Deepening and Connecting Geometry, Geometric Measurement, and Operations and Algebraic Thinking (OA) Within and Across the Grades

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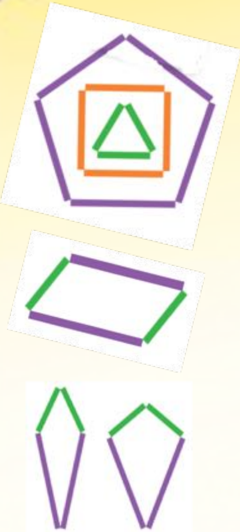
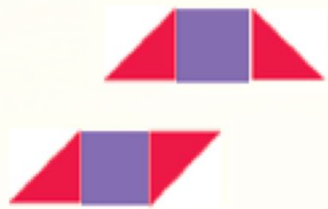
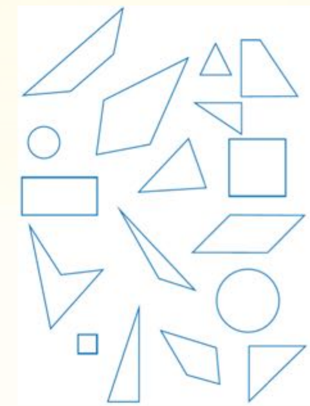
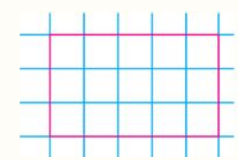






Paper presented at the Annual Conference of the National Council of Supervisors of Mathematics, 2019, San Diego, CA



Please see my website karenfusonmath.com for

- **the 18 hours of audio-visual Teaching Progressions for all CCSS domains I have made, and**
- **for my papers, classroom videos, and presentations including this one.**

Connecting Geometric Measurement, Other Measures, and Data Use Standards to Each Other and to Other CCSS Domains

Professor Emerita Karen C. Fuson

Northwestern University

Paper presented at the Annual Conference of the National Council of Supervisors of Mathematics, 2018, Washington, D.C.

For more details about all CCSS domains including Measurement and Data, please see the 18 hours of audio-visual Teaching Progressions I have made. You can find links to these and to papers and other presentations at karenfusonmath.com

This presentation is also posted there.

Geometry often is neglected in U.S. math classes.

Geometry understanding develops differently than for number and measures, which get larger and smaller and with more layers of units.

“Mile wide and inch deep” does not apply as strongly to geometry.

Geometry is a rich domain for exploring and articulating relationships.

Children who are weak in number may be strong in geometry.

We need to find ways to spend more time on geometry in ways that will build understandings for the major geometric measurement topics as well as for the geometric properties and classification of shapes.

I will discuss 4 such ways in this talk.

I am NOT NOT NOT suggesting that we need to change the standards but rather to focus on crucial learning paths.

Geometry Content in MD Measurement/Data

	K	1	2	3	4	5
	MD Measurement and Data: K to 5					
Length	**Geometric Measurement: K to 6** Use length to make area and volume units					
	(Describe attributes)	(Length)	Length	Area	Angles	Volume [G6 geometry: surface area and area of triangles, special quadrilaterals, and polygons
	Other Measures: K to 5					
Various	(Describe attributes)	(Time)	Time Money	Time Liq volume Mass	Larger to smaller units x	Convert units both ways x ÷
Base ten				Metric liq vol, mass are multiples of ten		
	Represent and interpret data: K to 5					
			Line plots	½ ¼	½ ¼ 1/8	Use fraction operations
Things	Classify into categories, count	Up to 3 categories compare	Picture & bar graphs all problems	Picture & bar graphs scale multiple unit 1- and 2-step compare		

Major Aspects of the Geometry Standards

G Geometry: K to G6 Units Are Shapes

K	1	2	3	4	5
K 1 2: Analyze, name, describe attributes and compose/decompose shapes (continues and used in higher grades)			Classify sub-categories	Classify using properties	Classify in a hierarchy
					Coordinate plane
	Partition a shape into equal shares/parts				[G6 draw polygons on coordinate plane]

Coordinating Geometric Measurement and Geometry Standards

K	1	2	3	4	5
MD Measurement and Data: K to 5					
Geometric Measurement: K to 6 Use length to make area and volume units					
(Describe attributes)	(Length)	Length	Area	Angles	Volume [G6 geometry: surface area and area of triangles, special quadrilaterals, and polygons

K	1	2	3	4	5
K 1 2: Analyze, name, describe attributes and compose/decompose shapes (continues and used in higher grades)			Classify sub-categories	Classify using properties	Classify in a hierarchy
					Coordinate plane [G6 draw polygons on coordinate plane]
	Partition a shape into equal shares/parts				

Use PK, K, 1, 2 Geometry to Prepare for Geometric Measurement

Help children form core geometric seeing frames

K 1 2: Analyze, name, describe attributes

a. Focus on length and area and their units

compose/decompose shapes

b. Focus on area units and on right-angles in shapes

c. See parallelograms and trapezoids as composed from rectangles and right triangles

d. Relate 3- and 4-sided shapes in two major ways:

Two right triangles make a rectangle and a related isosceles triangle

Two acute or two obtuse triangles make a parallelogram

These approaches can support interesting and productive activities.

Two Major Impediments in the Past

Two major impediments for this approach in the past has been

1. the lack of physical materials that support composition/decomposition of rectangles and right triangles and focusing on length and area

The National Research Council Report

Mathematics learning in early childhood: Paths toward excellence and equity

(National Research Council, 2009)

recommended increased use of such materials instead of the almost exclusive use of pattern blocks, which have very limited shapes.

2. The limited list of shapes specified in the geometry grade-level standards along with the reluctance of some programs and districts to go beyond grade-level standards even when it makes mathematical sense to do so.

These two impediments are related.

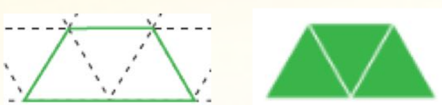
But they can be overcome.

Pattern Blocks Are Composed from Small Equilateral Triangles

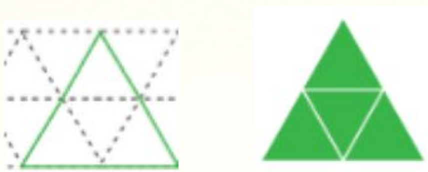
Triangles with all 3 sides of equal lengths are called equilateral triangles.

Equilateral triangles compose to make certain shapes.

2 triangles make a special parallelogram called a rhombus because all 4 sides have equal lengths.

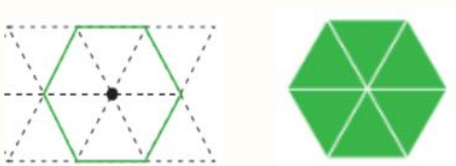


3 triangles make a trapezoid. A trapezoid has 1 pair of parallel sides. Parallel sides go on and on and never meet; they stay the same length apart.



4 triangles make a bigger equilateral triangle.

4 triangles also make a parallelogram. A parallelogram has 2 pairs of parallel sides; opposites sides are parallel.



6 triangles make a hexagon (kindergarteners saw these).

Pattern blocks make these restricted shapes.

Which 2-D Shapes Are in Which Grade Standards?

2-D shapes specifically named in grade-level standards:

K: squares, circles, triangles, rectangles, hexagons (why hexagons now?)

1: rectangles, squares, trapezoids, triangles; half-circles, quarter-circles (why trapezoids and not parallelograms?)

2: triangles, quadrilaterals, pentagons, hexagons (number of sides/angles)

3: rhombuses, rectangles, squares, quadrilaterals (why rhombuses and not parallelograms?)

These shapes are pattern block shapes which are very restricted.

If a shape is introduced, it should be treated generally.

Parallelograms should be introduced with trapezoids to see relationships.

4: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

5: Classify two-dimensional figures in a hierarchy based on properties.

K 1 2: Analyze, name, describe attributes

a. Focus on length and area units

2.MD.1: Measure Lengths in Standard Units Using Tools.

Length and length tools are visually difficult.

Children are wired to see things, so they see the marks on rulers.

Numbers by the marks draw the eye even more to marks.

All length tools share this problem. It is a HUGE PROBLEM and takes a lot of teaching and experience to overcome the problem.

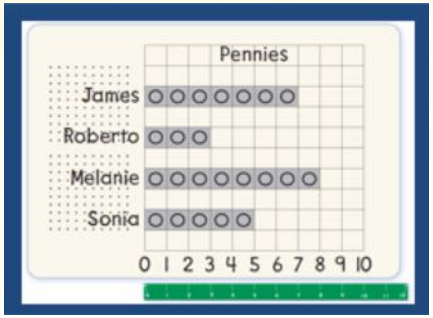
Conceptual Tools in Measurement, Data, and Fractions

Length Tools

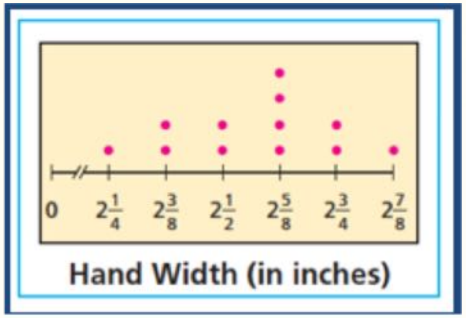
Rulers

Measurement scales for liquid volume and mass

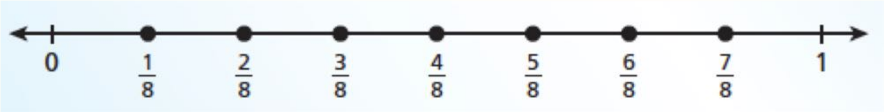
Bar graph scales



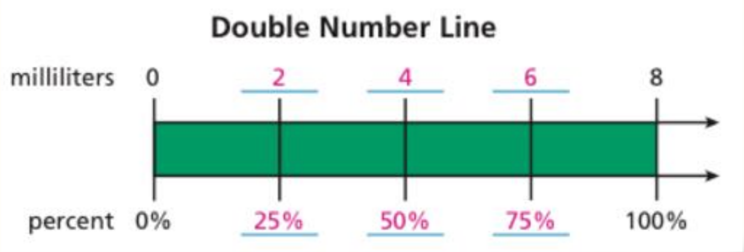
Line plots



Number-line diagrams



Double number-line diagrams



In PK, K, 1, 2 support concepts of lengths

K 1 2: Analyze, name, describe attributes

a. See lengths

b. See length units

Help students see lengths by using lengths to build shapes and modify shapes in different ways:

Do this with physical lengths and then by drawing.

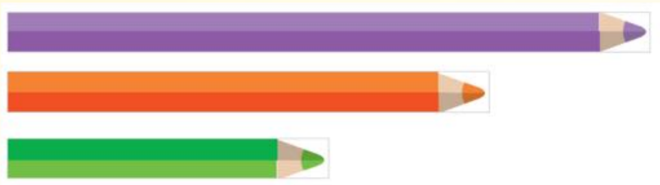
Later use length units to make lengths:
Build up the length units and
decompose lengths by drawing unit marks

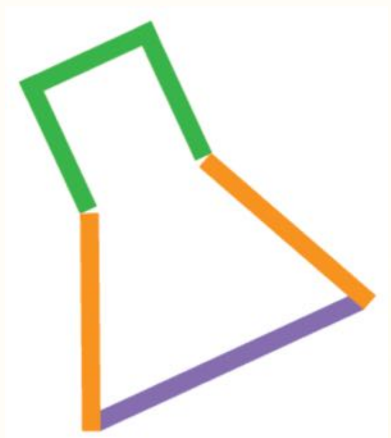
Do this with physical lengths and then by drawing.

Measure Lengths and Make Shapes

Children use punch out strips of 2”, 3”, and 4” to measure and make shapes.

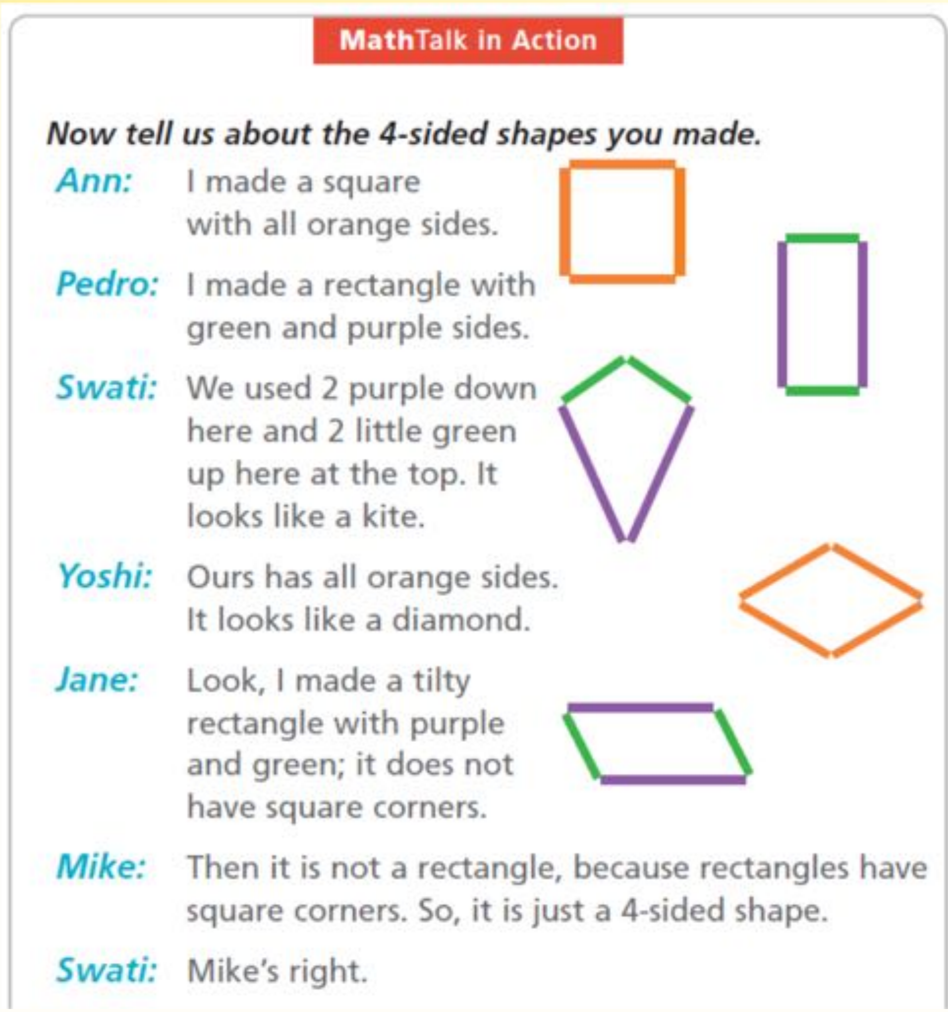
Pencils are on one side and colored strips on the other side.





Make Shapes and Discuss Attributes

The length strips allow children to see "empty" shapes and focus on the sides and their relationships.



MathTalk in Action

Now tell us about the 4-sided shapes you made.

Ann: I made a square with all orange sides.

Pedro: I made a rectangle with green and purple sides.

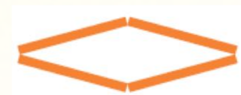
Swati: We used 2 purple down here and 2 little green up here at the top. It looks like a kite.

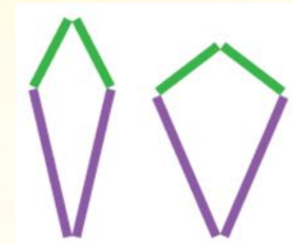
Yoshi: Ours has all orange sides. It looks like a diamond.

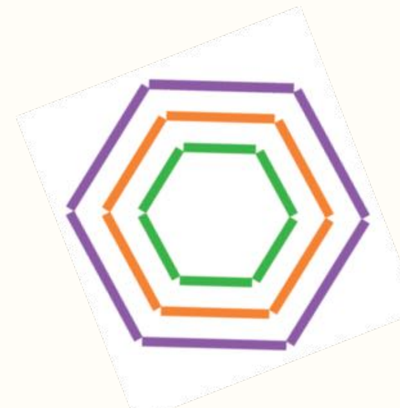
Jane: Look, I made a tilty rectangle with purple and green; it does not have square corners.

Mike: Then it is not a rectangle, because rectangles have square corners. So, it is just a 4-sided shape.

Swati: Mike's right.







Children can make shapes with 3, 4, 5, 6, 7, 8, 9, 10 sides.

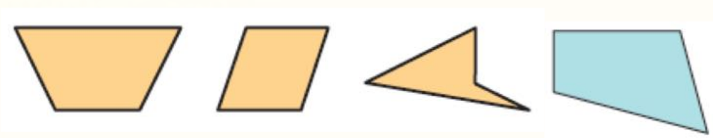
These can have equal or unequal sides.

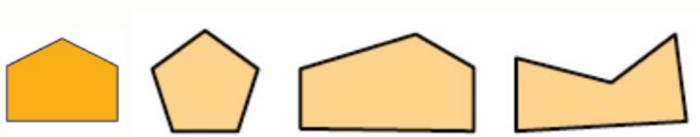
G.2.1: Draw and Discuss Triangles, Quadrilaterals, Pentagons, Hexagons

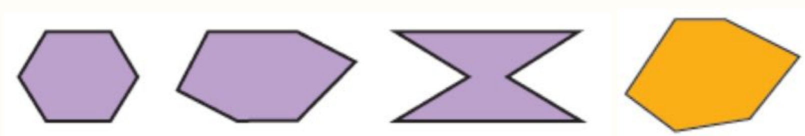
Students draw and discuss shapes with 3, 4, 5, and 6 sides and angles.

triangle	3 sides	3 angles
quadrilateral	4 sides	4 angles
pentagon	5 sides	5 angles
hexagon	6 sides	6 angles

Tri means 3. So triangle is 3 angles.
Quad means 4. **Lateral means side.**
So quadrilateral has 4 sides.
Quad**rangle** has 4 angles.
Gon means angle. **Penta means 5.**
So pentagon means 5 angles.
Hex means 6. So hexagon means 6 angles.







Students enjoy drawing **concave** shapes that have a little cave inside. Students do not have to use the word concave.

Students Open and Close Straws to See Angles Getting Larger and Smaller

Students open straws to see angles getting larger.

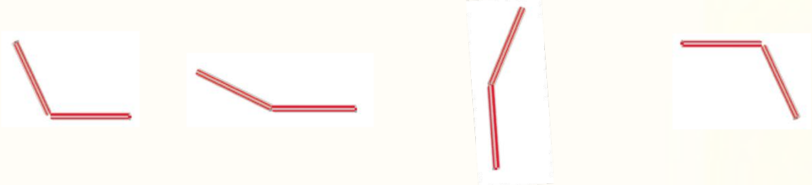
They close straws to see angles getting smaller.

They make right angles and

(and with their arms at the elbow)

angles smaller than right angles

and larger than right angles.



Focus on Area and Area Units

K 1 2: Analyze, name, describe attributes

a. Focus on area units

compose/decompose shapes

b. Focus on area units and on right-angles in shapes

a. See areas

b. See area units (square units)

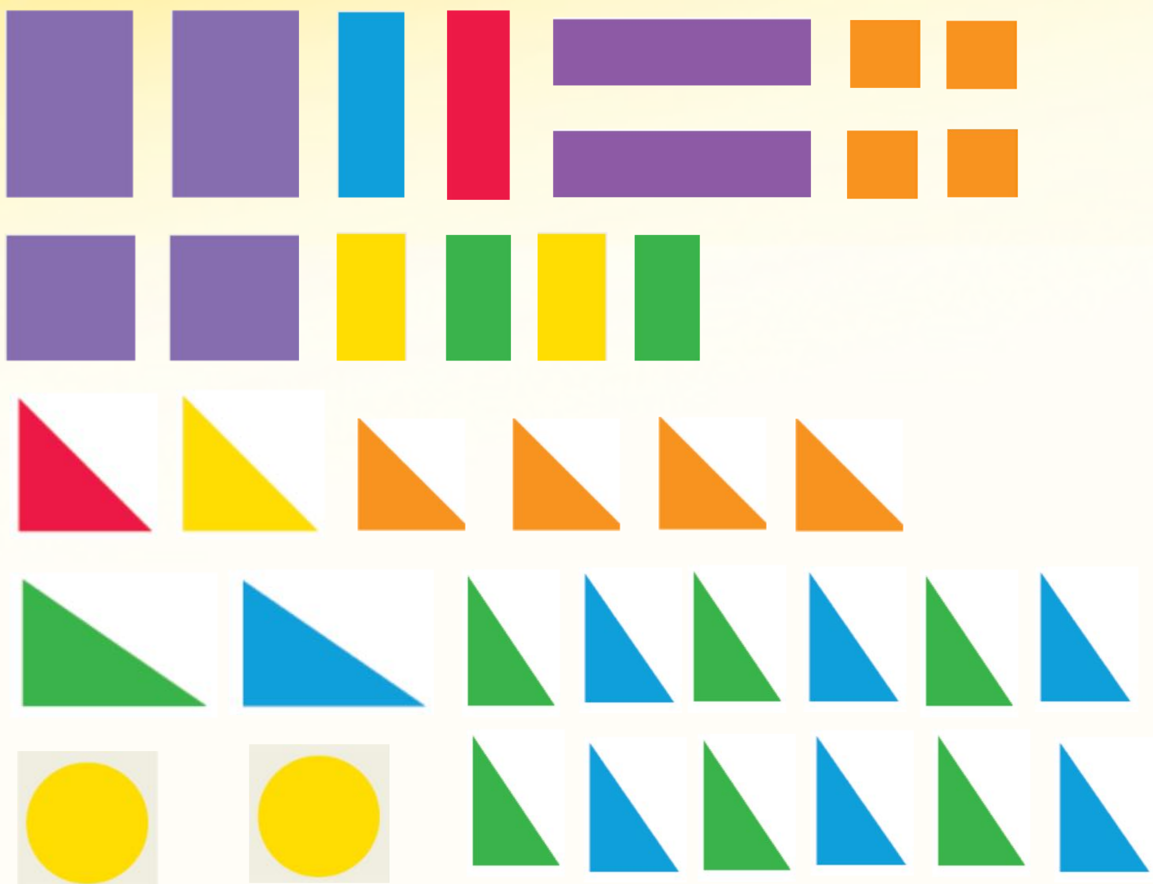
Help students see areas by composing/decomposing 2D shapes using especially right-angled shapes.

Later use square units to make areas:

Do this with physical area units and then by drawing.

Draw on square grids and then build up the area units by drawing on sides of shapes unit length marks that can be connected to draw square area units

An example 2D Shape Set based on 1 inch squares



Rectangles were selected to compose to make each other in multiple ways and to make many other shapes.

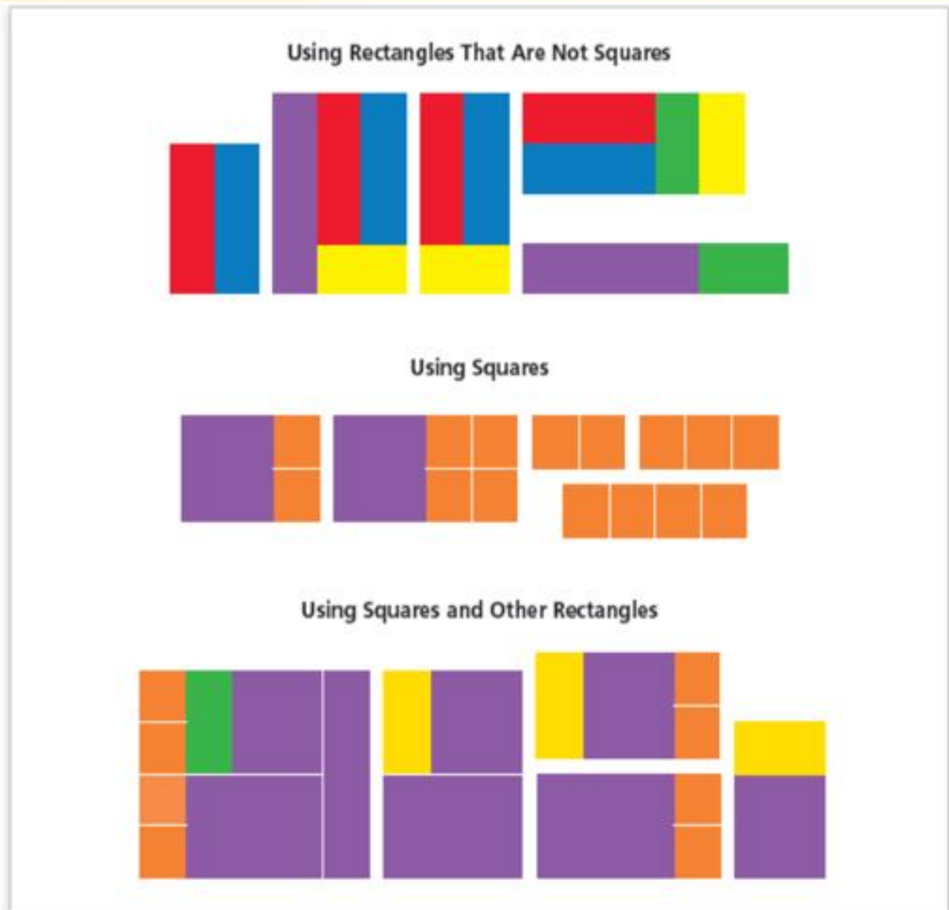
Right triangles compose to make rectangles and other shapes. They also work with the rectangles and with the 1”x 1” right triangles of 6 colors.

Square inches and right triangles made from square inches
are very helpful for various activities including number activities.
I had these made in matching 6 colors.

Cubes are useful for counting and building including seeing layers for volume.

PK, K, 1: Children Compose 2D Shapes to Create a Composite Shape

Children compose to make rectangles that are not square.



Using triangles

MathTalk in Action

What did you find out about making rectangles with the shapes?

Rashida: We can make lots of different rectangles. We made long rectangles and short rectangles.

Pat: I only used the triangles with a square corner to make my rectangles.

Chun: Why couldn't you use the other triangles?

Pat: Rectangles have square corners, so I needed to use triangles with a square corner.

Chun: When you put some triangles together they make a square corner.

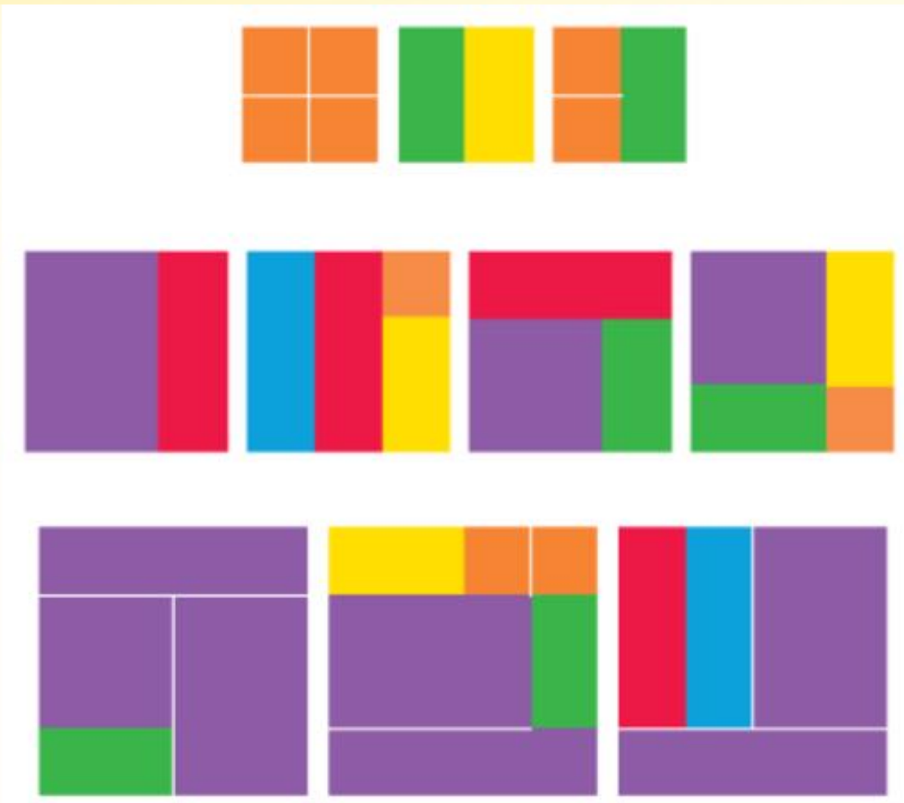
Tara: Yes, but when I put other triangles together, I do not get square corners. Some corners are larger than a square corner, and some corners are smaller than a square corner.

You can only compose rectangles with shapes that have square corners or that make square corners when you put them together.

PK, K, 1: Children Compose 2D Shapes to Make Squares

Children compose to make squares.

Using rectangles



Tyrette: We made fourths with the little orange squares.
Ziv: And halves with the green and yellow small rectangles. Two halves.
Nina: And two fourths and one of the halves can also make the square.

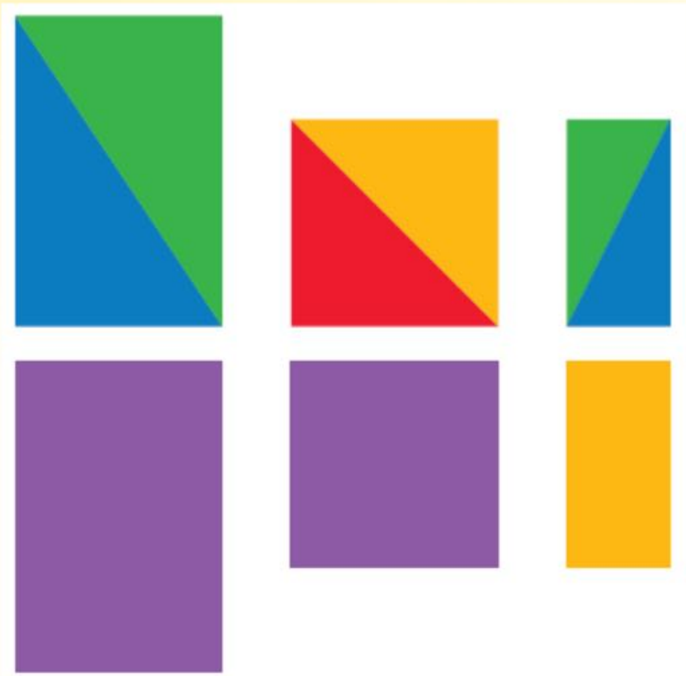
Using triangles

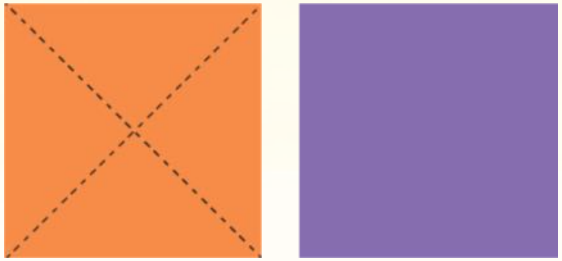
Luz: Look!!! I made fourths and then halves and then fourths in a different way!!!

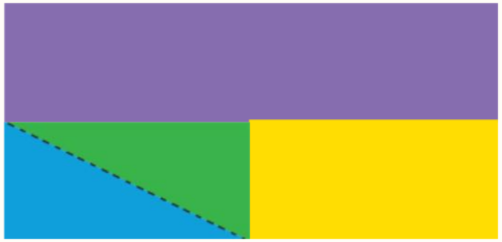
Logan: Wow!!! Look at all of our different squares we all made.

Compose Right Triangles to Make Rectangles

Composing is like subitizing shapes hiding inside other shapes.



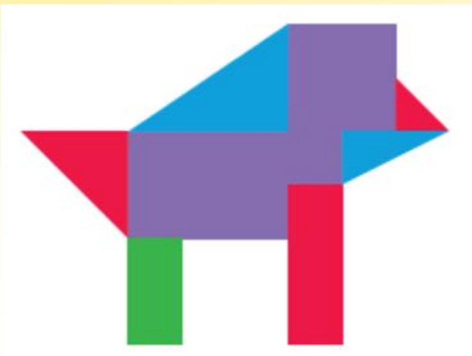




Composing right angled shapes prepares for many future math topics.

We show shapes beside each other, but children can put shapes on top.

Composing with right-angled shapes

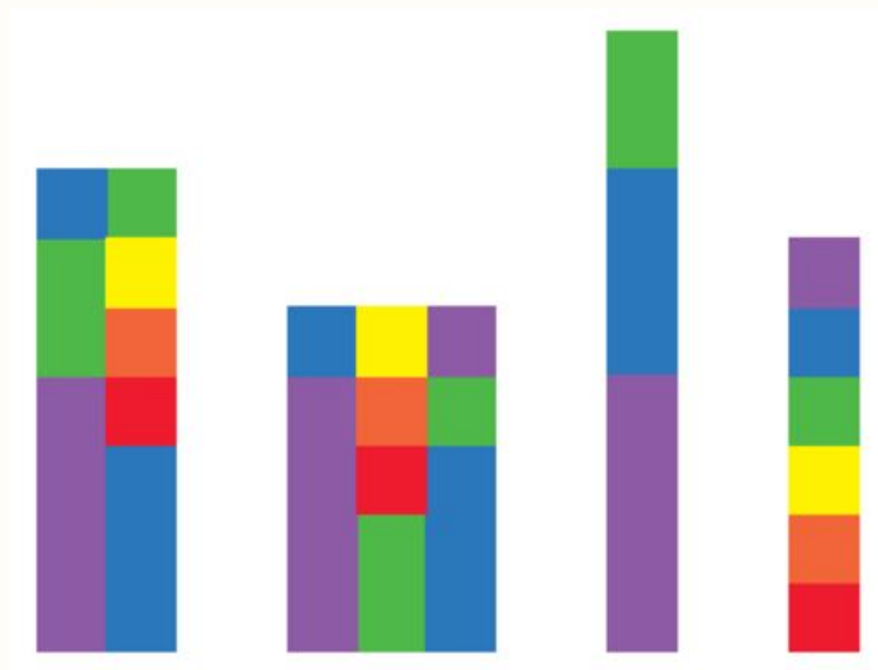


I made a long rectangle.

Look at my tall rectangles.

This is my pet dog.

And this is my submarine.



Compose 2 Identical Right Triangles

Children work together to discuss how they can put 2 identical right triangles together.

They describe what shapes they made.

Look, we made two different triangles. One is tall and one is short.

They look like tents.

And we have a kind of tilty rectangle. It has 4 sides but it does not have any right angles. It maybe is going to fall over.

We made a rectangle. It has 4 sides and 4 square corners.

This has 4 sides and 2 right angles. It looks like a kite.

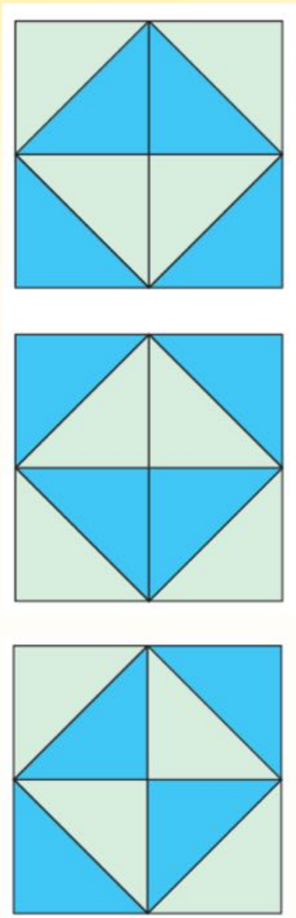
And another tilty shape with 4 sides.

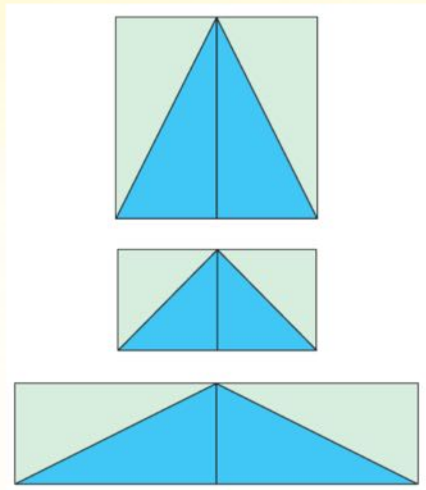
Compose Shapes with Square Corners

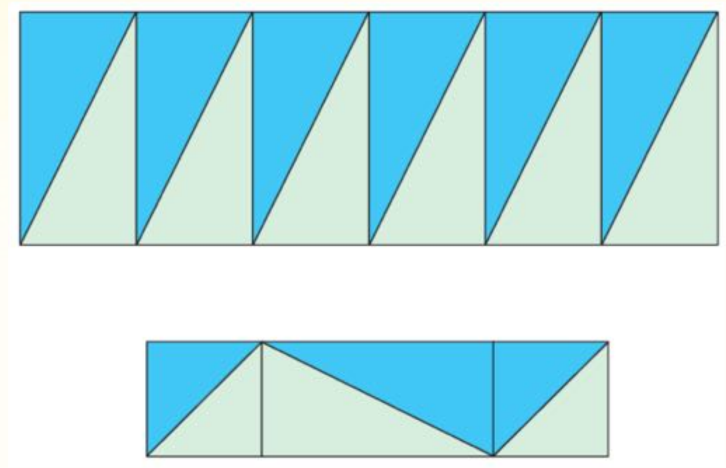
Children use the right triangle shapes to match puzzles.

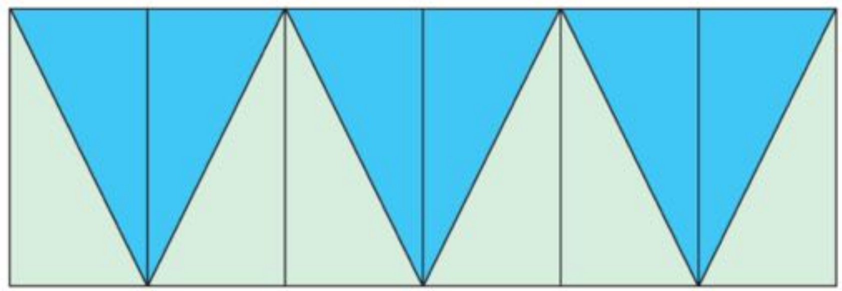
They discuss what they see.

They make their own designs.



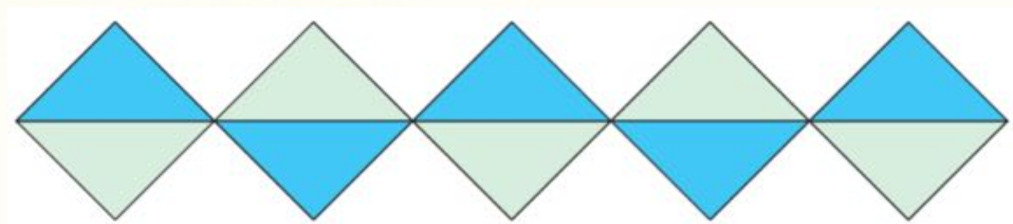


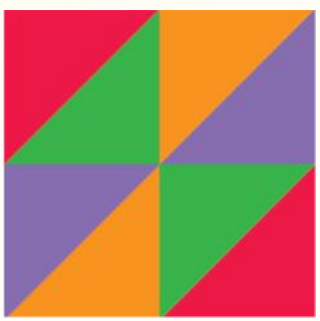


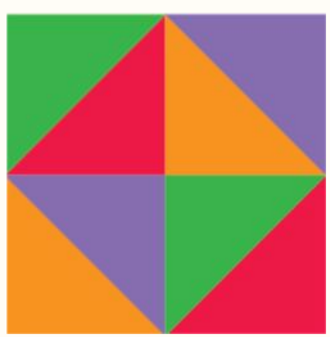


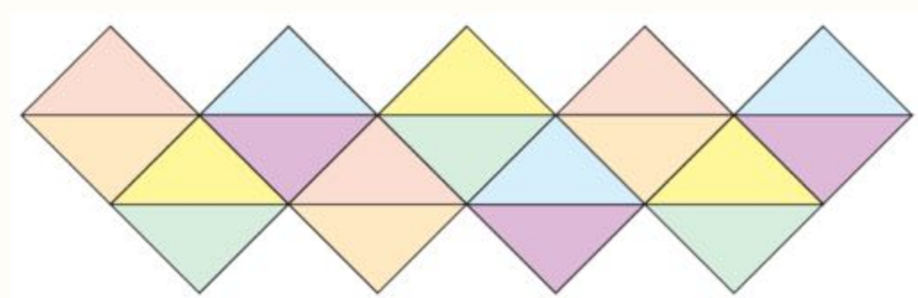
Make Designs with Small Right Triangles of 6 Colors

Children use the small right triangles of 6 colors on puzzles, and they make their own designs.





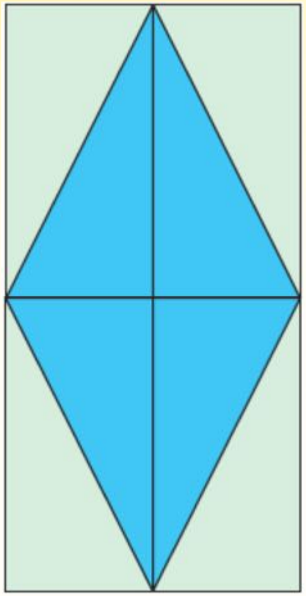


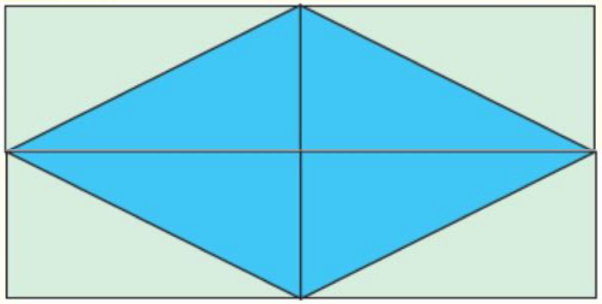


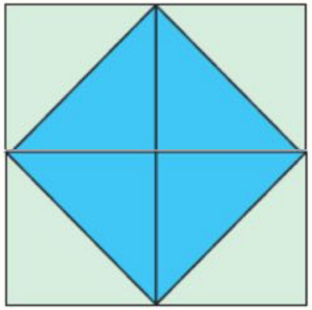
Composing identical right triangles: Noticing relationships

What are these top blue shapes? Why?

What do you notice about the diagonals? How do you know?







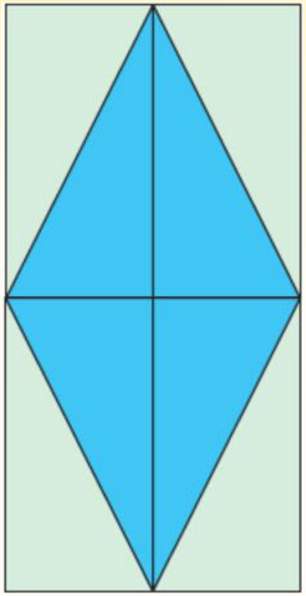
What is this blue shape? Why?

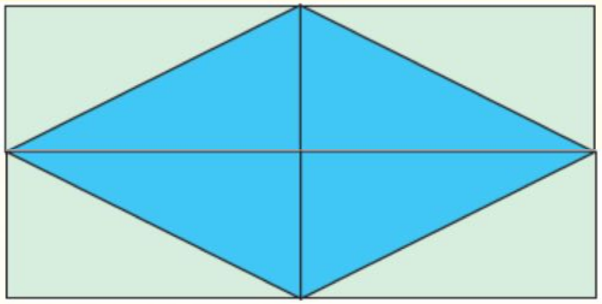
What do you notice about the diagonals? How do you know?

How to generalize?

Do the diagonals of all rhombuses make equal lengths on (bisect) each other?

Do they always make square corners (right angles)?

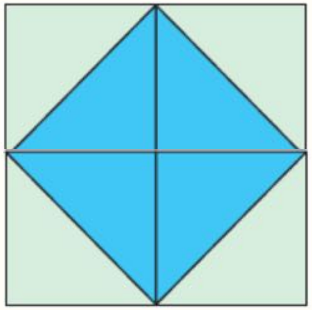




How could you find out?

Draw on grid paper. Try different right triangles to make my rhombus.

Cut out different right triangles and put them together.



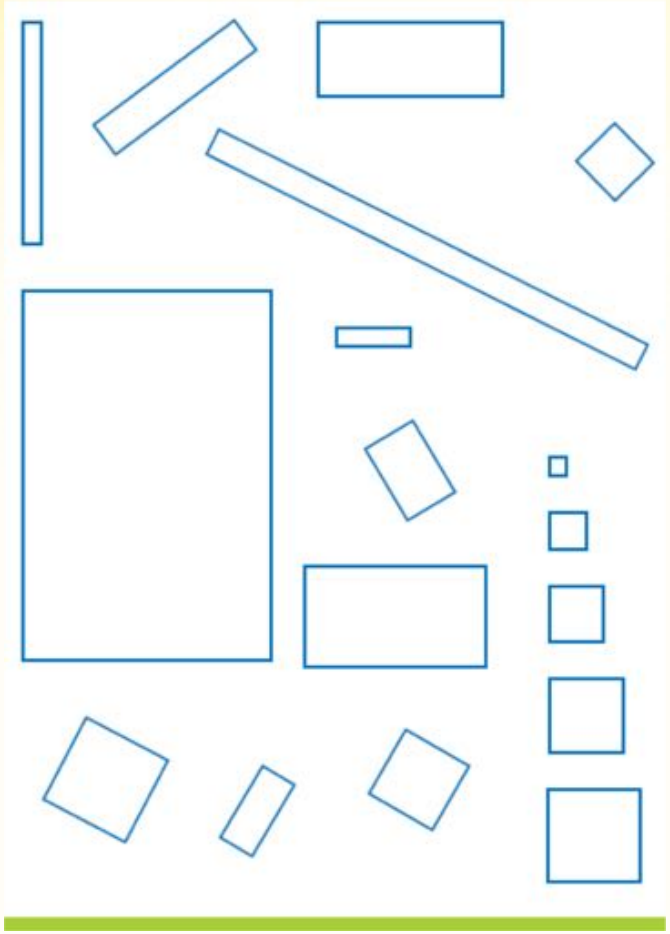
Use an app that changes the 4 corners of my rhombus.

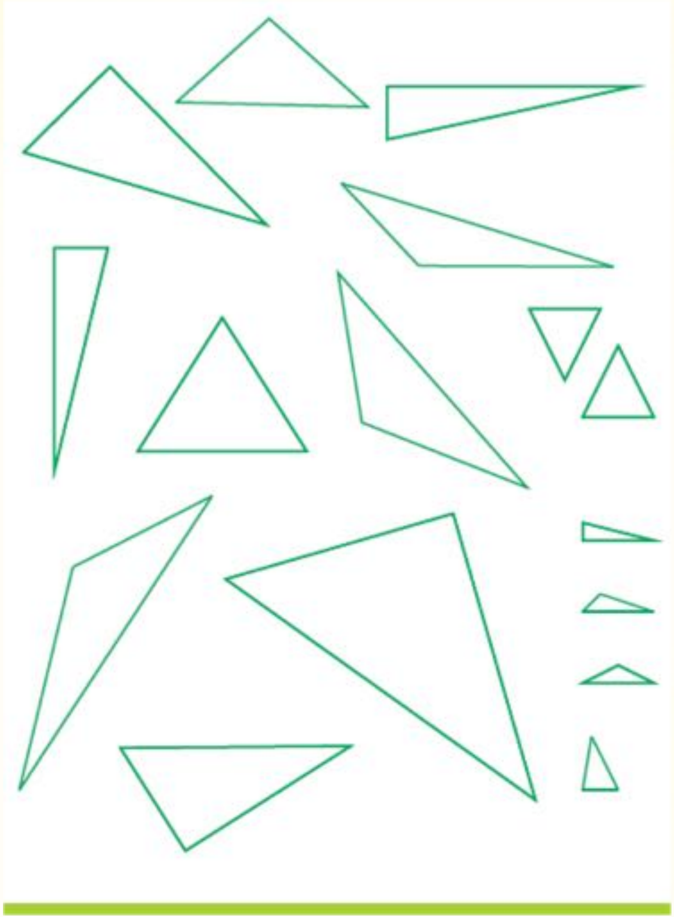
How do you make a rhombus be a square?

Generalize Rectangles and Triangles

Children discuss the different examples on Discussion Cards.

They relate these shapes to examples in the real world.

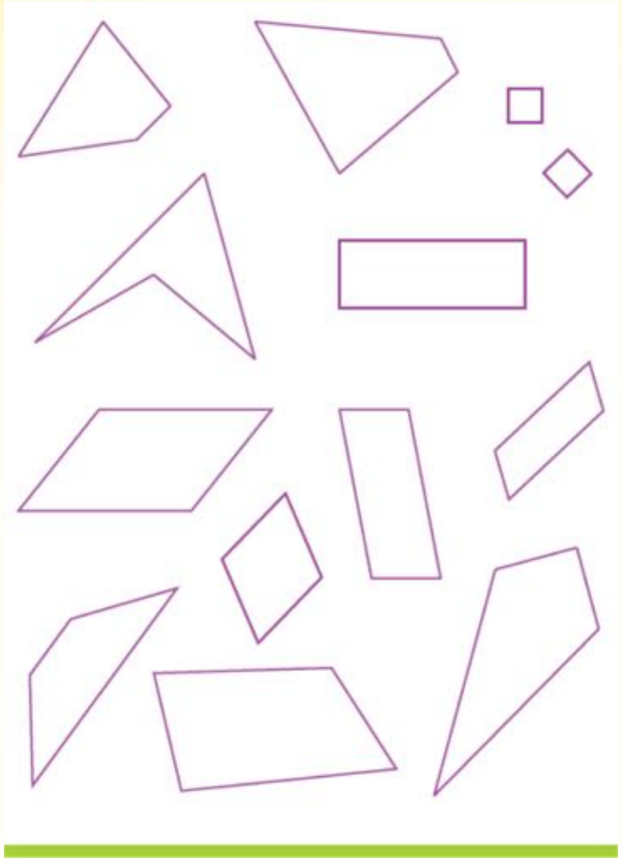


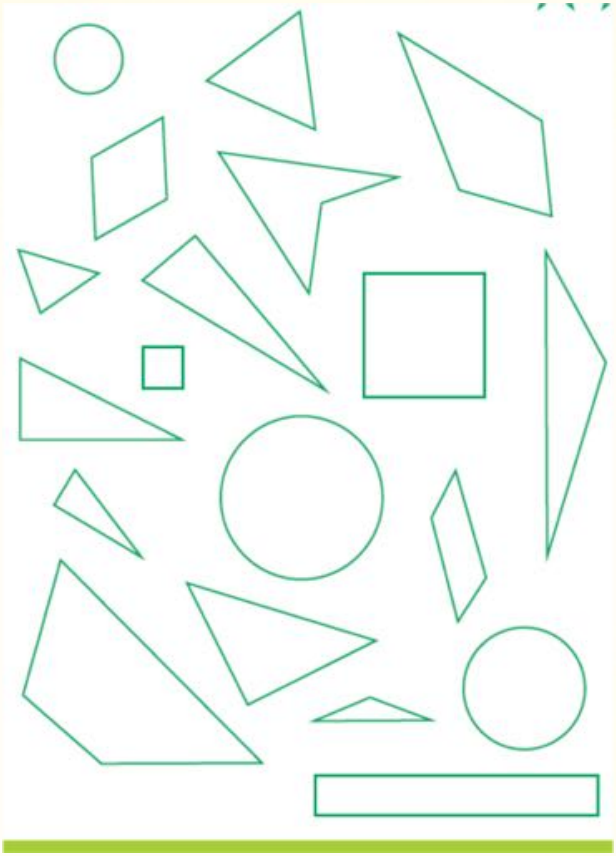


Generalize 4-Sided Shapes and Identify 3-Sided and 4-Sided Shapes

Children discuss the different examples on these Discussion Cards.

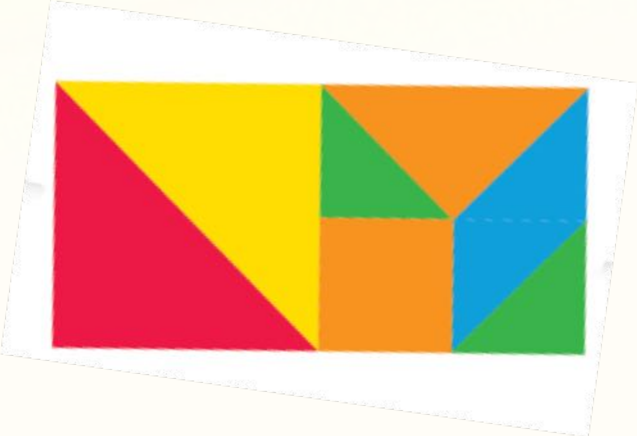
They relate these shapes to examples in the real world.

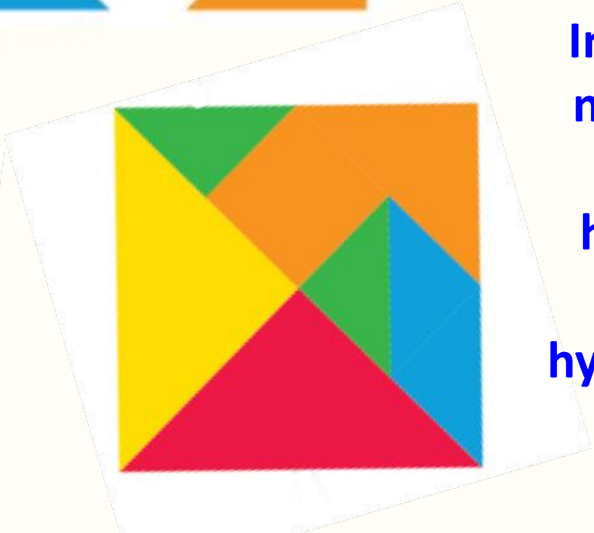




Tangram pieces can be a way to see composing and area.

The 2-D shape set has tangram pieces if 2 small right triangles are put together to make a parallelogram (here we show this in blue).





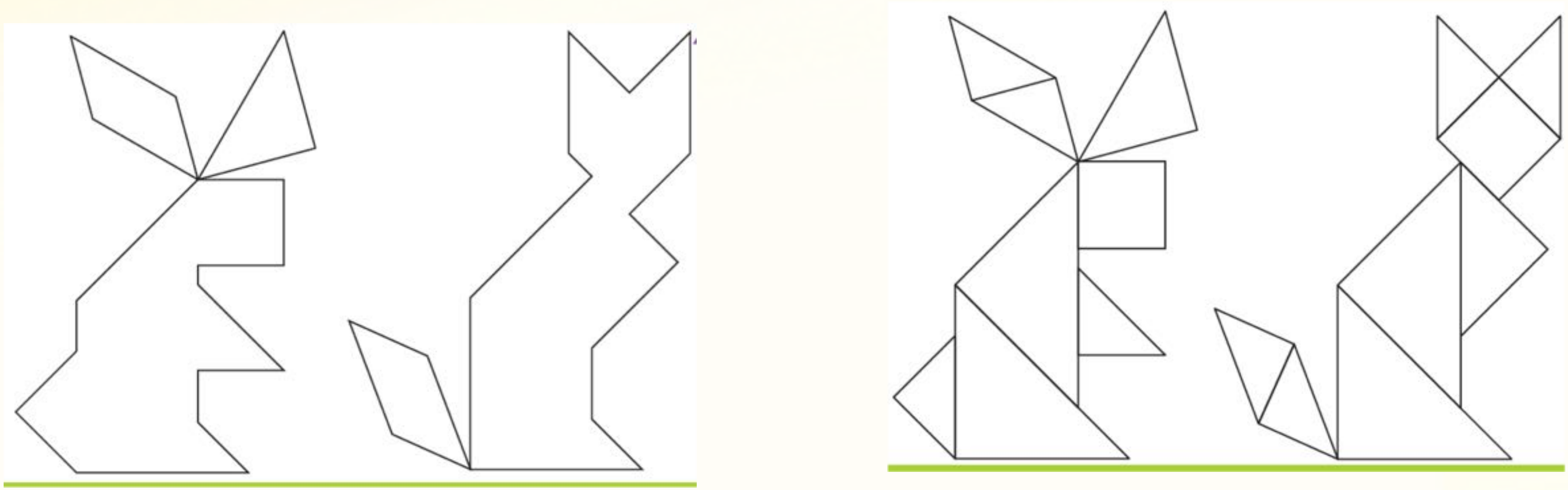
In tangram pieces, the medium right triangle (orange, here) has a hypotenuse of 2”. Its sides are the hypotenuse of the small right triangles.

1” right triangles can compose all of the tangram shapes.

From picture puzzles to composing shapes to area with right triangle units

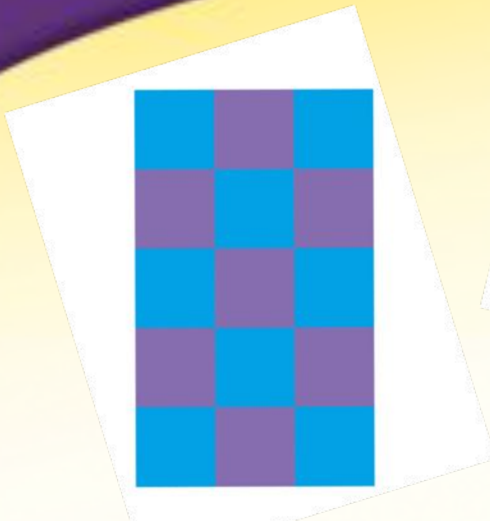
1. Tangram puzzle are often given without internal lines.

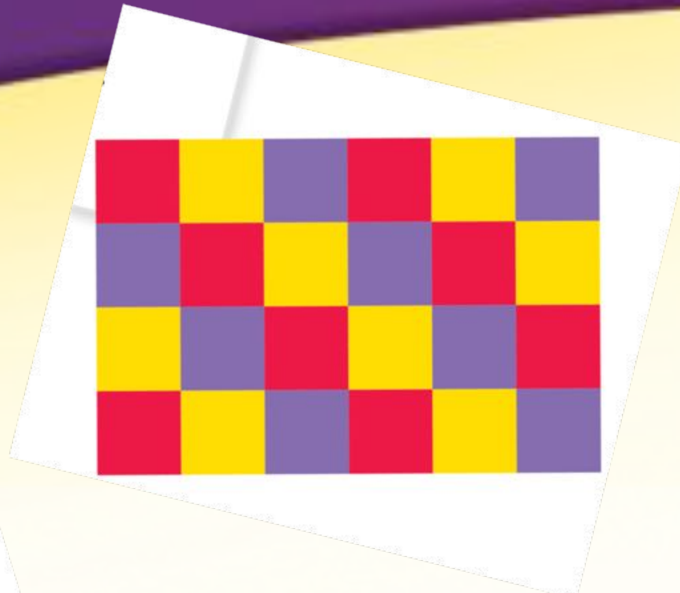
2. Internal lines show the composing shapes.

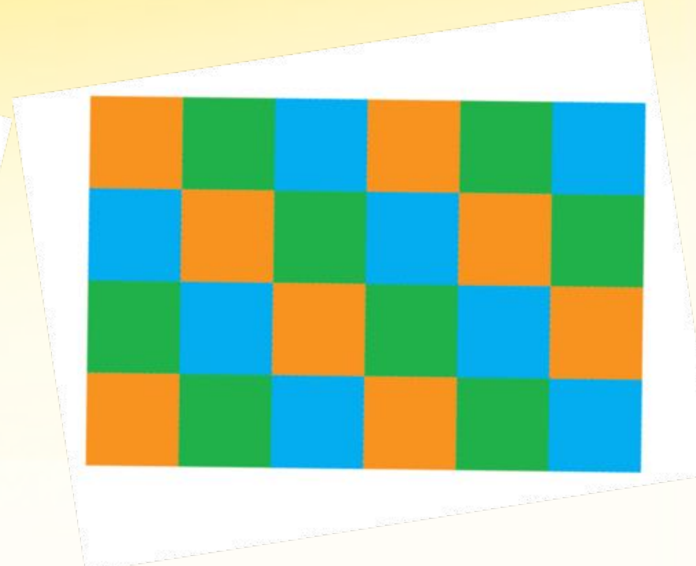


3. Cover the composing shapes with right triangle units to find the area in these units.

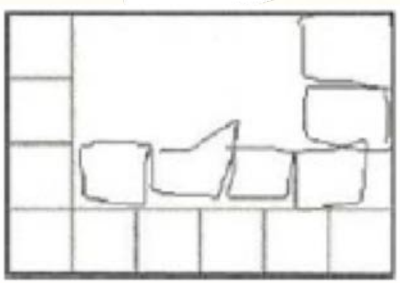
Make Patterns with Square Inches on Square Inch Grid Cards

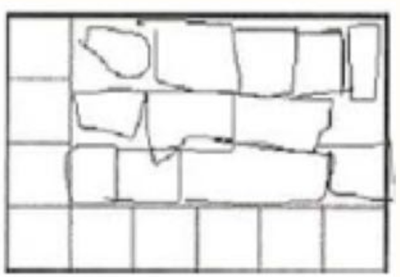


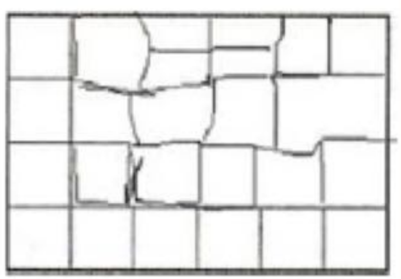




Clements, D. H., Sarama, J.. Van Dine, D. W., Barrett, J. E., Cullen, C. J., Hudyma, A., Dolgin, R., Cullen, A., Eames, C. L. (2018). Evaluation of three interventions teaching area measurement as spatial structuring to young children. *Journal of Mathematical Behavior, 50*, 23-41. https://doi.org/10.1016/j.jmathb.2017.12.004







Elementary school children have difficulties visualizing and drawing an area grid. Drawing around squares or drawing rows or columns helps.

Working with square inches on grids earlier can build visualizations.

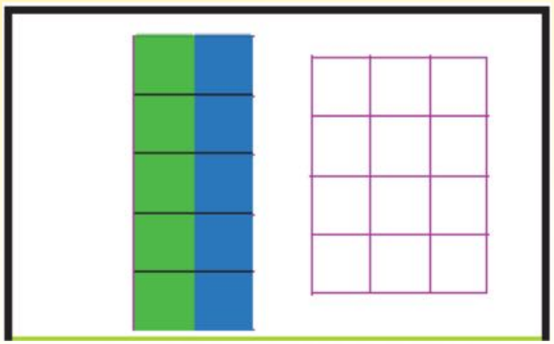
P, K, 1, 2: Experiencing Area as Filling with Squares

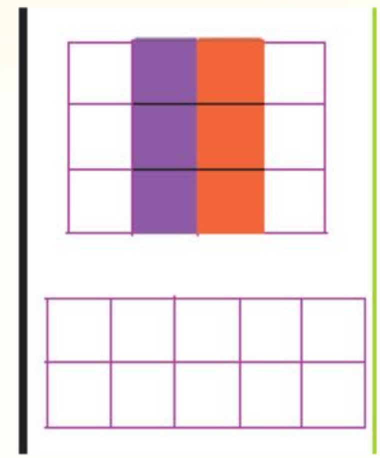
Give them square grids. Give them square inches.

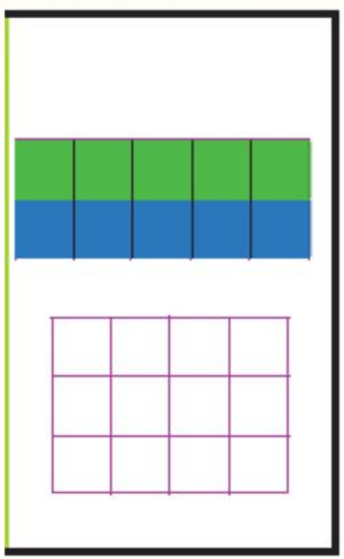
This helps them build visual images of square grids.

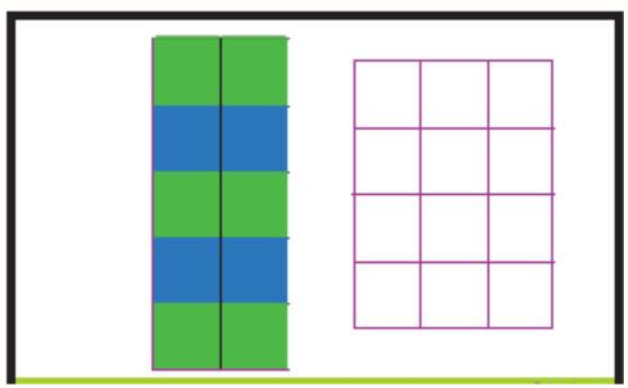
They can discuss patterns they make and see.

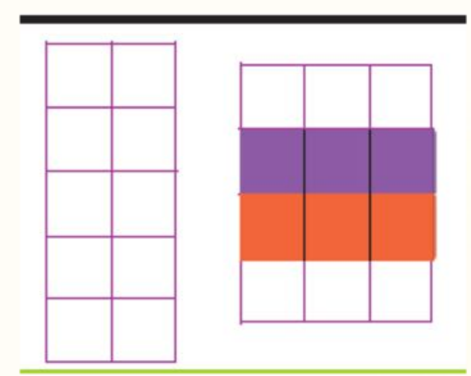
They can rotate the square grids to make columns be rows and vice versa.





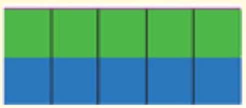






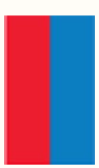
Prepare for partitioning rectangles by building them up in PK, K, 1 with rectangles or right triangles

2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

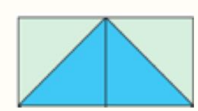


This is easier if rectangles have already been built up by squares.

2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths.

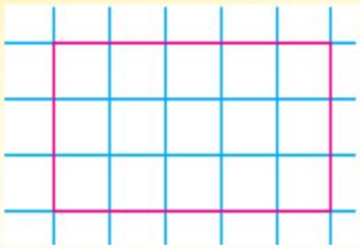


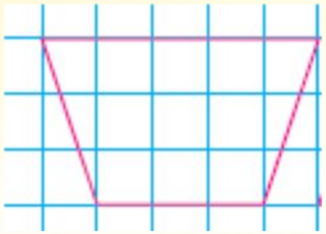
This is easier if rectangles have already been built up by other rectangles or squares or right triangles.

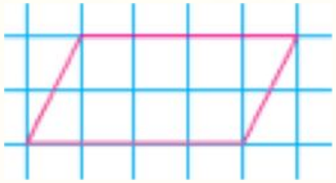


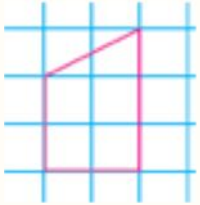
Draw on grid paper to see the square units.

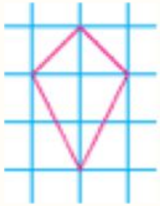
Also draw on grid paper to see squares inside rectangles and other shapes.











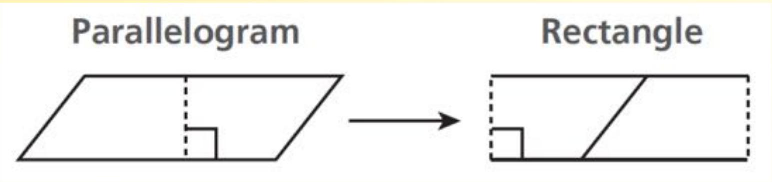
After working with composing rectangles from right triangles, students can count the number of squares (the area) making each of these shapes.

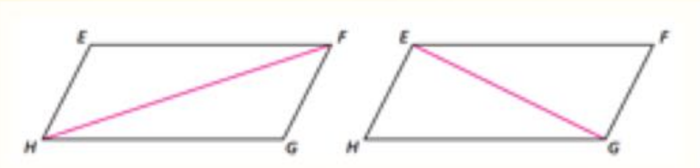
Students can use multiple recomposing or "surround and subtract" strategies to count square units. These strategies can be used later to find formulas for the area of these shapes.

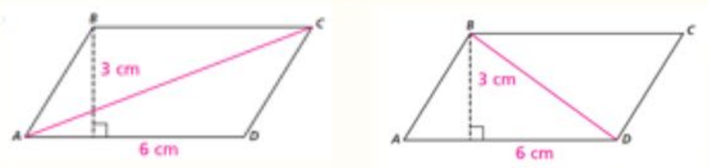
Students can cut and recompose or visualize and describe.

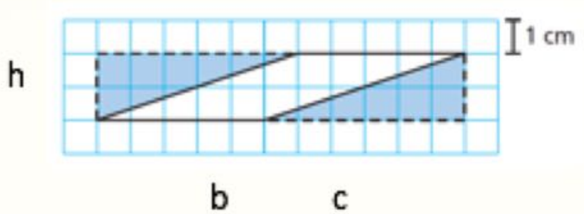
Seeing heights in shapes; recomposing to make a rectangle

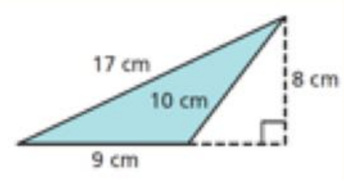
Grade 6 area of parallelograms, triangles, trapezoids, regular polygons.





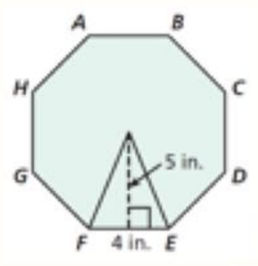


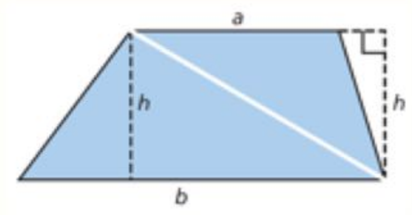


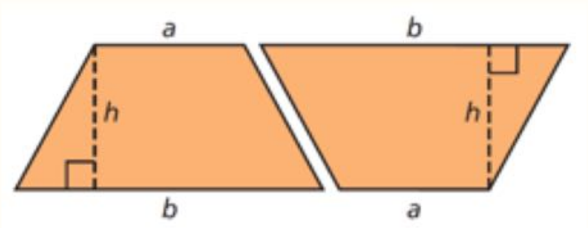


Octagon

8 x 1/2 h s

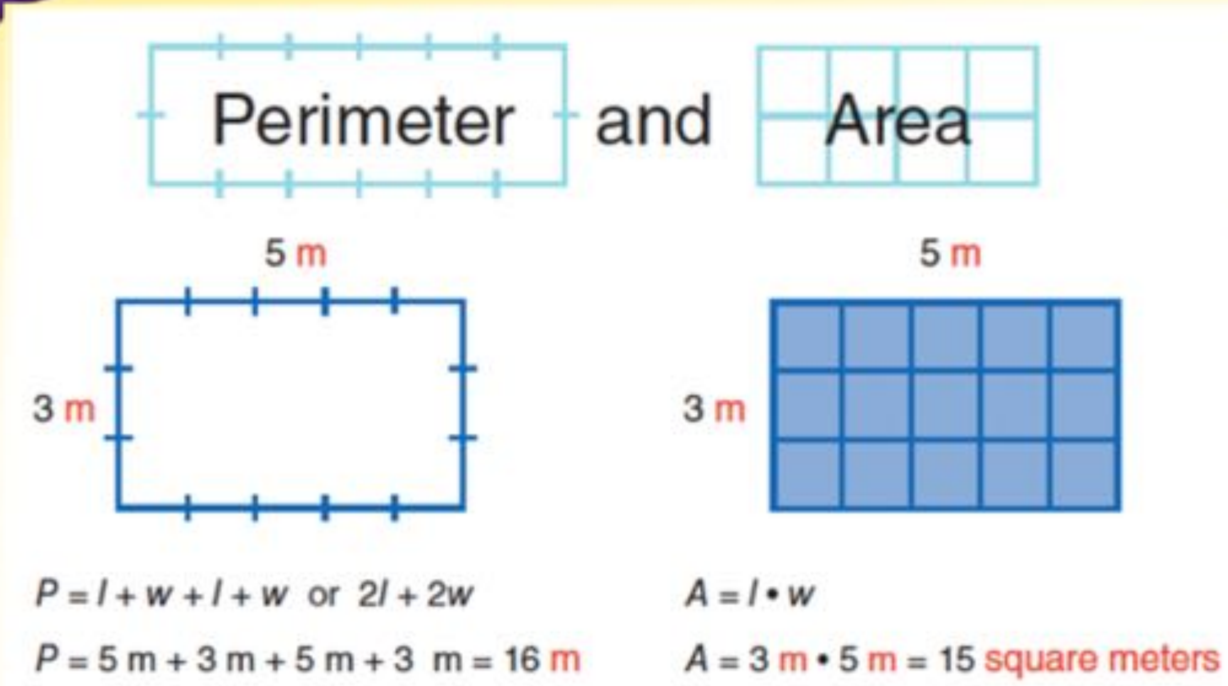






Work toward G3: Emphasize the length units for perimeter and the square units for area

Differentiate
the length units in perimeter
and the square units in area.

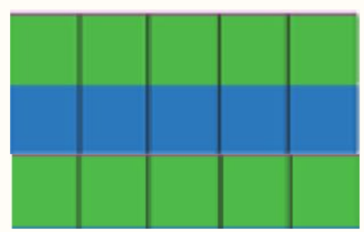


On blank paper:

Draw along all sides and make each endmark for the length units in the perimeter. Count these.

Make the length endpoints and then connect these length unit marks to make vertical and horizontal line segments that create squares inside the rectangle.

Softly shade or color alternate rows and count the square units. Discuss why this is 3 groups of 5.



Work toward G3: Emphasize the length units for perimeter and the square units for area

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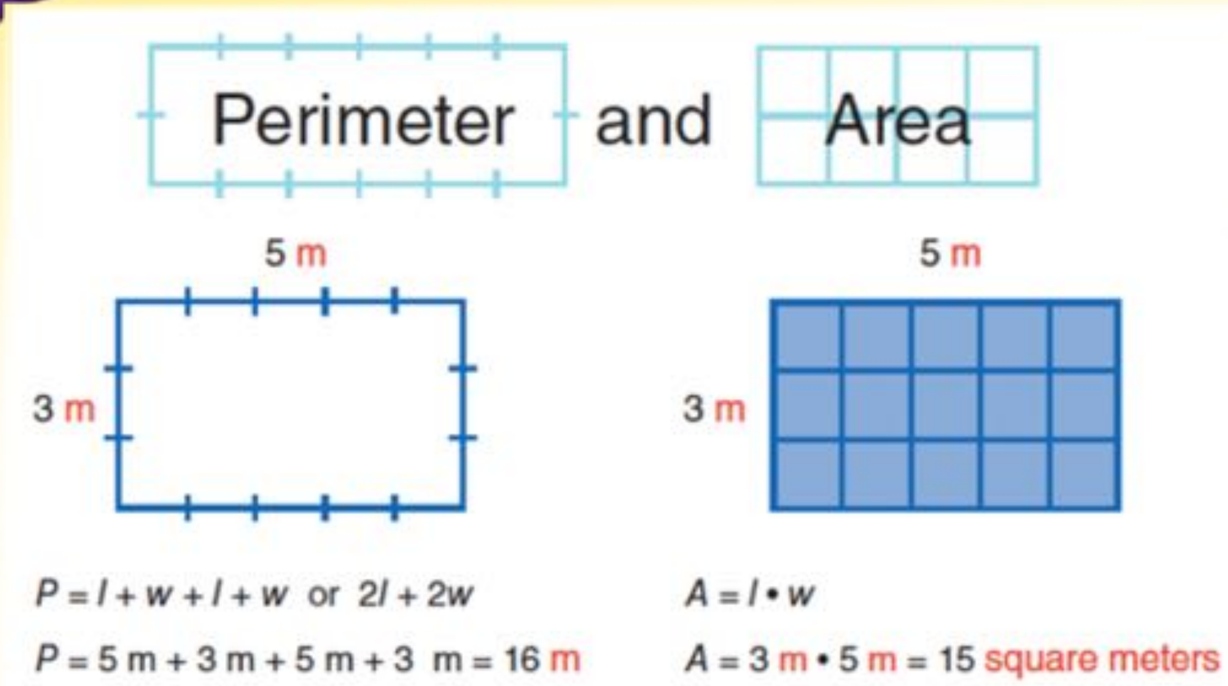
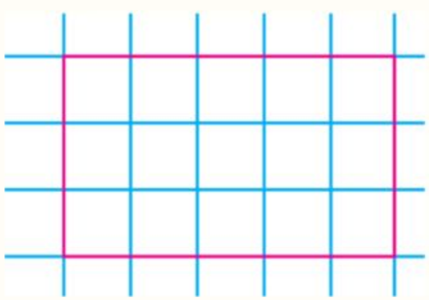
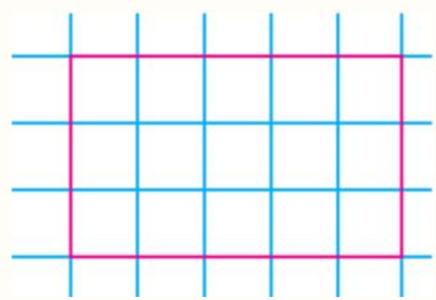




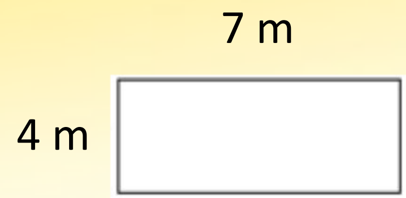


On grid paper:
Emphasize the perimeter. Draw along all sides and make each endmark for the length units in the perimeter. Count these.

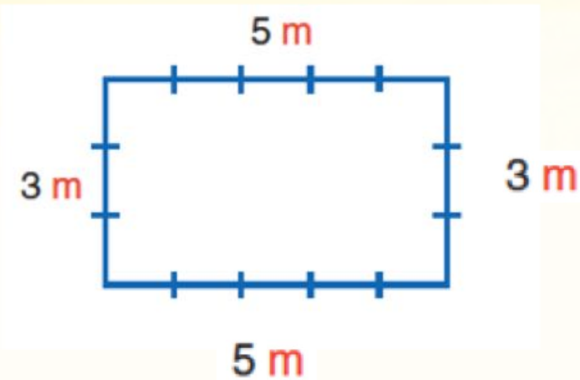
For area, the squares are already inside the rectangle. Softly shade alternate rows and count the square units. Discuss why this is 3 groups of 5.

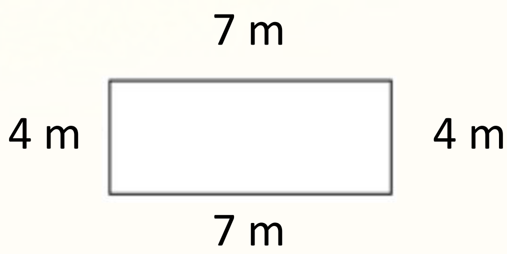
Avoiding common errors

Find the perimeter.



A. Initially students must see all 4 sides for any perimeter problem, so label rectangles for perimeter on all 4 sides to show what must be added.





Then discuss strategies:

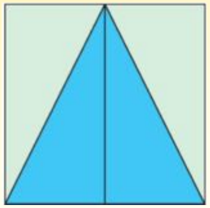
3 + 5 + 3 + 5

(3 + 5) x 2

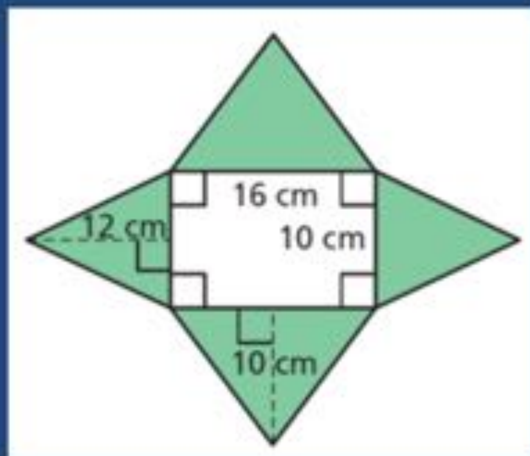
2 x 5 + 2 x 3

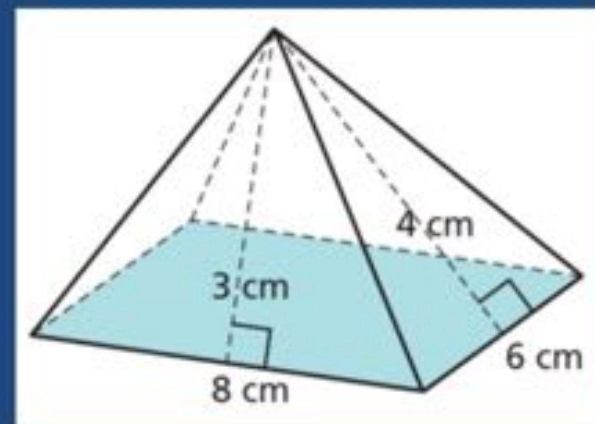
B. For perimeter and area, check that the side lengths have the same units so you can add them or make the square units.

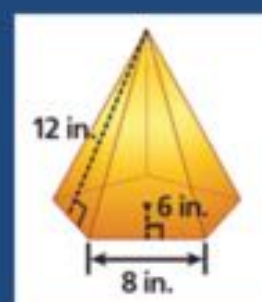
Seeing square units on different shapes and composing shapes



Surface Area Grade 6



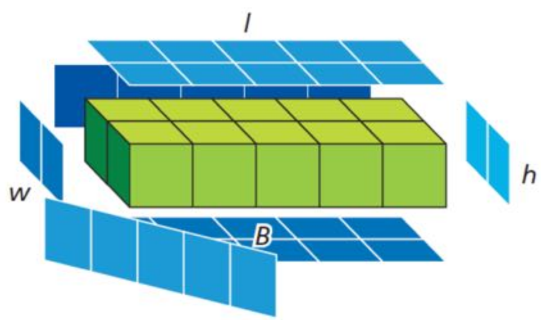




G6: Students Differentiate Surface Area and Volume of Prisms

Students see and identify the kinds of units used to measure surface area and volume.

- They see the **square units** that make the surface area and review that they write the answer as $unit^2$.
- They see the **cubic units** that make the volume and review that they write the answer as $unit^3$.

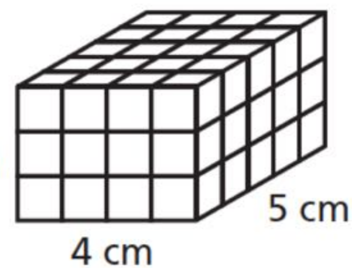


11. What is the surface area and volume of the prism you made?

$SA =$ $34\ cm^2$ $\quad 2x5 + 2x2 + 2x2x5\ cm^2$

$V =$ $10\ cm^3$ $\quad 2x5x1\ cm^3$

16.



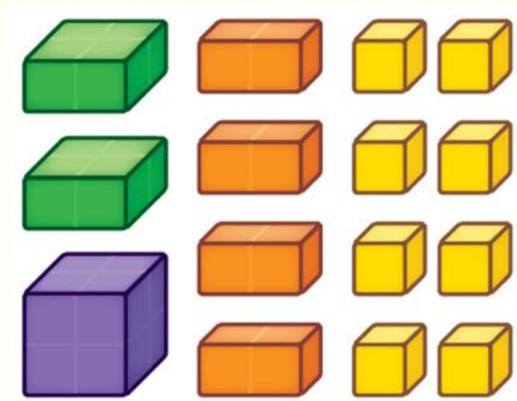
$SA =$ $94\ cm^2$ $\quad 2(3x4 + 3x5 + 4x5)\ cm^2$

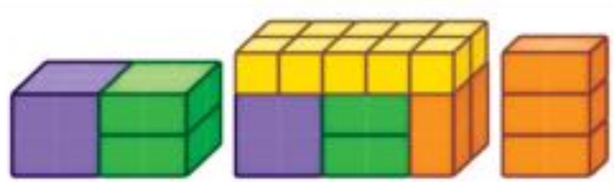
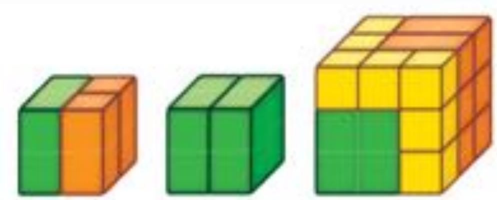
$V =$ $60\ cm^3$ $\quad 3x4x5\ cm^3$

Su

With squares and cubes, children can build ideas of surface area and volume beginning in PK.

In later grades they can draw nets on grid paper and fold up to see surface area.



Compose Rectangular Prisms That Are Not Cubes	Compose Rectangular Prisms That Are Cubes
	

compose/decompose shapes

c. How can you compose parallelograms and trapezoids from rectangles and right triangles?
And how do parallelograms and trapezoids relate?

Which 2-D Shapes Are in Which Grade Standards?

2-D shapes specifically named in grade-level standards:

K: squares, circles, triangles, rectangles, hexagons (why hexagons now?)

1: rectangles, squares, trapezoids, triangles; half-circles, quarter-circles (why trapezoids and not parallelograms?)

2: triangles, quadrilaterals, pentagons, hexagons (number of sides/angles)

3: rhombuses, rectangles, squares, quadrilaterals (why rhombuses and not parallelograms?)

These shapes are pattern block shapes which are very restricted.

If a shape is introduced, it should be treated generally.

Parallelograms should be introduced with trapezoids to see relationships.

4: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

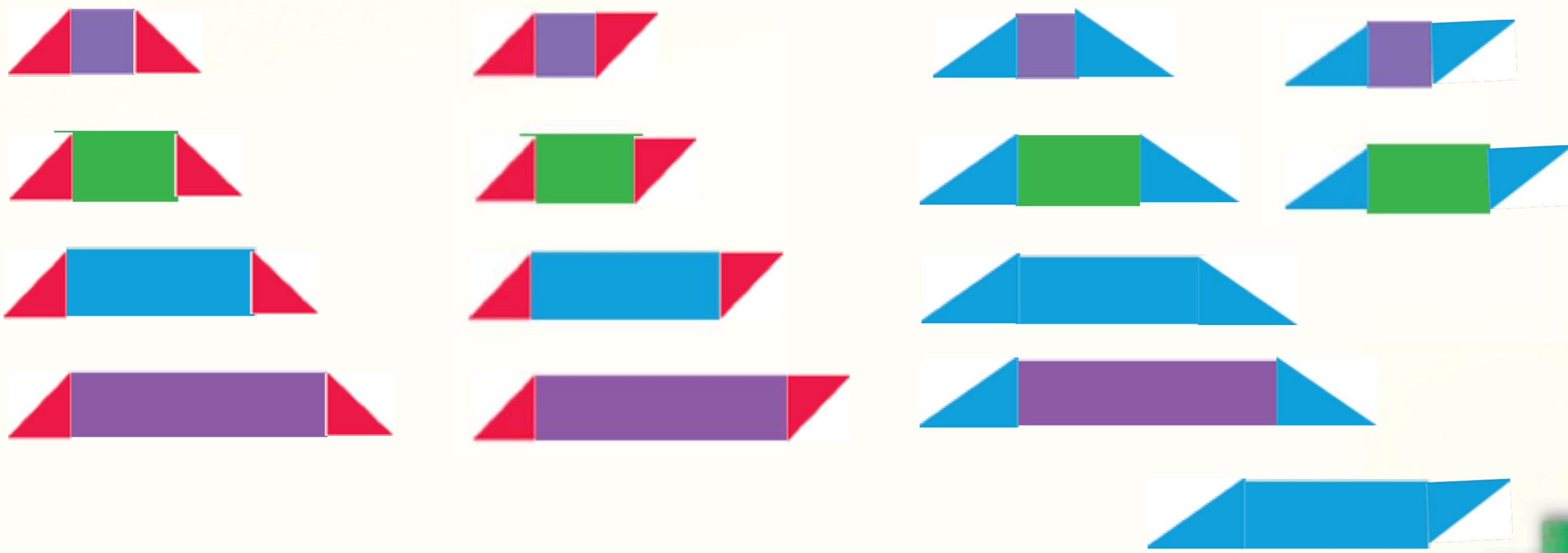
5: Classify two-dimensional figures in a hierarchy based on properties.

G1: Relating Trapezoids and Parallelograms by Flipping One Right Triangle

Parallelograms are more important shapes than trapezoids, so it is not clear why parallelograms are not listed with trapezoids in Grade 1. These two shapes have a lovely relationship that children can discover by making these shapes and also discussing related drawings of these shapes.

Children can see and discuss trapezoids and parallelograms that have the same rectangles in the middles and have one triangle flipped to extend the top of the rectangle to the right.

trapezoids parallelograms trapezoids parallelograms

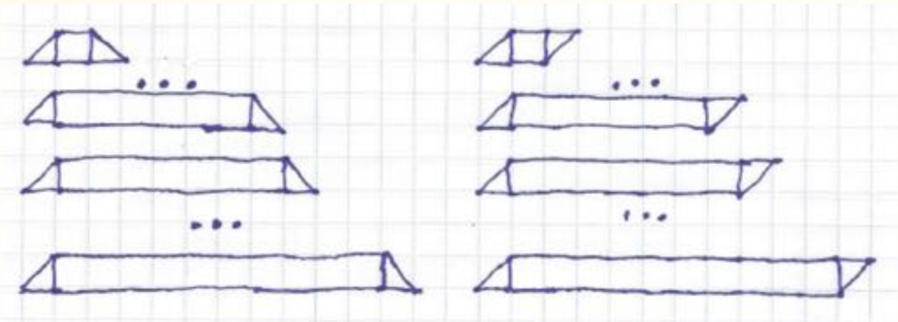


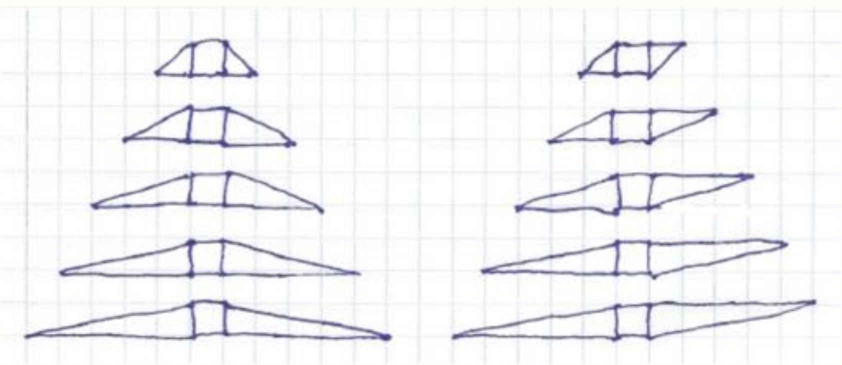
G1: Generalize by drawing on grid paper

Children can generalize and, by drawing on grid paper, see even more trapezoids and parallelograms made by flipping the right triangle.

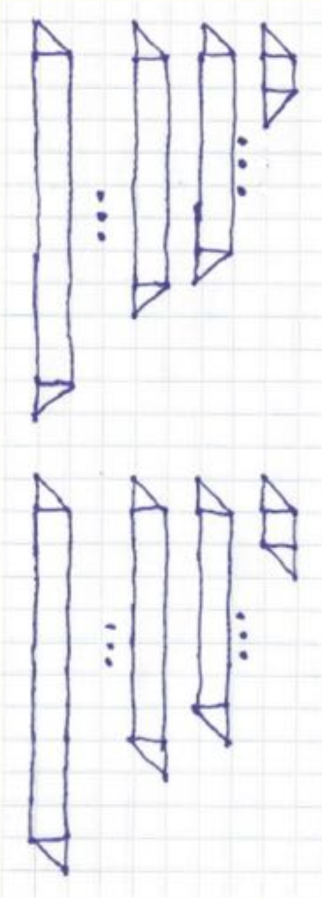
They can draw extreme cases with long rectangles or long sides on the right triangles to see many different cases of these shape.

They can turn around the grid paper to see the shapes in different orientations.

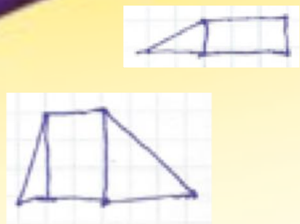




Children are counting length units as they draw.

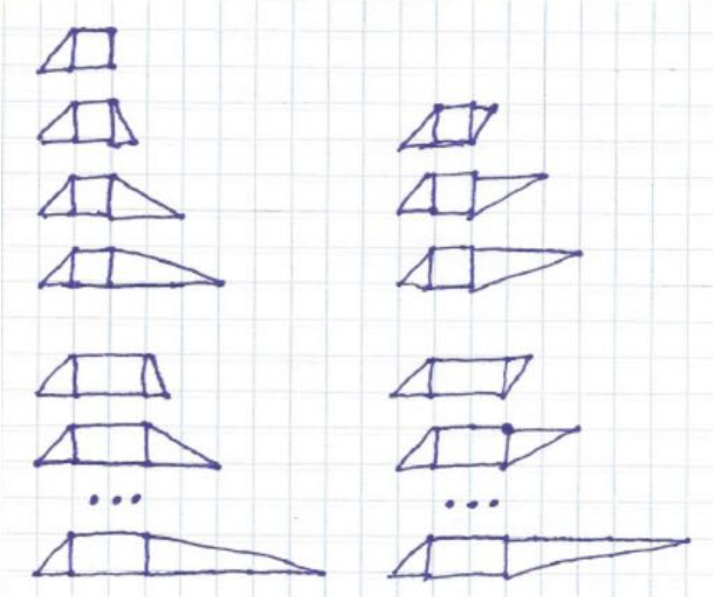


G1: Trapezoids but not parallelograms can have 2 different right triangles.



Trapezoids can have slanting sides of different lengths. But if you flip one unequal triangle, you **do not get a parallelogram.**
This is an early experience of **slope: over n and up m. You count these lengths as you draw.**
The slants are different so those sides are not parallel.

trapezoids **not parallelograms**

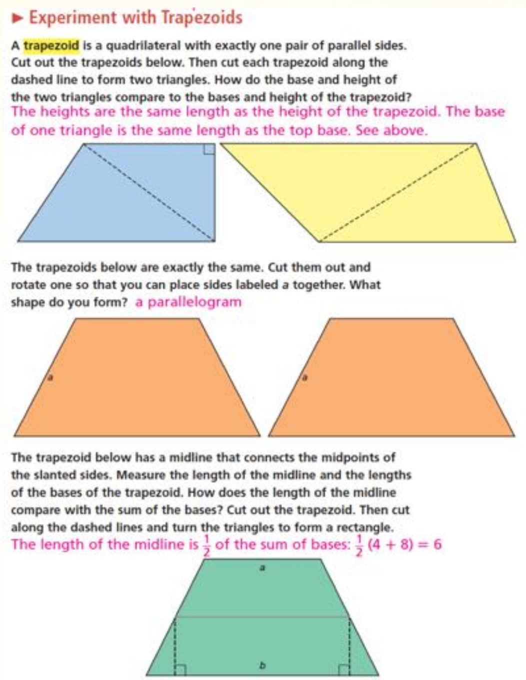


So a trapezoid is composed from a rectangle and one or two right triangles. Seeing the right triangles gives early experiences of the **height** of a trapezoid. The compositions help to prepare for all of the fun ways to show the area of a trapezoid that depend on various decompositions or compositions.

G6: Decompose Trapezoids to Find the Area in Different Ways

Students cut out trapezoids and cut along given line segments or rearrange shapes to decompose trapezoids in different ways.

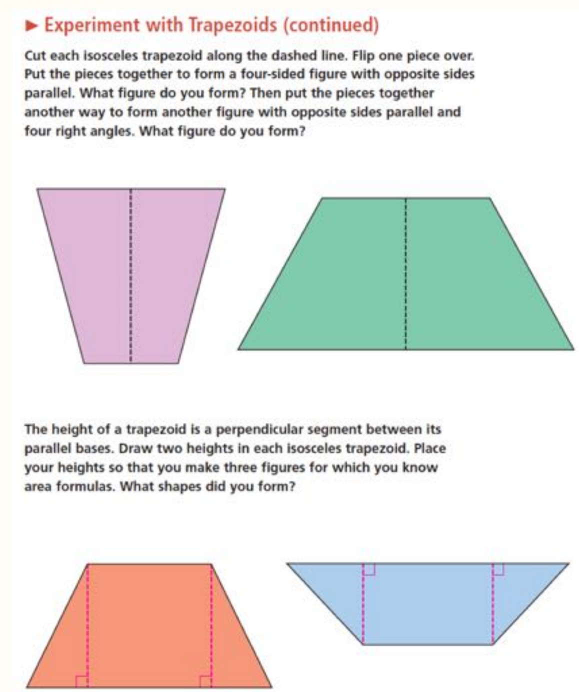
They use reasoning to express the area. We will look next at each of the 5 strategies whose trapezoids are shown below.


► Experiment with Trapezoids

A trapezoid is a quadrilateral with exactly one pair of parallel sides. Cut out the trapezoids below. Then cut each trapezoid along the dashed line to form two triangles. How do the base and height of the two triangles compare to the bases and height of the trapezoid?
The heights are the same length as the height of the trapezoid. The base of one triangle is the same length as the top base. See above.

The trapezoids below are exactly the same. Cut them out and rotate one so that you can place sides labeled a together. What shape do you form? a parallelogram

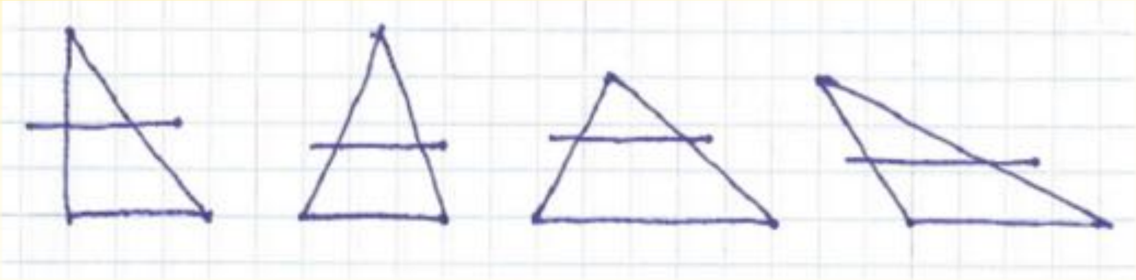
The trapezoid below has a midline that connects the midpoints of the slanted sides. Measure the length of the midline and the lengths of the bases of the trapezoid. How does the length of the midline compare with the sum of the bases? Cut out the trapezoid. Then cut along the dashed lines and turn the triangles to form a rectangle.
The length of the midline is $\frac{1}{2}$ of the sum of bases: $\frac{1}{2}(4 + 8) = 6$


► Experiment with Trapezoids (continued)

Cut each isosceles trapezoid along the dashed line. Flip one piece over. Put the pieces together to form a four-sided figure with opposite sides parallel. What figure do you form? Then put the pieces together another way to form another figure with opposite sides parallel and four right angles. What figure do you form?

The height of a trapezoid is a perpendicular segment between its parallel bases. Draw two heights in each isosceles trapezoid. Place your heights so that you make three figures for which you know area formulas. What shapes did you form?

Trapezoids can come from slicing off a triangle parallel to any base.



These truncated triangles also form trapezoids.

A truncated isosceles triangle forms an isosceles trapezoid (one whose non-parallel sides are of equal length)

There is no agreement about how to define a trapezoid. Some mathematicians define a trapezoid as having exactly one pair of opposite sides parallel, in which case a parallelogram is not a trapezoid. I prefer this definition to give students separate terms for parallelograms and trapezoids during all of the early exploratory work.

Others define a trapezoid as having at least one pair of opposite sides parallel, in which case a parallelogram is a special kind of trapezoid.

Students can discuss this difference and look at the 2 different classifications in grade 5 or earlier.

compose/decompose shapes

d. Relate 3- and 4-sided shapes in two major ways:
Two right triangles make a rectangle and an isosceles triangle
Two acute or two obtuse triangles make a parallelogram

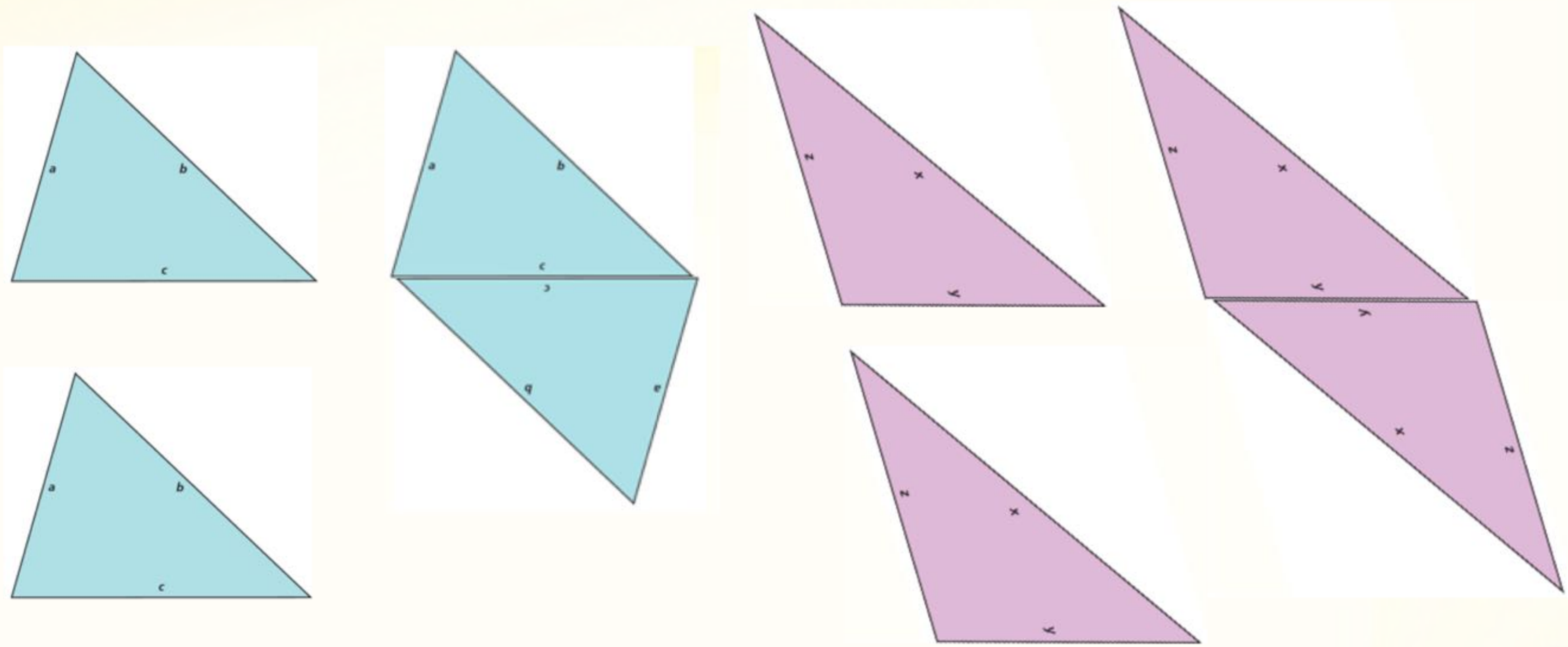
Compose 2 Acute or 2 Obtuse Triangles to Make a Parallelogram

Compose to make a parallelogram

Students cut out and compose 2 identical acute triangles to make a parallelogram.

- They can match the sides a or b or c, so the two green acute triangles make 3 different parallelograms.

Then they do the same for 2 identical obtuse triangles (below in purple).



Draw diagonals in two identical parallelograms to see two acute and two obtuse triangles

See the two acute triangles and two obtuse triangles that compose a parallelogram by drawing each diagonal

two acute triangles

two obtuse triangles

Make a right angle with your arm bent at the elbow.

Bend your hand toward your head to show the acute angles.

Open the angle to show the obtuse angles.

Or show angles with your hands.

Coordinating Geometric Measurement and Geometry Standards

K	1	2	3	4	5
MD Measurement and Data: K to 5					
Geometric Measurement: K to 6 Use length to make area and volume units					
(Describe attributes)	(Length)	Length	Area	Angles	Volume [G6 geometry: surface area and area of triangles, special quadrilaterals, and polygons

K	1	2	3	4	5
K 1 2: Analyze, name, describe attributes and compose/decompose shapes (continues and used in higher grades)			Classify sub-categories	Classify using properties	Classify in a hierarchy
					Coordinate plane [G6 draw polygons on coordinate plane]

Partition a shape into equal shares/parts

Formal angle measurement is in Grade 4, but much understanding can prepare for formal measurement

4.MD.C.5,6,7 is formal angle measurement.
It is in MD because these standards parallel those for length in Grade 2 and area in Grade 3.

4.G.1, 2, 3 uses angle measurement with shapes:
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

We have seen several ways to prepare for the formal ideas in Grade 4.

4.MD.3.5, 6, 7. Geometric Measurement: Understand Concepts of Angle and Measure Angles

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.G.1, 2, 3. Draw and Identify Lines and Angles, and Classify Shapes by Properties of Their Lines and Angles.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

OA, Geometric Measurement, and Geometry

The Operations and Algebraic Thinking (OA) word problems for addition and subtraction are similar for discrete objects and for measures.

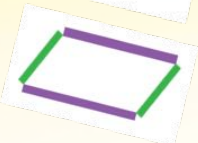
The activities proposed in this talk can help students understand and work better with the units of length and of area to avoid errors.

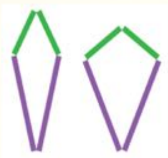
For multiplication/division equal groups and compare problems, measurement and discrete object problems are similar but students always have to be clear about which answers they label with a measurement unit and which they only label with a number telling how many times as much.

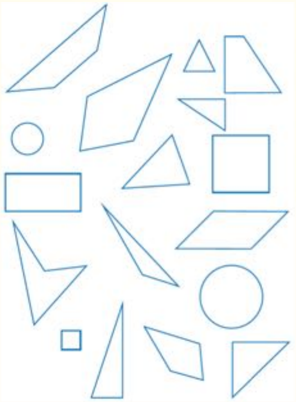
Area problems use length and area units. Differentiating length and area units as suggested in this talk is very important for solving these problems.

Making a drawing can also help.

Deepening and Connecting Geometry, Geometric Measurement, and Operations and Algebraic Thinking (OA) Within and Across the Grades







Professor Emerita Karen C. Fuson

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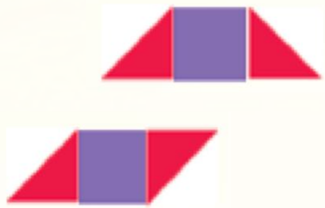
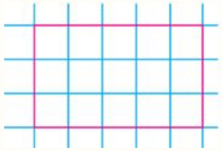


Paper presented at the Annual Conference of the National Council of Supervisors of Mathematics, 2019, San Diego, CA



Please see my website karenfusonmath.com for

- **the 18 hours of audio-visual Teaching Progressions for all CCSS domains I have made, and**
- **for my papers, classroom videos, and presentations including this one.**

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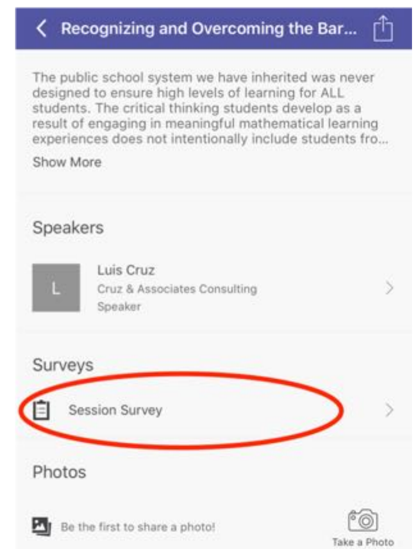

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