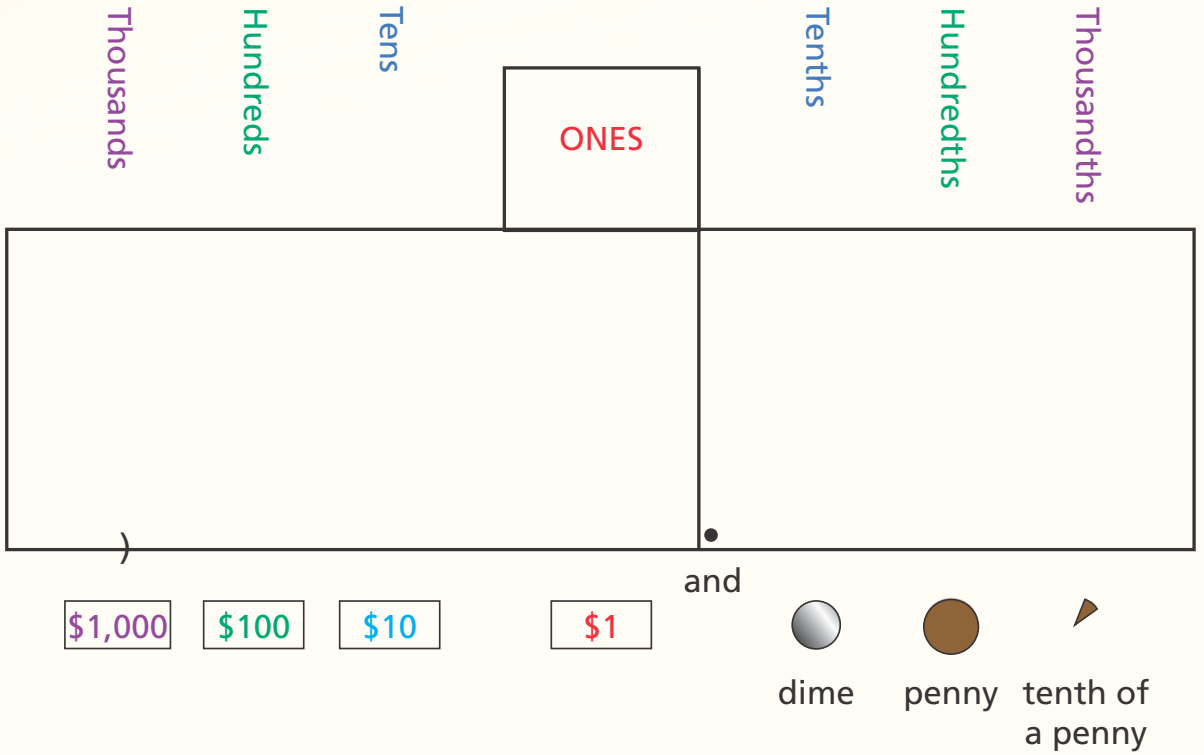




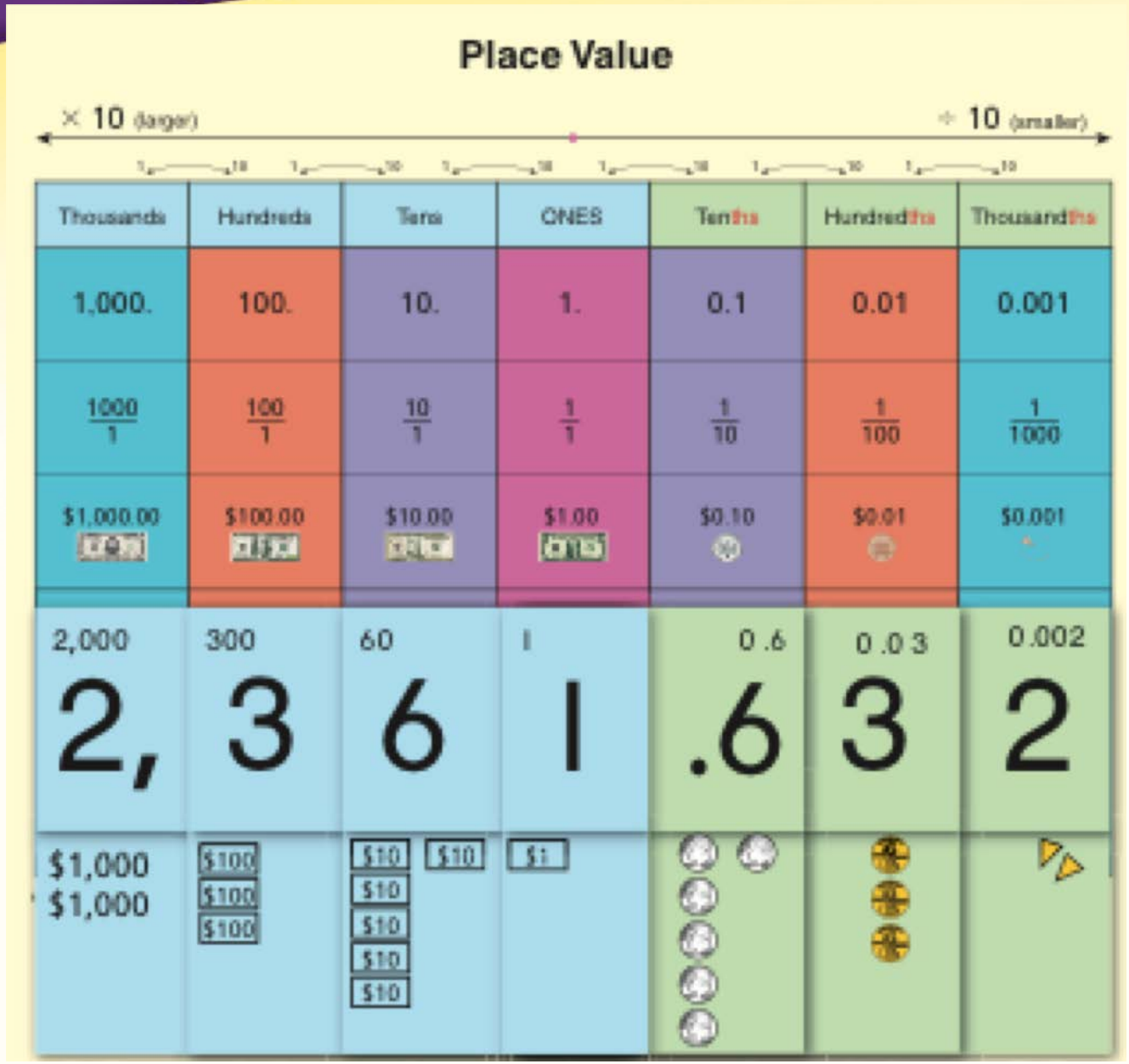
G5 Secret Code Cards Whole Numbers and Decimals



Use your Secret-Code Cards to make numbers on the frame.



G6 Relating Money Values, Decimals, and Fractions



G6 Relating Money Values, Decimals, and Fractions

$2{,}361.632 = 2{,}000 + \underline{300} + \underline{60} + \underline{1} + \underline{0.6} + \underline{0.03} + 0.002$

$0 \quad .6 \quad 3 \quad 2$

$+ \frac{6}{10} + \frac{3}{100} + \frac{2}{1{,}000}$

$+ \frac{600}{1{,}000} + \frac{30}{1{,}000} + \frac{2}{1{,}000}$

$$\begin{array}{rcccc} & 0 & .6 & 0 & 0 \\ + & 0 & .0 & 3 & 0 \\ + & 0 & .0 & 0 & 2 \\ \hline & 0 & .6 & 3 & 2 \end{array}$$

G5 A Whole Number Times a Decimal

Cost of a Red Phantom Marble

$\_\_\_\_ \_\_\_\_ \_\_\_\_ . 4 1 2$

× 1

1 × $0.412 = $0.412

×

10 Red Phantom Marbles

$\_\_\_\_ \_\_\_\_ 4 . 1 2 \_\_\_\_$

× 10

10 × $0.412 = $4.12

100 Red Phantom Marbles

$\_\_\_\_ 4 1 . 2 0 \_\_\_\_$

× 100

100 × $0.412 = $41.20

1,000 Red Phantom Marbles

$ 4 1 2 . 0 0 ____

× 1,000

1,000 × $0.412 = $412.00

OA Problem Situations

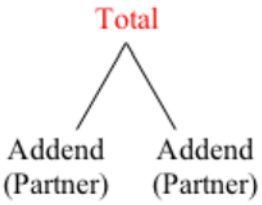
The 6 Situations

K — Add To / Take From

Start + Change = Result
Start – Change = Result

⟶ (becomes)

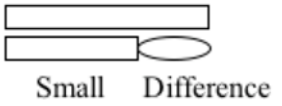
K — Put Together / Take Apart



T = A + A

$\overset{i}{=}$ (identical)

Gr1 — Additive Comparison



Small + Difference = Big
Big – Difference = Small
Big – Small = Difference

$\overset{n}{=}$ (same number)

Gr3 — Equal Groups

G + G + G + G = P
M times

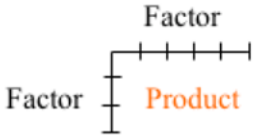
M x G = P

⟶ (becomes)

Gr3 — Rectangular Everything Times Everything

Array

Area



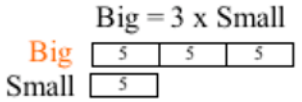
(Long Division Format)

F x F = P
P ÷ F = F

(table)

$\overset{i}{=}$ (identical)

Gr4 — Multiplicative Comparison



Small = $\frac{1}{3}$ x Big

Big ÷ 3 = Small

$\overset{n}{=}$ (same number)

Connecting Geometric Measurement, Other Measures, and Data Use Standards to Each Other and to Other CCSS Domains

Professor Emerita Karen C. Fuson

Northwestern University

Paper presented at the Annual Conference of the National Council of Supervisors of Mathematics, 2018, Washington, D.C.

For more details about all CCSS domains including Measurement and Data, please see the 18 hours of audio-visual Teaching Progressions I have made. You can find links to these and to papers and other presentations at karenfusonmath.com

This presentation is also posted there.