Student drawings of length models can support understanding of fraction computation

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Please see the Teaching Progressions, Classroom Videos, and Publications on my website karenfusonmath.com for fractions and for other CCSS-M math topics. There are 18 hours of Teaching Progressions for the various math domains in the CCSS-M.

The Math Practices in action
in a Nurturing Math Talk Community

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

Can I do some part of this better tomorrow?

[SMP 1 & 6; 7 & 8; 4 & 5; 2 & 3]

K	1	2	3	4	5
NF Number and Operations–Fractions: 3 to 5					
		General:	**Unit fractions**	**Compare any frs** **find eq frs**	**+- any fr eq frs** **fr x WN, fr** **[G6 fr ÷ fr]**
		Special cases:	**Compare like n or d**	**+- like denom** **WN x fr**	**n ÷ d = fr** **WN÷fr** **fr÷WN**

G3 NF Standards: 3.NF.1, 2, 3

Grade 3: Develop understanding of fractions as numbers.

1. Understand a unit fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3. Explain equivalence of fractions in special cases [small numbers], and compare fractions by reasoning about their size [same denominator or same numerator].

G3 Difficulty Seeing the Parts Within the Total

Most fraction drawings are intended to be seen as the total with the part embedded within it. Seeing the embedded part is difficult.

Error: See the first part and the second part and write those numbers: 3/2 instead of 3/5

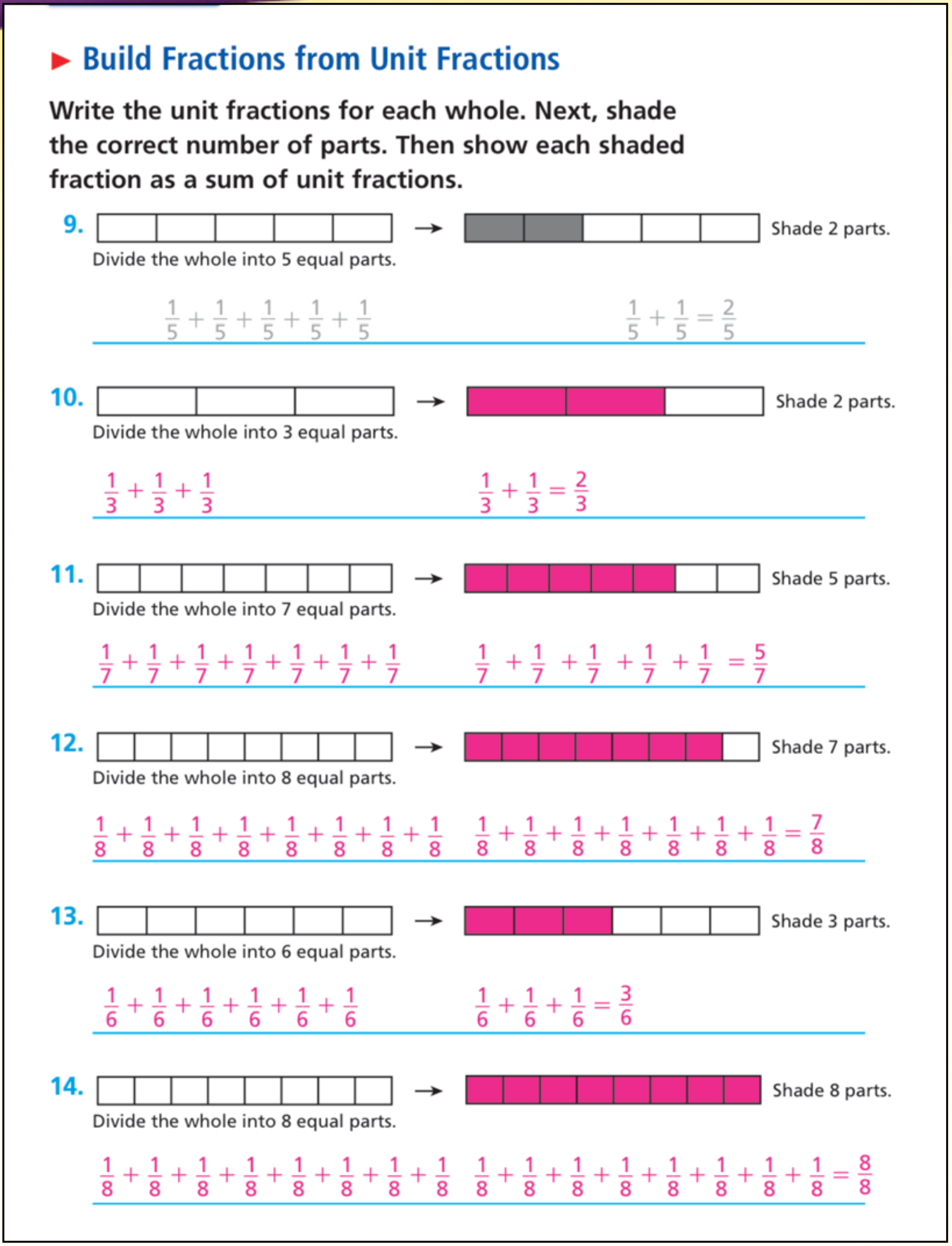
The *Children's Math Worlds* solution:

A. Initially make fraction drawings in two steps:
  1. Divide the whole into unit fractions.
  2. Circle/shade the number of unit fractions you want.

B. Write unit fractions numerically with each part to see a numerical unit fraction and to emphasize the meaning of the denominator as the number of equal parts of the whole.

First see all the unit fractions visually and in fraction notation.

Then shade or circle the unit fractions you need.



Build Fractions from Unit Fractions

Write the unit fractions for each whole. Next, shade the correct number of parts. Then show each shaded fraction as a sum of unit fractions.

9. Divide the whole into 5 equal parts. → Shade 2 parts.

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ $\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$

10. Divide the whole into 3 equal parts. → Shade 2 parts.

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

11. Divide the whole into 7 equal parts. → Shade 5 parts.

$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$ $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{5}{7}$

12. Divide the whole into 8 equal parts. → Shade 7 parts.

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{7}{8}$

13. Divide the whole into 6 equal parts. → Shade 3 parts.

$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$

14. Divide the whole into 8 equal parts. → Shade 8 parts.

$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{8}{8}$

G3 Number Line Standard 3.NF.2

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

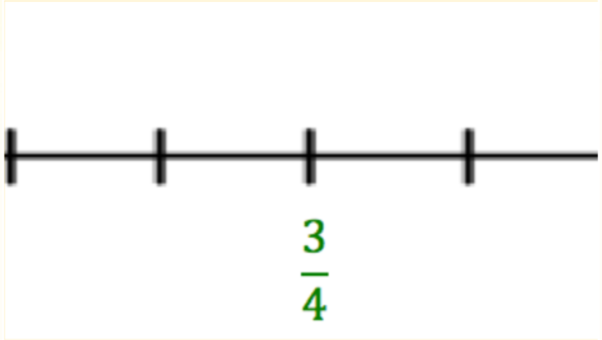
a. Represent a fraction 1/b on a number line diagram by defining the **interval** from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the **endpoint** of the part based at 0 locates the number 1/b on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its **endpoint** locates the number a/b on the number line.

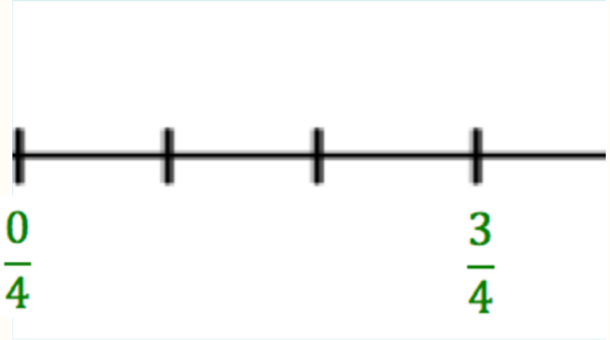
Not Enough Unit Lengths

Errors when drawing or using number lines

**Error: Not enough unit lengths-
student counts marks rather than lengths.**

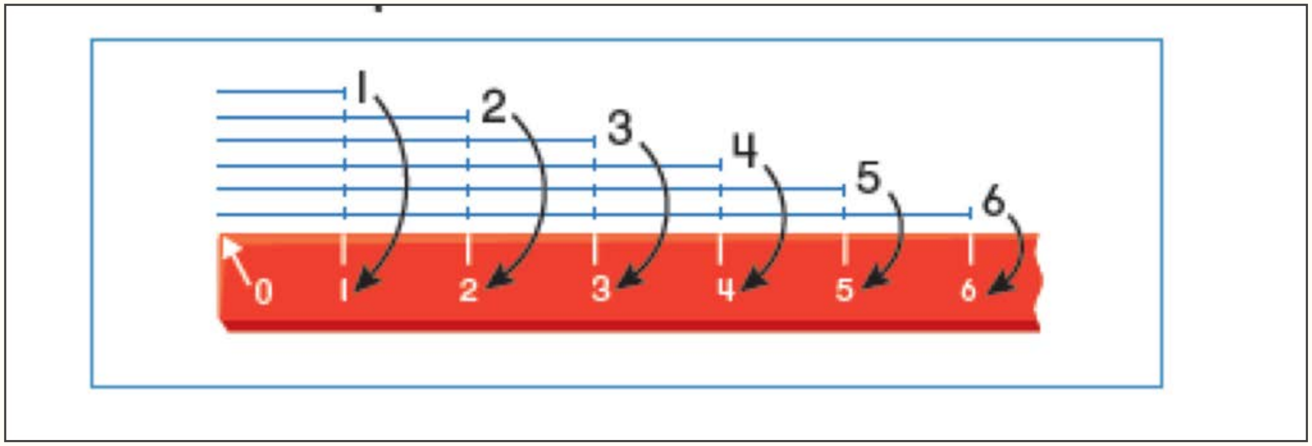


Correct: Count 3 unit lengths



Grade 2: Students have to see the length units on a ruler and number line diagram and on bar graph and line plot scales

Students need to see how length units are composed to make a ruler



G3 See Lengths on Number Lines

You can help students see the lengths on number lines by

a) folding fraction strips (can be of two lengths)

b) relating unit lengths on number lines to lengths more easily seen in fraction bars,

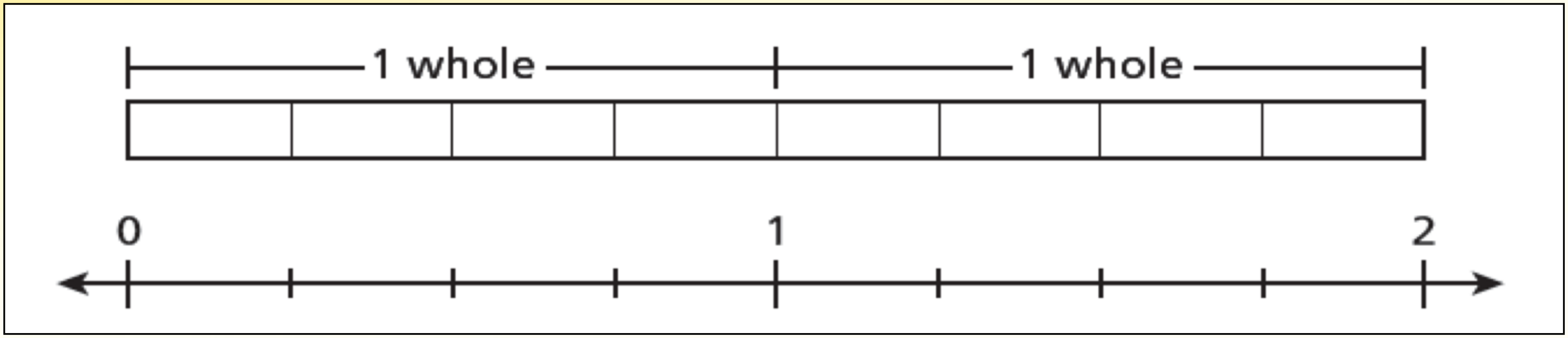
c) encircling the lengths on number lines,

d) having students run a finger along each length to count the unit fractions on the number line.

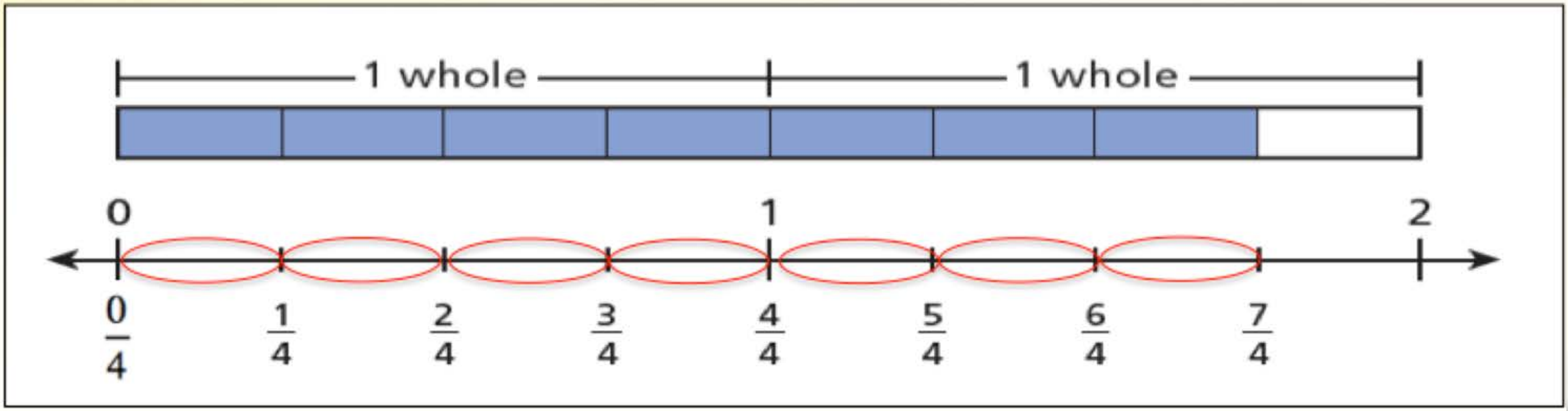
It is vital for the teacher and students to emphasize the lengths in all work with number lines.

Seeing the Fraction Lengths

Step 1: Make the 4 unit fractions ¼ within each 1 whole.

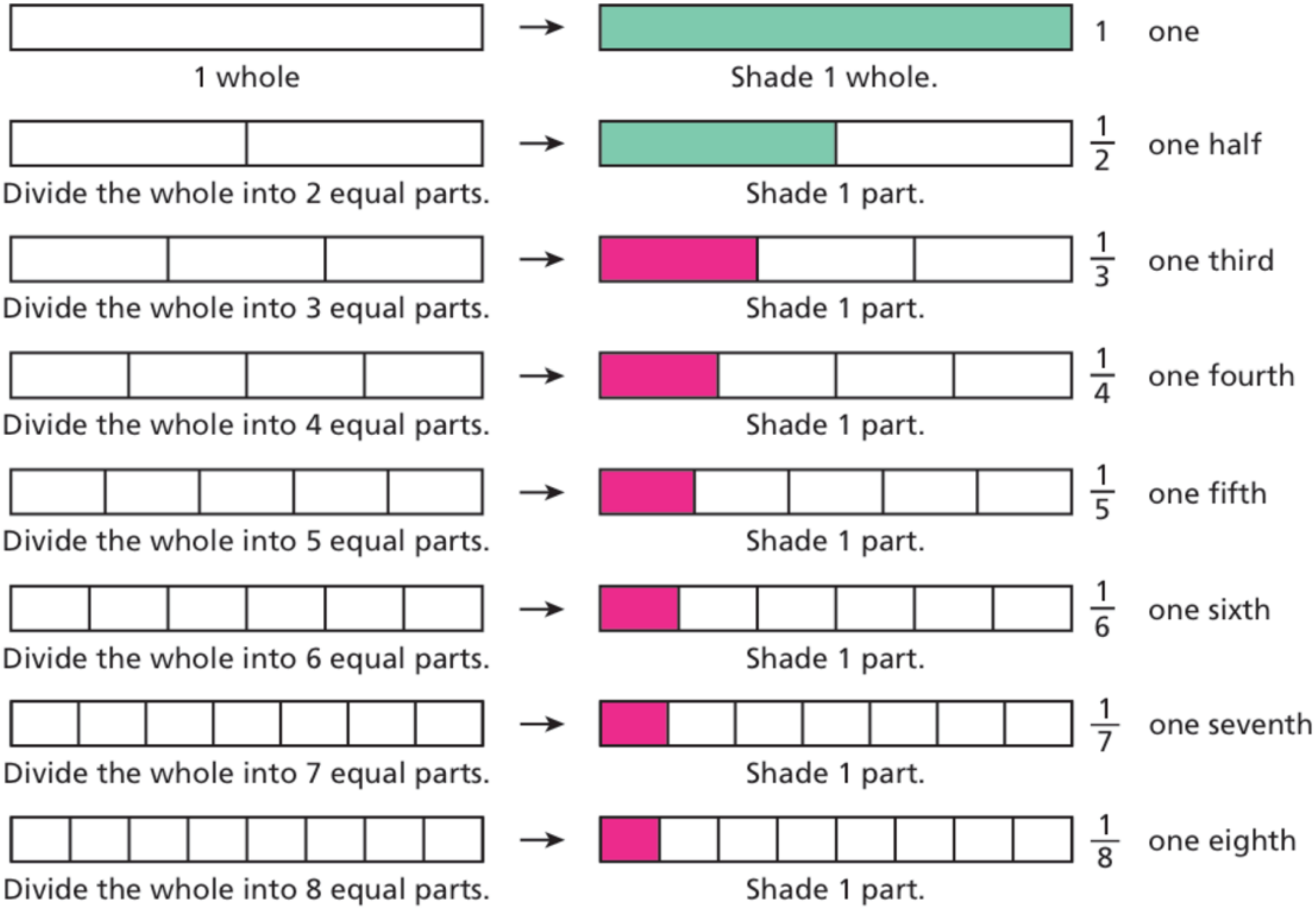


Step 2: Shade or encircle 7 unit fractions and label the number line.



G3 Compare Unit Fractions With Same Numerators

A **unit fraction** has a numerator of 1. Shade the rest of the fraction bars at the right below to represent unit fractions. What patterns do you see?



1 whole	Shade 1 whole.	1	one
Divide the whole into 2 equal parts.	Shade 1 part.	$\frac{1}{2}$	one half
Divide the whole into 3 equal parts.	Shade 1 part.	$\frac{1}{3}$	one third
Divide the whole into 4 equal parts.	Shade 1 part.	$\frac{1}{4}$	one fourth
Divide the whole into 5 equal parts.	Shade 1 part.	$\frac{1}{5}$	one fifth
Divide the whole into 6 equal parts.	Shade 1 part.	$\frac{1}{6}$	one sixth
Divide the whole into 7 equal parts.	Shade 1 part.	$\frac{1}{7}$	one seventh
Divide the whole into 8 equal parts.	Shade 1 part.	$\frac{1}{8}$	one eighth

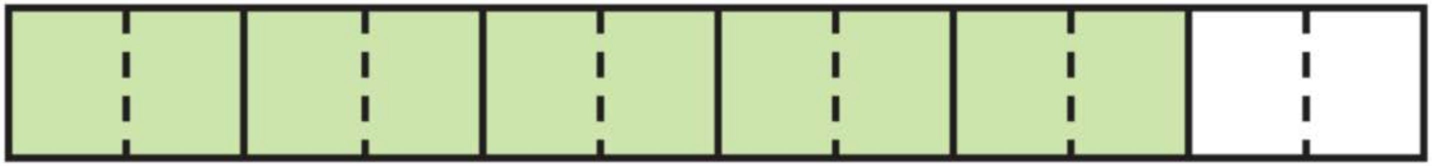
G4 Make Related Opposite Changes for Equivalent Fractions

Equivalent fractions are made by **dividing physically**
but **multiplying numerically**:

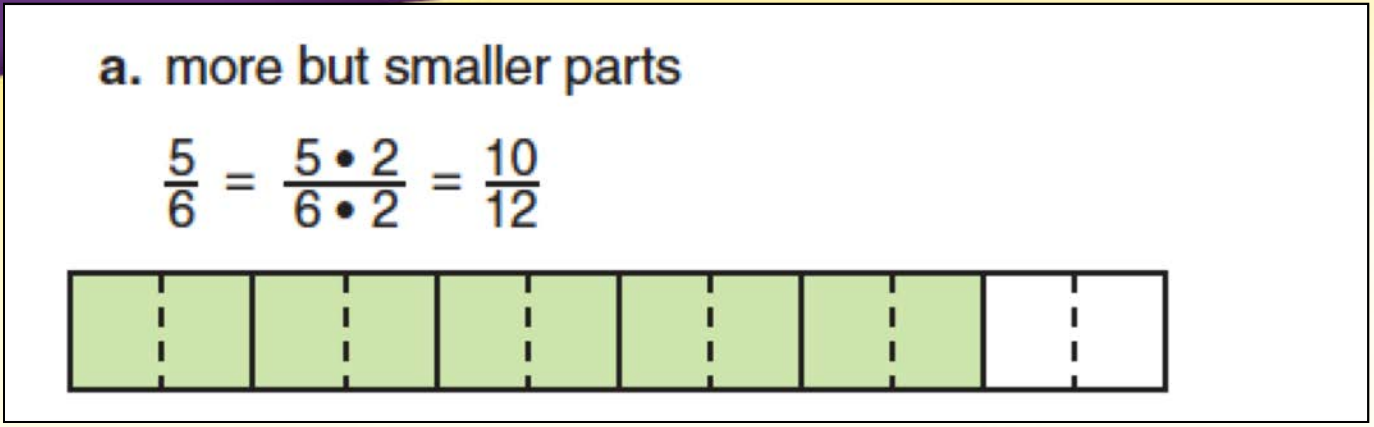
Physically divide (equal-fracture) each unit fraction **in the visual model** to get **more** but smaller unit fractions in the numerator and the denominator. **Numerically multiply** the top and bottom of the written fraction to get **more but smaller** unit fractions.

You see the **numbers** in the **written fraction getting bigger,**
but you **do not see** the unit fractions getting smaller except in visual models.
You have to remember that a larger denominator is a smaller unit fraction.

$$\frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$$



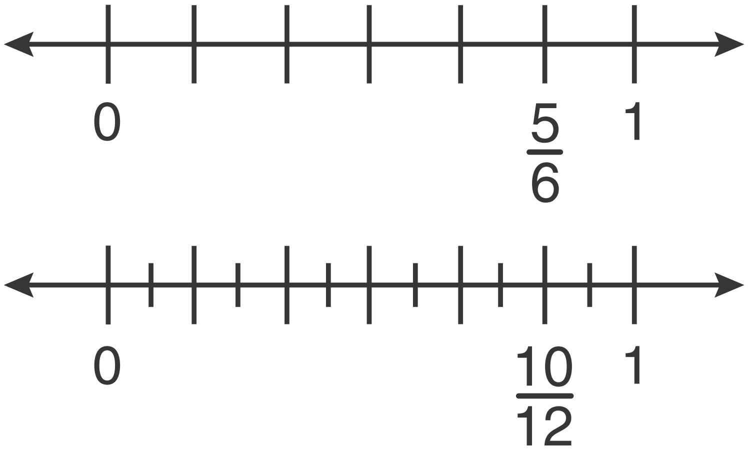
G4 More Visual Models for Equivalent Fractions



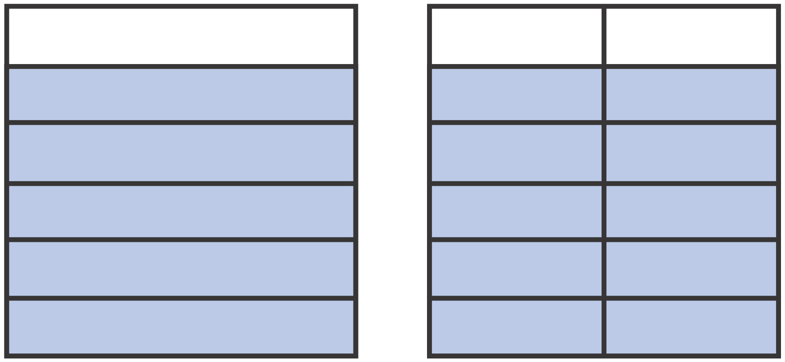
a. more but smaller parts

$$\frac{5}{6} = \frac{5 \bullet 2}{6 \bullet 2} = \frac{10}{12}$$

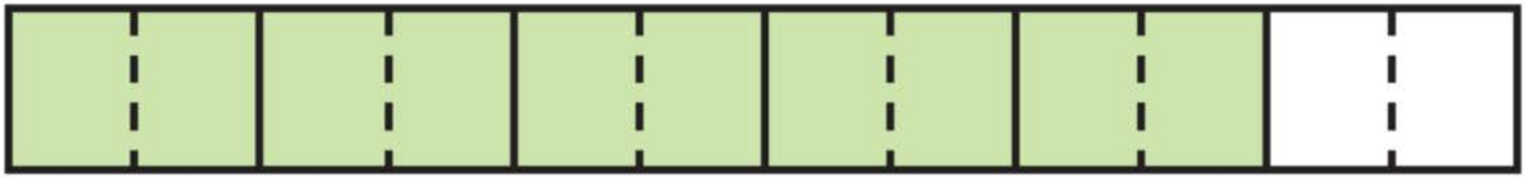
Number Line Model



Area Model

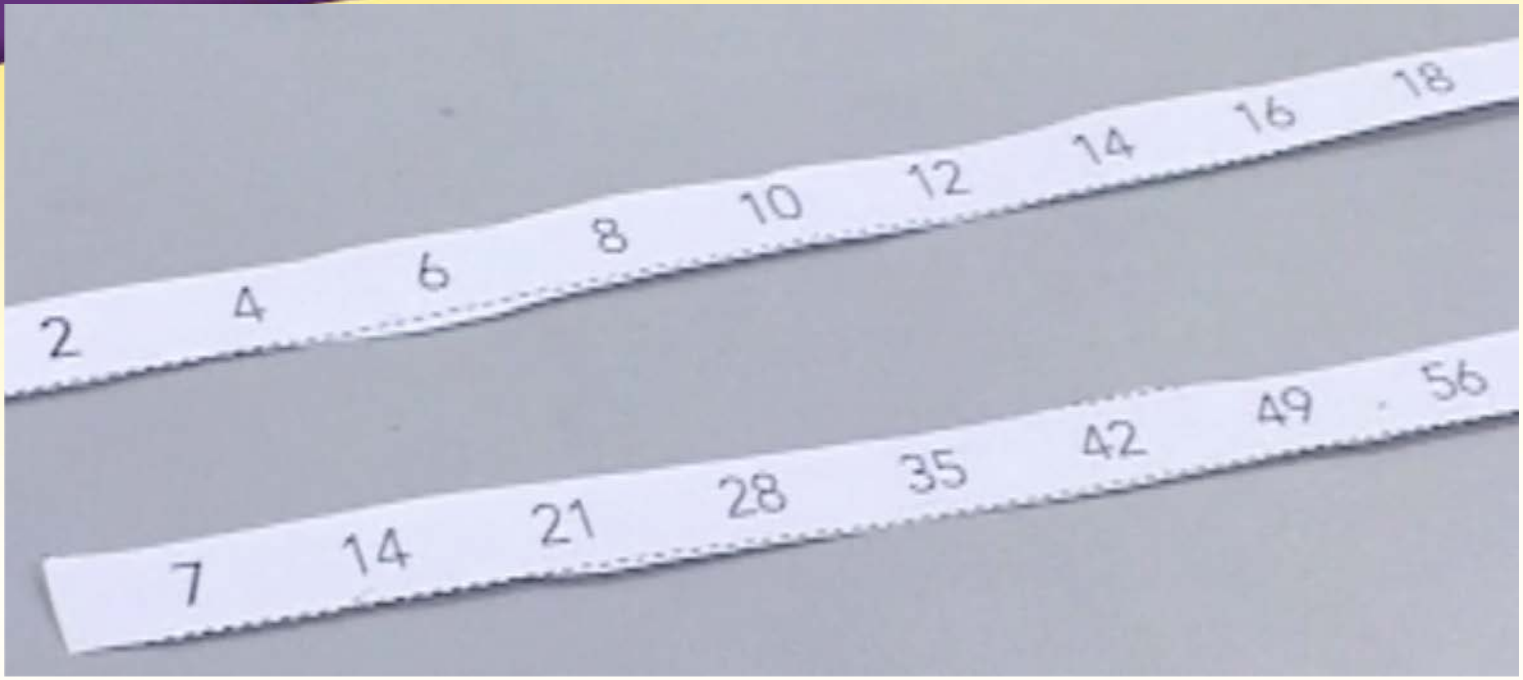


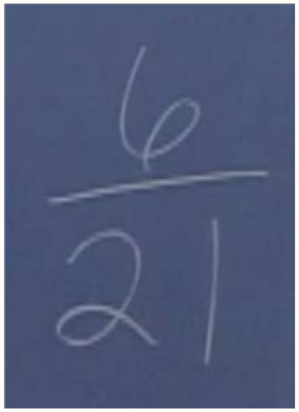
G4 Many Equivalent Fractions



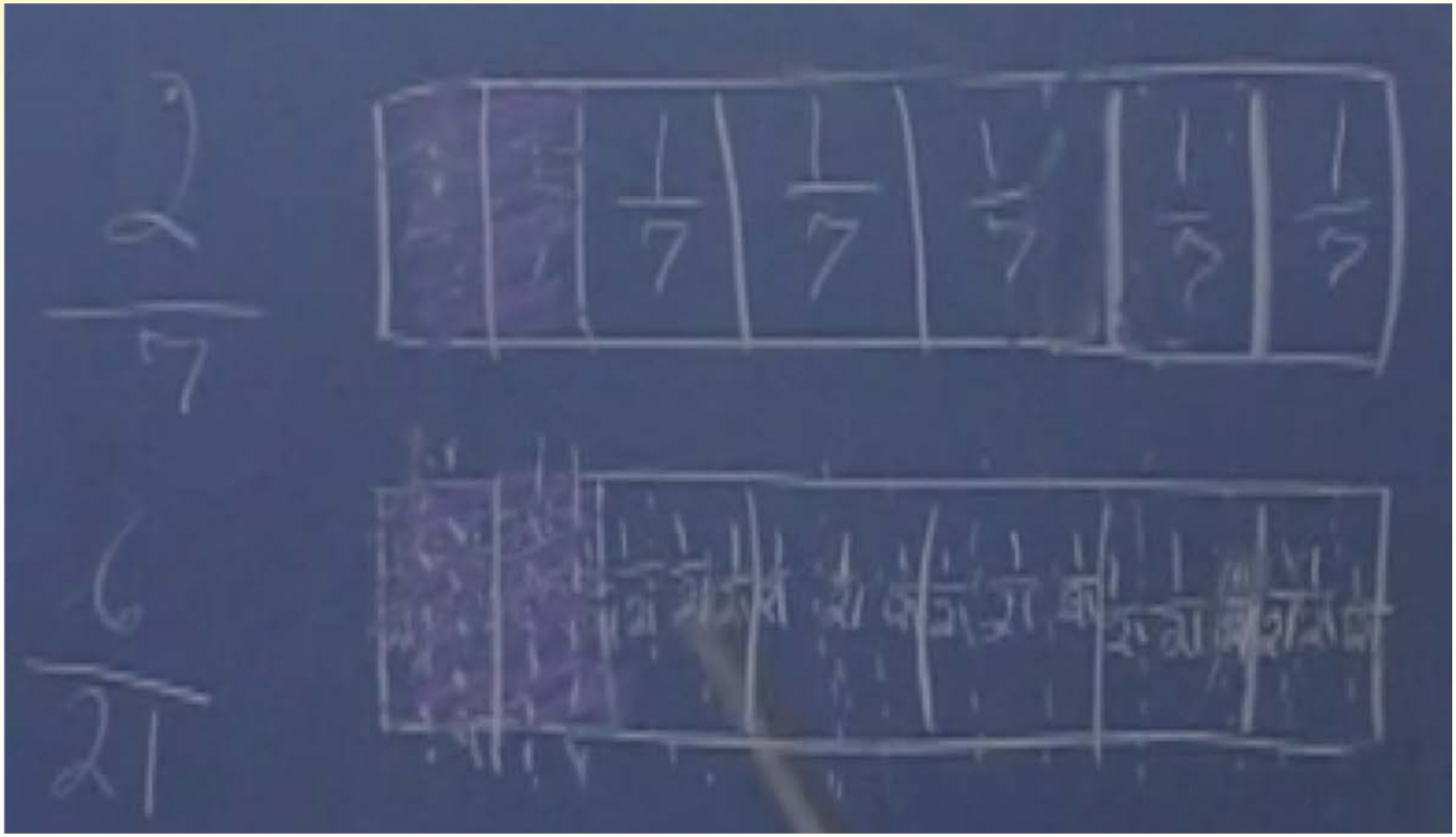
$$\frac{5}{6} = \overset{\cdot 2}{\underset{\cdot 2}{\frac{10}{12}}} = \overset{\cdot 3}{\underset{\cdot 3}{\frac{15}{18}}} = \overset{\cdot 4}{\underset{\cdot 4}{\frac{20}{24}}} = \overset{\cdot 5}{\underset{\cdot 5}{\frac{25}{30}}} = \overset{\cdot 6}{\underset{\cdot 6}{\frac{30}{36}}} = \overset{\cdot 7}{\underset{\cdot 7}{\frac{35}{42}}} = \overset{\cdot 8}{\underset{\cdot 8}{\frac{40}{48}}} = \overset{\cdot 9}{\underset{\cdot 9}{\frac{45}{56}}}$$

Fraction Strips for 2/7: Multiplication Table Rows

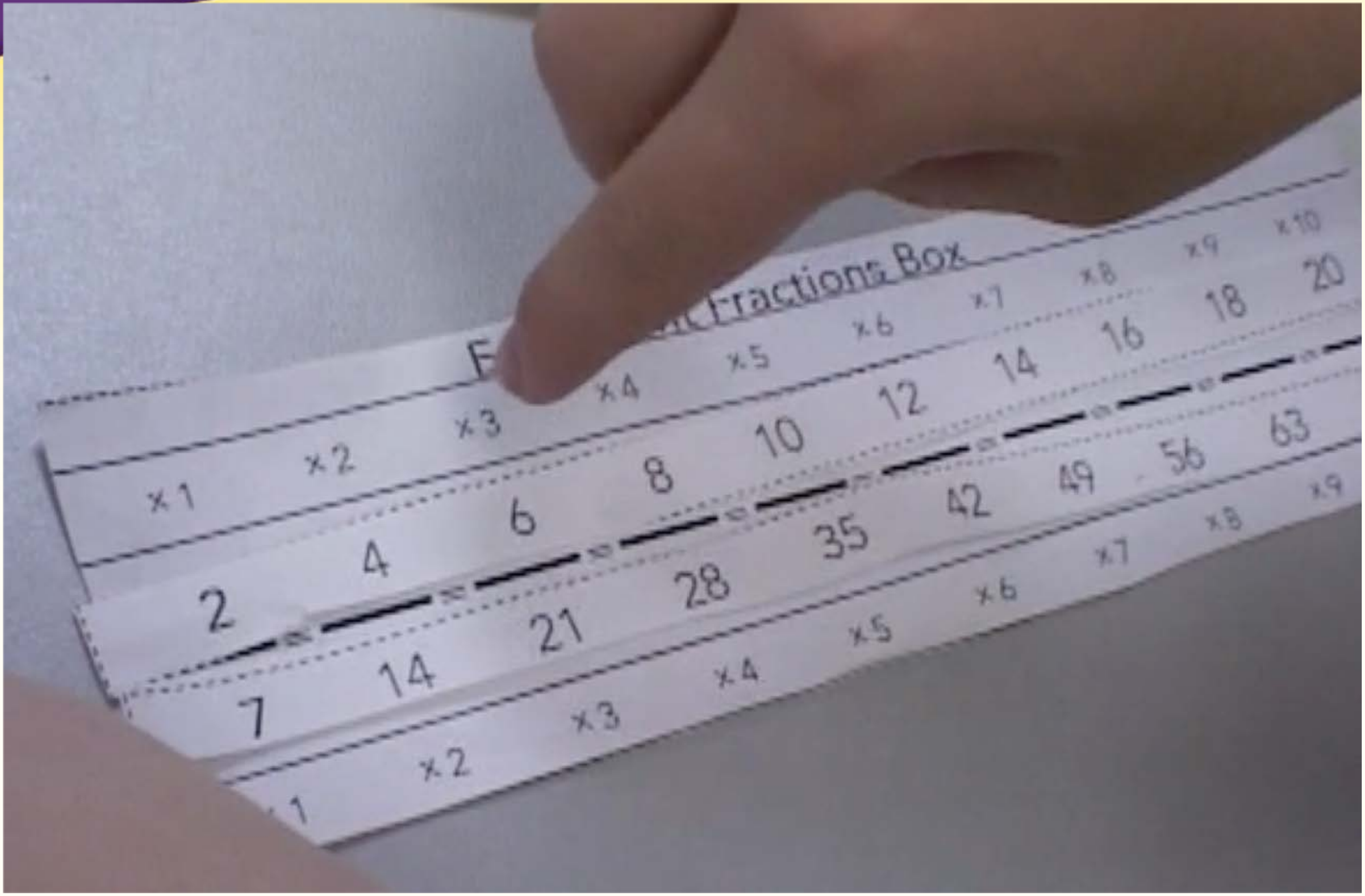




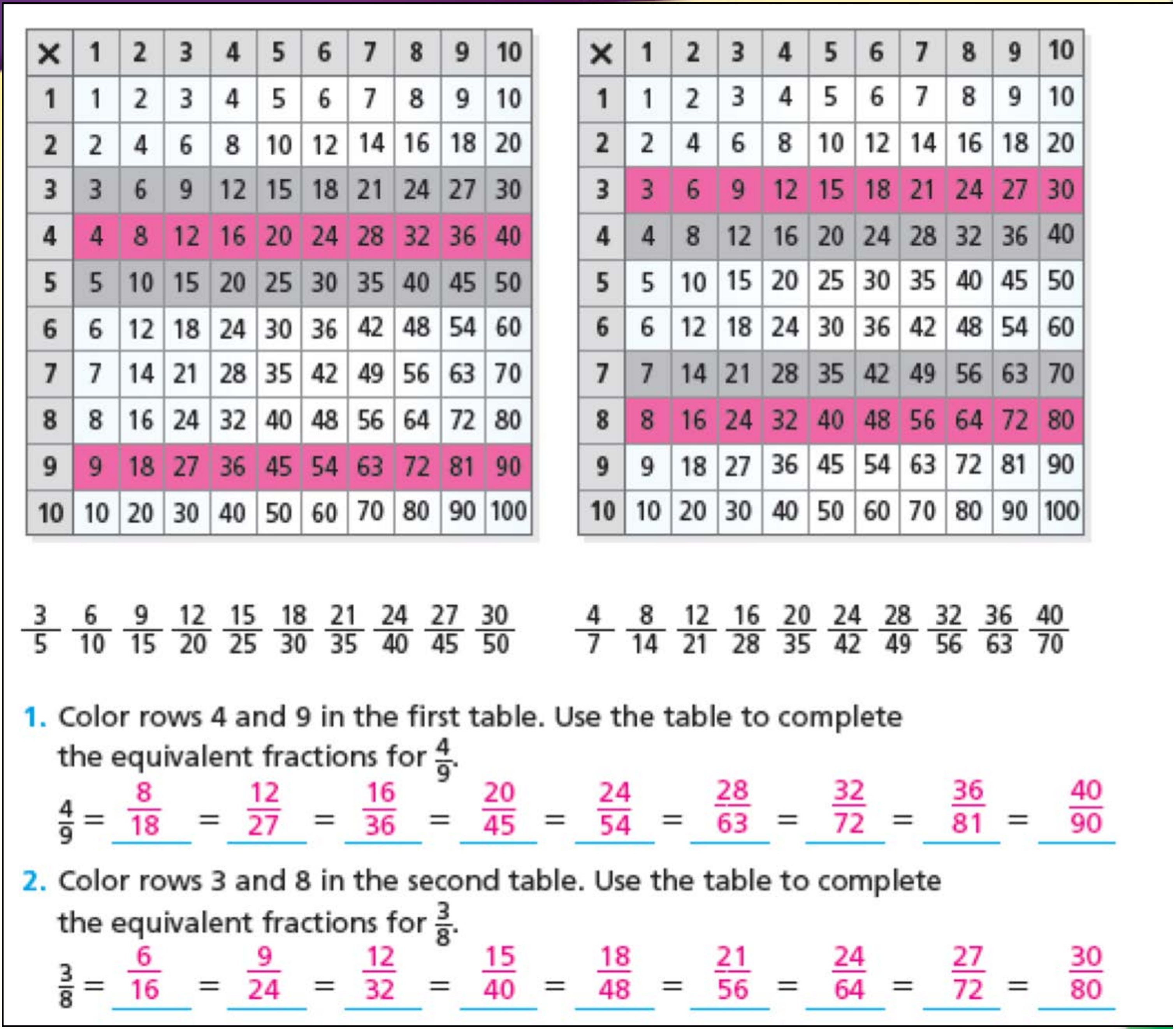
Drawing to show why the fractions are equivalent



Equivalent fractions box



Equivalent Fractions in the Multiplication Table



×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

$\frac{3}{5}$ $\frac{6}{10}$ $\frac{9}{15}$ $\frac{12}{20}$ $\frac{15}{25}$ $\frac{18}{30}$ $\frac{21}{35}$ $\frac{24}{40}$ $\frac{27}{45}$ $\frac{30}{50}$

$\frac{4}{7}$ $\frac{8}{14}$ $\frac{12}{21}$ $\frac{16}{28}$ $\frac{20}{35}$ $\frac{24}{42}$ $\frac{28}{49}$ $\frac{32}{56}$ $\frac{36}{63}$ $\frac{40}{70}$

1. Color rows 4 and 9 in the first table. Use the table to complete the equivalent fractions for $\frac{4}{9}$.

$\frac{4}{9} = \frac{8}{18} = \frac{12}{27} = \frac{16}{36} = \frac{20}{45} = \frac{24}{54} = \frac{28}{63} = \frac{32}{72} = \frac{36}{81} = \frac{40}{90}$

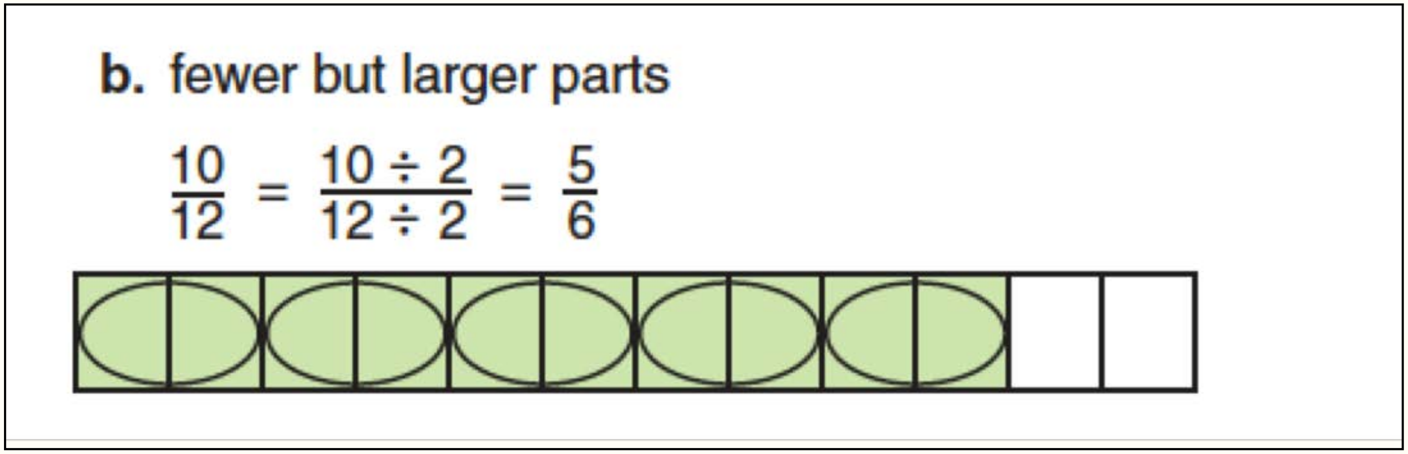
2. Color rows 3 and 8 in the second table. Use the table to complete the equivalent fractions for $\frac{3}{8}$.

$\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{12}{32} = \frac{15}{40} = \frac{18}{48} = \frac{21}{56} = \frac{24}{64} = \frac{27}{72} = \frac{30}{80}$

G4 Make Related Opposite Changes for Simplifying to an Equivalent Fraction

Equivalent fractions are made by **grouping physically**
but **dividing numerically**:

You see the **numbers** in the **written fraction getting smaller,**
but you **do not see** the unit fractions getting bigger except in visual models.
You have to remember that a smaller denominator is a larger unit fraction.



b. fewer but larger parts

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

Common error when adding fractions is?

$$\frac{2}{7}+\frac{3}{7}=$$

G4 Adding and Subtracting Common Error

Common error:
Add fractions by adding the tops and adding the bottoms

$$\frac{2}{7}+\frac{3}{7}=\frac{5}{14}$$

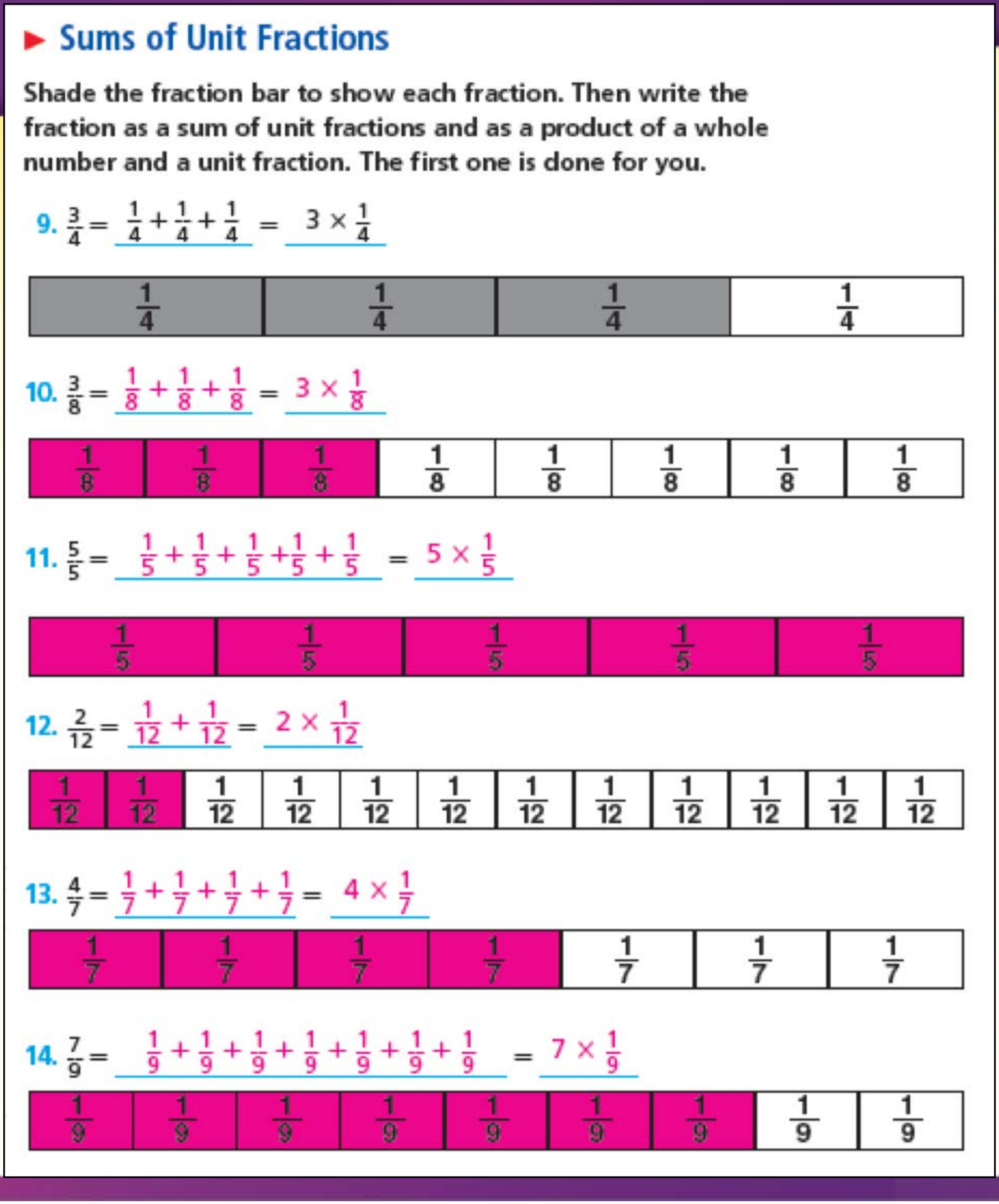
Children's Math Worlds Solutions

1. Offer many experiences writing sums of unit fractions where the denominator stays the same number and include drawings.

2. Find partners of a whole to practice this concept and use drawings.

1. Show addition and subtraction models. Write the sum or difference above the unit fraction as a middle step.

G4 Write Sums of Unit Fractions



Sums of Unit Fractions

Shade the fraction bar to show each fraction. Then write the fraction as a sum of unit fractions and as a product of a whole number and a unit fraction. The first one is done for you.

9. $\frac{3}{4} = \frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 3 \times \frac{1}{4}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

10. $\frac{3}{8} = \frac{1}{8}+\frac{1}{8}+\frac{1}{8} = 3 \times \frac{1}{8}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

11. $\frac{5}{5} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5} = 5 \times \frac{1}{5}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

12. $\frac{2}{12} = \frac{1}{12}+\frac{1}{12} = 2 \times \frac{1}{12}$

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

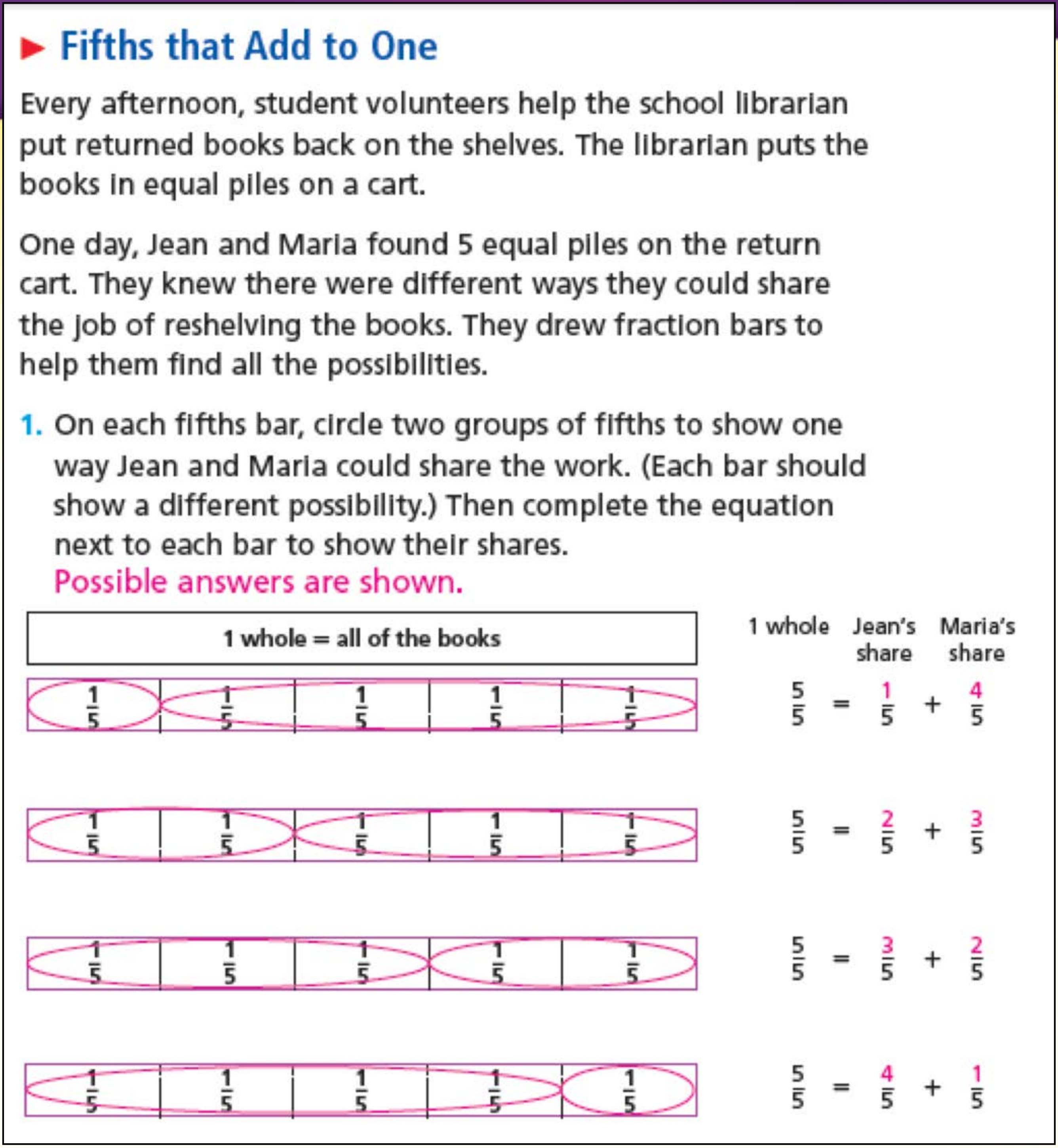
13. $\frac{4}{7} = \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7} = 4 \times \frac{1}{7}$

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

14. $\frac{7}{9} = \frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9} = 7 \times \frac{1}{9}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

G4 Decompose Fractions



► Fifths that Add to One

Every afternoon, student volunteers help the school librarian put returned books back on the shelves. The librarian puts the books in equal piles on a cart.

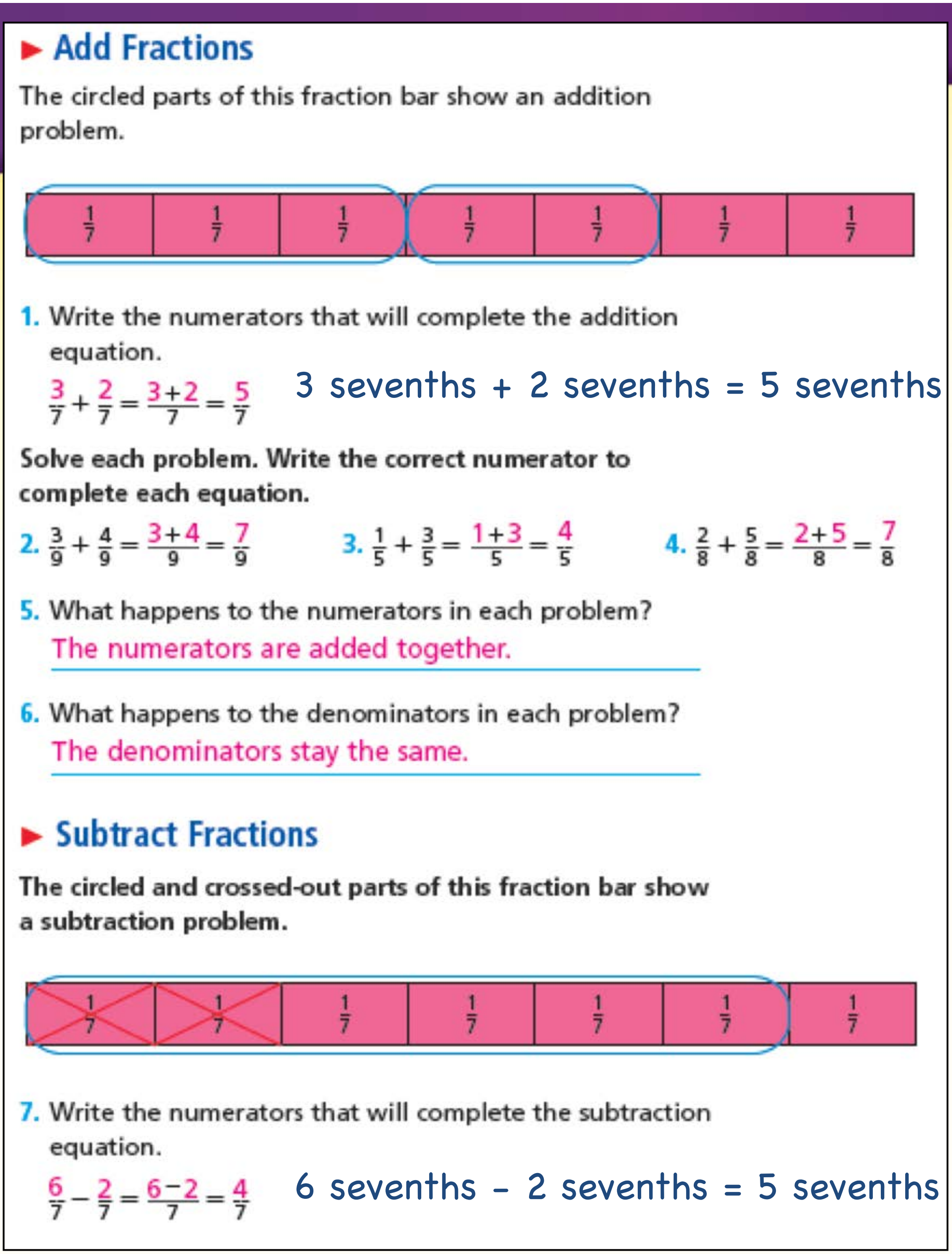
One day, Jean and Maria found 5 equal piles on the return cart. They knew there were different ways they could share the job of reshelving the books. They drew fraction bars to help them find all the possibilities.

1. On each fifths bar, circle two groups of fifths to show one way Jean and Maria could share the work. (Each bar should show a different possibility.) Then complete the equation next to each bar to show their shares.
   Possible answers are shown.

1 whole = all of the books

					1 whole		Jean's share		Maria's share
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{5}{5}$	=	$\frac{1}{5}$	+	$\frac{4}{5}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{5}{5}$	=	$\frac{2}{5}$	+	$\frac{3}{5}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{5}{5}$	=	$\frac{3}{5}$	+	$\frac{2}{5}$
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{5}{5}$	=	$\frac{4}{5}$	+	$\frac{1}{5}$

G4
Add and Subtract Fractions



▶ Add Fractions

The circled parts of this fraction bar show an addition problem.

1. Write the numerators that will complete the addition equation.

$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

3 sevenths + 2 sevenths = 5 sevenths

Solve each problem. Write the correct numerator to complete each equation.

2. $\frac{3}{9}+\frac{4}{9}=\frac{3+4}{9}=\frac{7}{9}$

3. $\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{4}{5}$

4. $\frac{2}{8}+\frac{5}{8}=\frac{2+5}{8}=\frac{7}{8}$

5. What happens to the numerators in each problem?
The numerators are added together.

6. What happens to the denominators in each problem?
The denominators stay the same.

▶ Subtract Fractions

The circled and crossed-out parts of this fraction bar show a subtraction problem.

7. Write the numerators that will complete the subtraction equation.

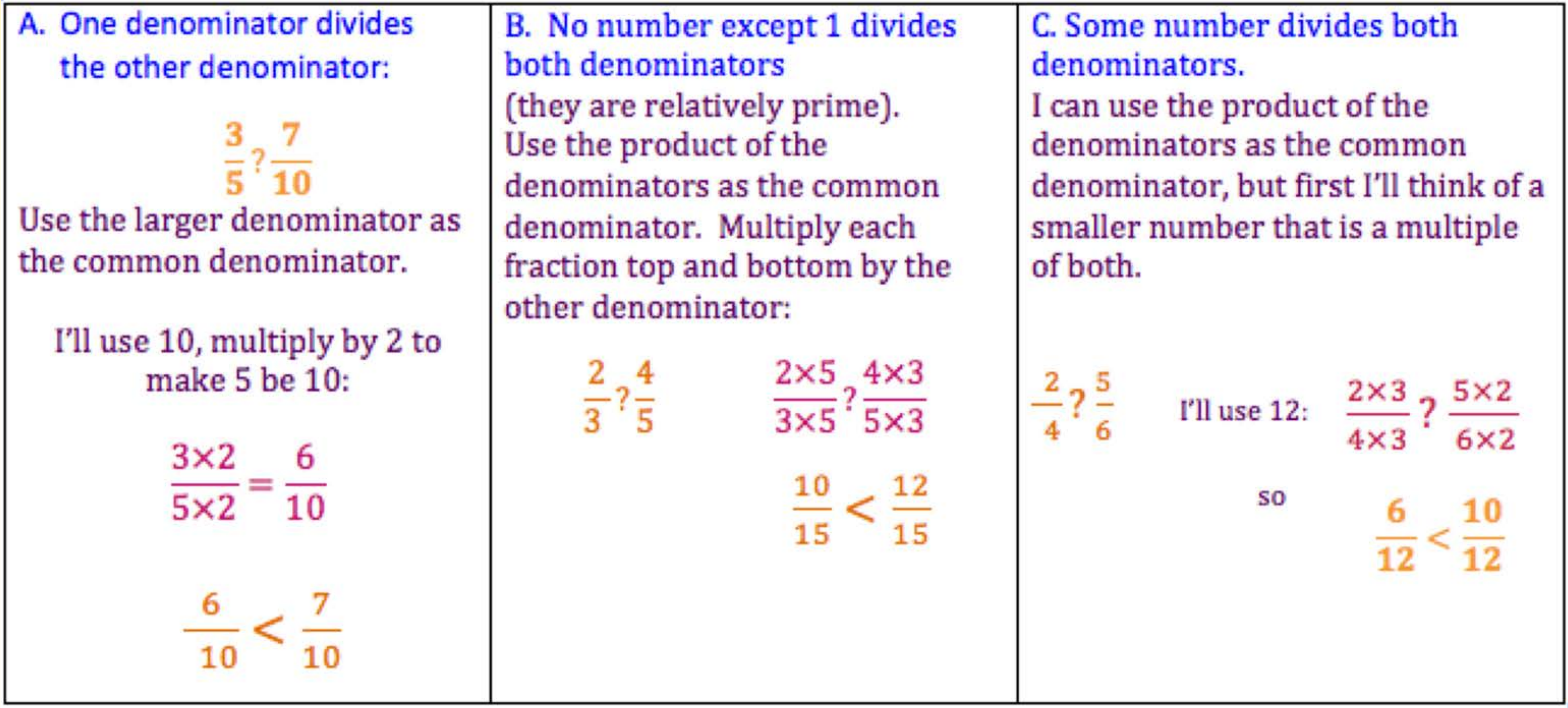
$\frac{6}{7}-\frac{2}{7}=\frac{6-2}{7}=\frac{4}{7}$

6 sevenths - 2 sevenths = 5 sevenths

G4 and G5 Cases for Finding a Common Denominator

Strategies for finding the common denominator.

Analyze pairs of fractions into three classes:

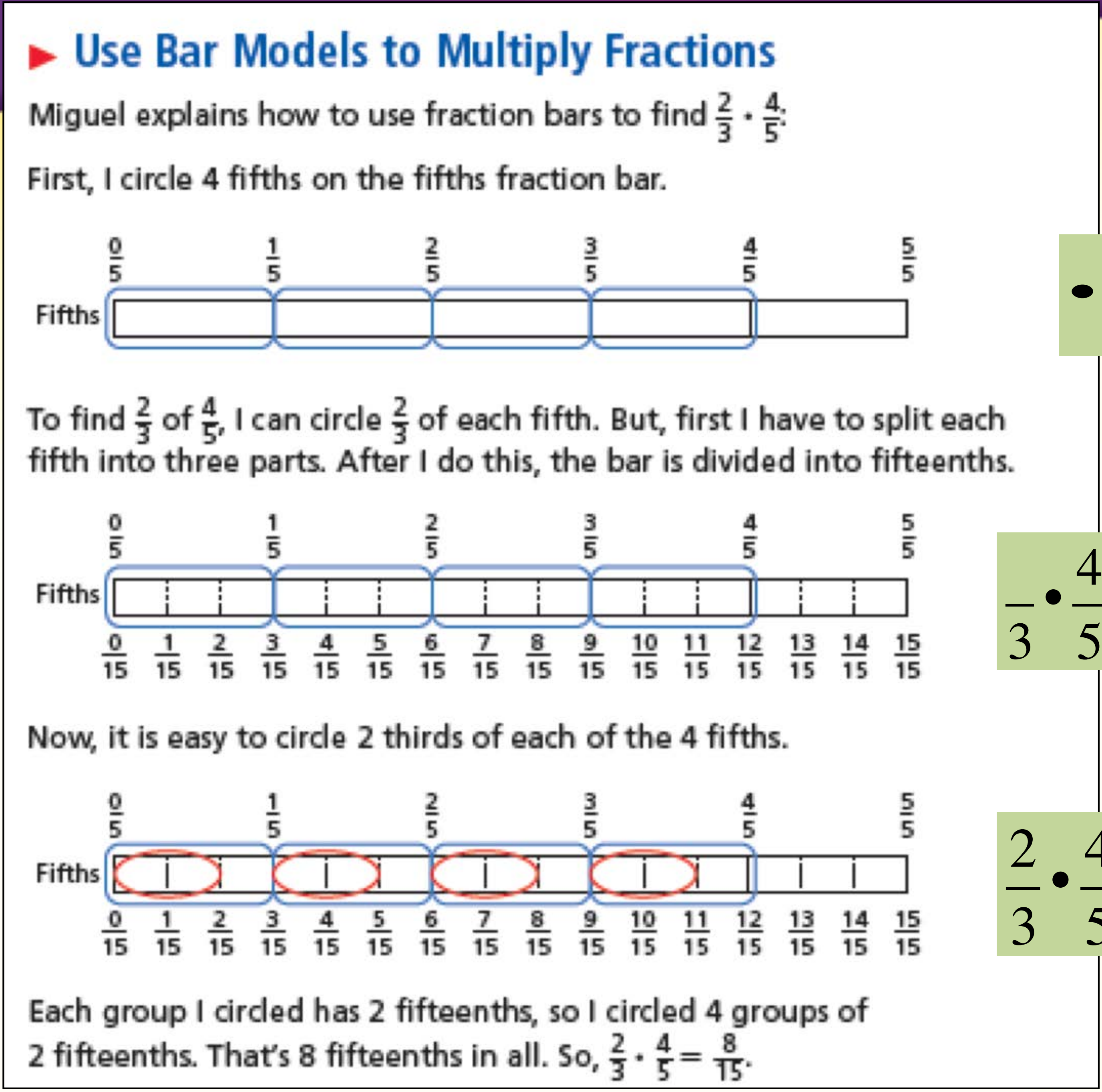


A. One denominator divides the other denominator:	B. No number except 1 divides both denominators	C. Some number divides both denominators.
$\frac{3}{5} ? \frac{7}{10}$ Use the larger denominator as the common denominator. I'll use 10, multiply by 2 to make 5 be 10: $\frac{3\times2}{5\times2} = \frac{6}{10}$ $\frac{6}{10} < \frac{7}{10}$	(they are relatively prime). Use the product of the denominators as the common denominator. Multiply each fraction top and bottom by the other denominator: $\frac{2}{3} ? \frac{4}{5}$ $\frac{2\times5}{3\times5} ? \frac{4\times3}{5\times3}$ $\frac{10}{15} < \frac{12}{15}$	I can use the product of the denominators as the common denominator, but first I'll think of a smaller number that is a multiple of both. $\frac{2}{4} ? \frac{5}{6}$ I'll use 12: $\frac{2\times3}{4\times3} ? \frac{5\times2}{6\times2}$ so $\frac{6}{12} < \frac{10}{12}$

G5 5.NF.4

Any Fraction Times Any Fraction

Length Model Using Unit Fractions



▶ Use Bar Models to Multiply Fractions

Miguel explains how to use fraction bars to find $\frac{2}{3} \cdot \frac{4}{5}$.

First, I circle 4 fifths on the fifths fraction bar.

To find $\frac{2}{3}$ of $\frac{4}{5}$, I can circle $\frac{2}{3}$ of each fifth. But, first I have to split each fifth into three parts. After I do this, the bar is divided into fifteenths.

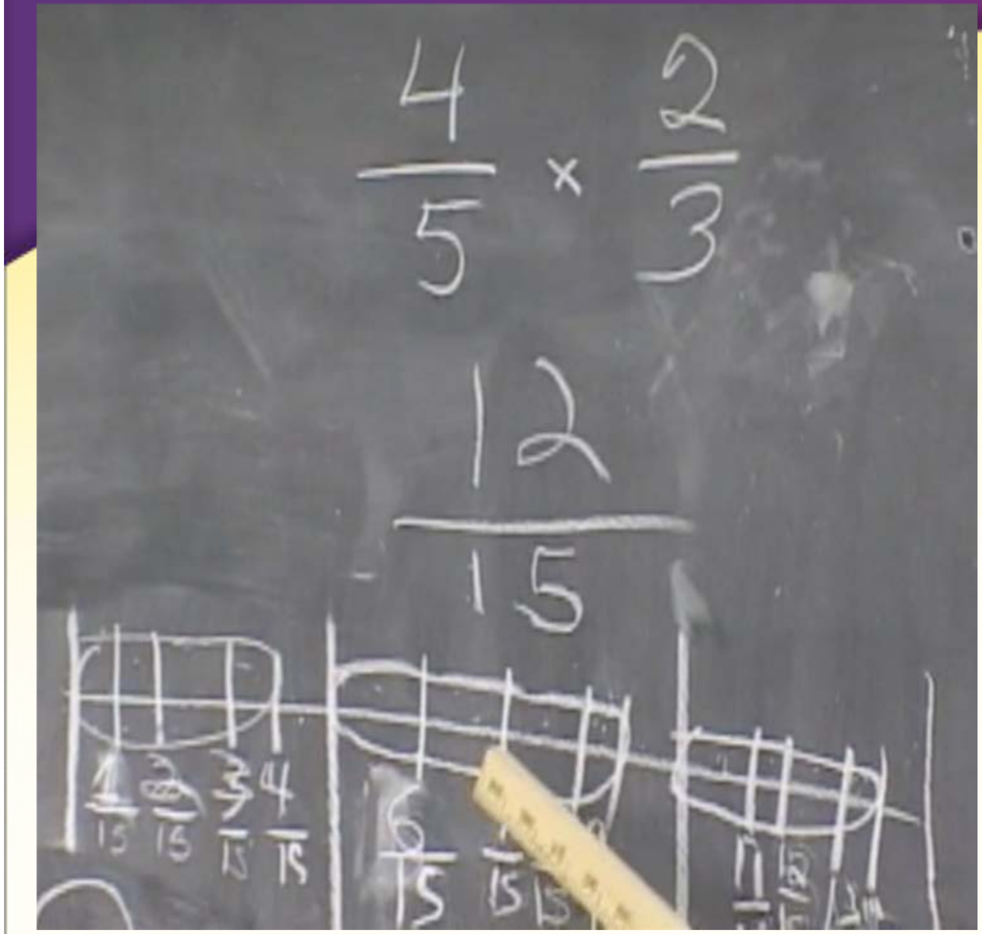
Now, it is easy to circle 2 thirds of each of the 4 fifths.

Each group I circled has 2 fifteenths, so I circled 4 groups of 2 fifteenths. That's 8 fifteenths in all. So, $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$.

$$\bullet \frac{4}{5}$$

$$\frac{}{3} \bullet \frac{4}{5} = \frac{}{15}$$

$$\frac{2}{3} \bullet \frac{4}{5} = \frac{8}{15}$$

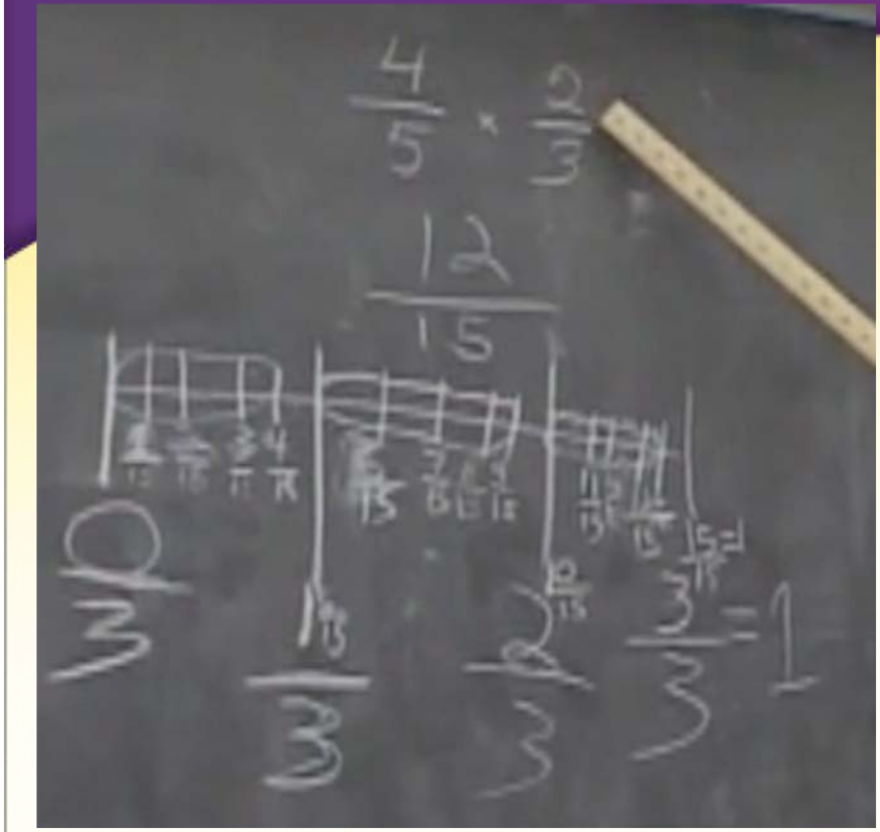


Student correctly 5-fractured each of the 1/3 to get the new unit fractions 15ths.

But she then incorrectly took 4 of these 15^{ths} for 3 thirds instead of only for 2 of the thirds. Here she is explaining that she took 4 of the 5 equal parts of each third.

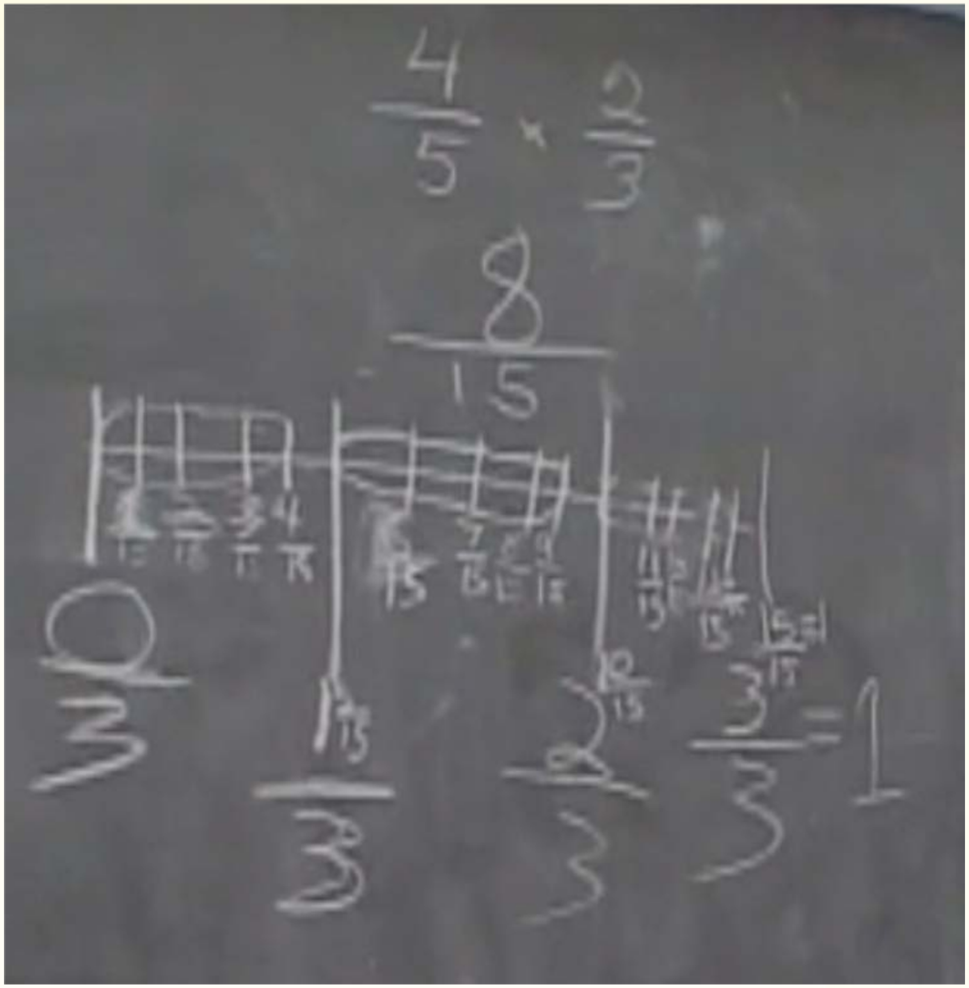
She then sees her mistake and erases the circle for the third third (next slide).


4/5 × 2/3
12/15
2/3
1/15
2/3
2/3
12/15
=1



She here points to the 2 in the 2/3 to explain why she should only have circled 4/5 in 2 of the 1/3ds.

She then says the answer 12/15 is also wrong and it should be 8 of the 15ths (4 and 4) and not 12, and she changes the 12 to 8.



The area model shown next is efficient, but it gives the answer and students only need to recognize it. The length model (fraction bar or fraction number line) demands more thinking from students.

All models require students to reflect on a given problem and see patterns that produce an answer so that the student can see and understand the general pattern:

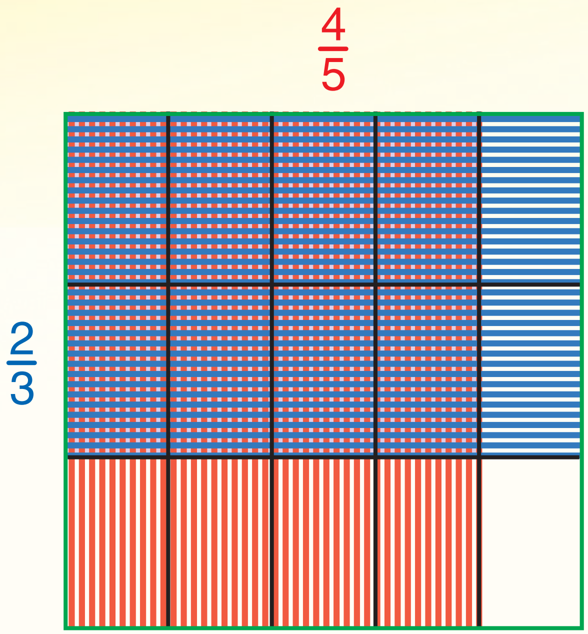
multiply tops and multiply bottoms

G5 5.NF.4

Any Fraction Times Any Fraction

Area Model

Area Model



$\frac{2}{3}$ times $\frac{4}{5}$

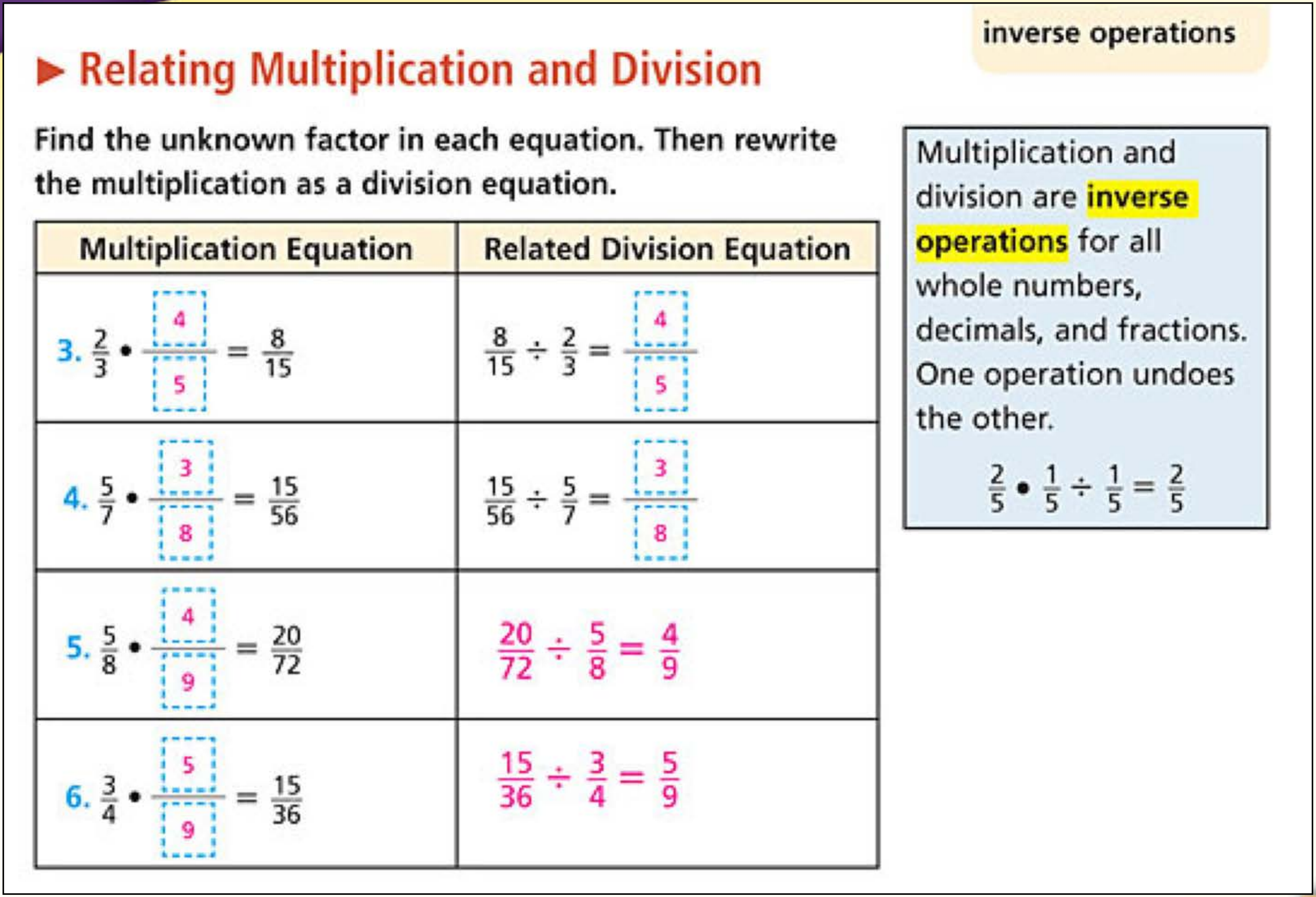
$\frac{2}{3}$ of each of the $\frac{4}{5}$

$\frac{2}{3} \bullet \frac{4}{5} = \frac{2 \bullet 4}{3 \bullet 5} = \frac{8}{15}$

How would you want to divide fractions?

What would make sense from multiplying fractions?

G6 Seeing Division as Finding the Unknown Factor Numerically



inverse operations

▶ Relating Multiplication and Division

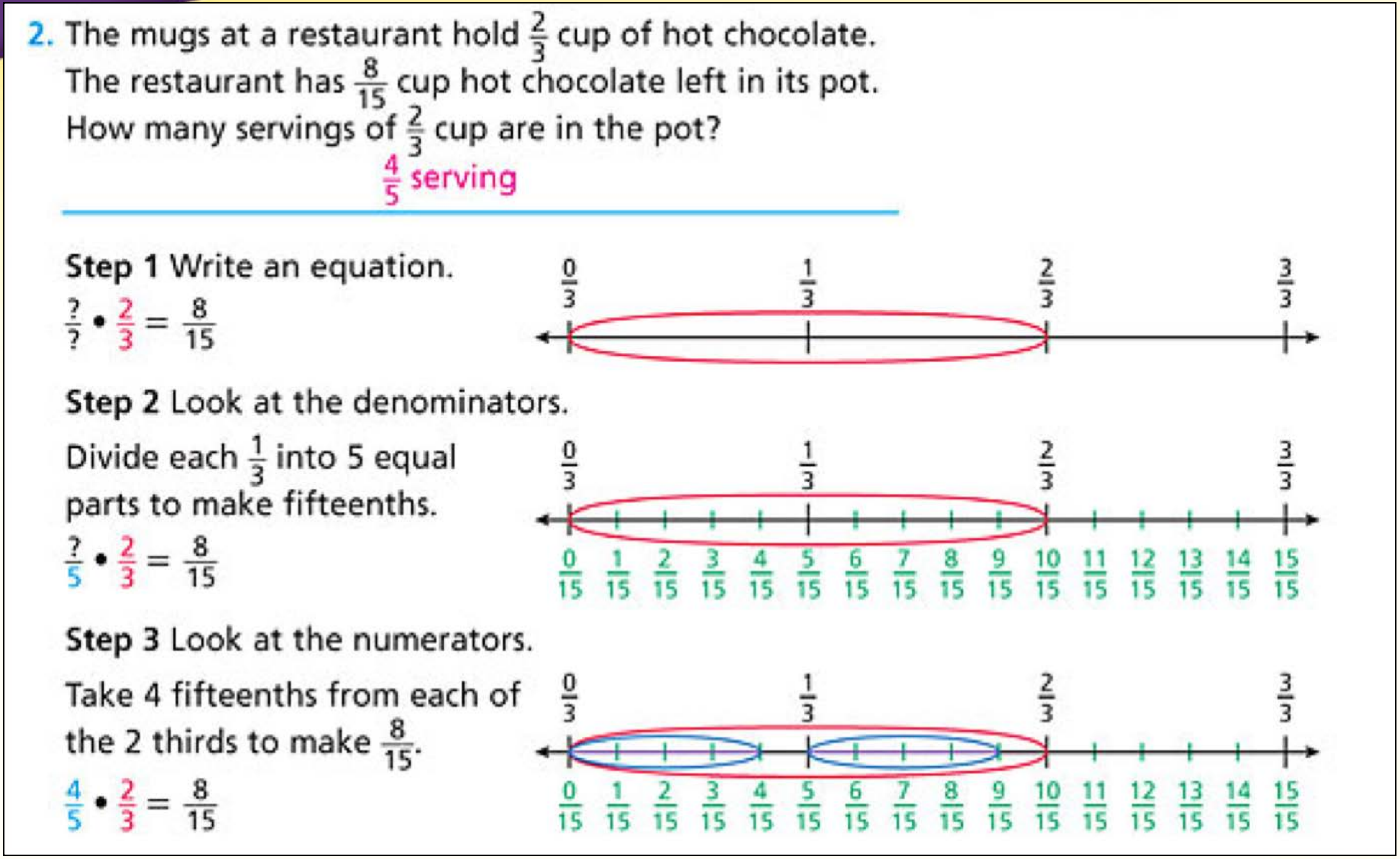
Find the unknown factor in each equation. Then rewrite the multiplication as a division equation.

Multiplication Equation	Related Division Equation
3. $\frac{2}{3} \cdot \frac{\boxed{4}}{\boxed{5}} = \frac{8}{15}$	$\frac{8}{15} \div \frac{2}{3} = \frac{\boxed{4}}{\boxed{5}}$
4. $\frac{5}{7} \cdot \frac{\boxed{3}}{\boxed{8}} = \frac{15}{56}$	$\frac{15}{56} \div \frac{5}{7} = \frac{\boxed{3}}{\boxed{8}}$
5. $\frac{5}{8} \cdot \frac{\boxed{4}}{\boxed{9}} = \frac{20}{72}$	$\frac{20}{72} \div \frac{5}{8} = \frac{4}{9}$
6. $\frac{3}{4} \cdot \frac{\boxed{5}}{\boxed{9}} = \frac{15}{36}$	$\frac{15}{36} \div \frac{3}{4} = \frac{5}{9}$

Multiplication and division are **inverse operations** for all whole numbers, decimals, and fractions. One operation undoes the other.

$$\frac{2}{5} \cdot \frac{1}{5} \div \frac{1}{5} = \frac{2}{5}$$

G6 Seeing Division as Finding the Unknown Factor in an Equal Groups Situation



2. The mugs at a restaurant hold $\frac{2}{3}$ cup of hot chocolate. The restaurant has $\frac{8}{15}$ cup hot chocolate left in its pot. How many servings of $\frac{2}{3}$ cup are in the pot?

$\frac{4}{5}$ serving

Step 1 Write an equation.

$\frac{?}{?} \bullet \frac{2}{3} = \frac{8}{15}$

Step 2 Look at the denominators.

Divide each $\frac{1}{3}$ into 5 equal parts to make fifteenths.

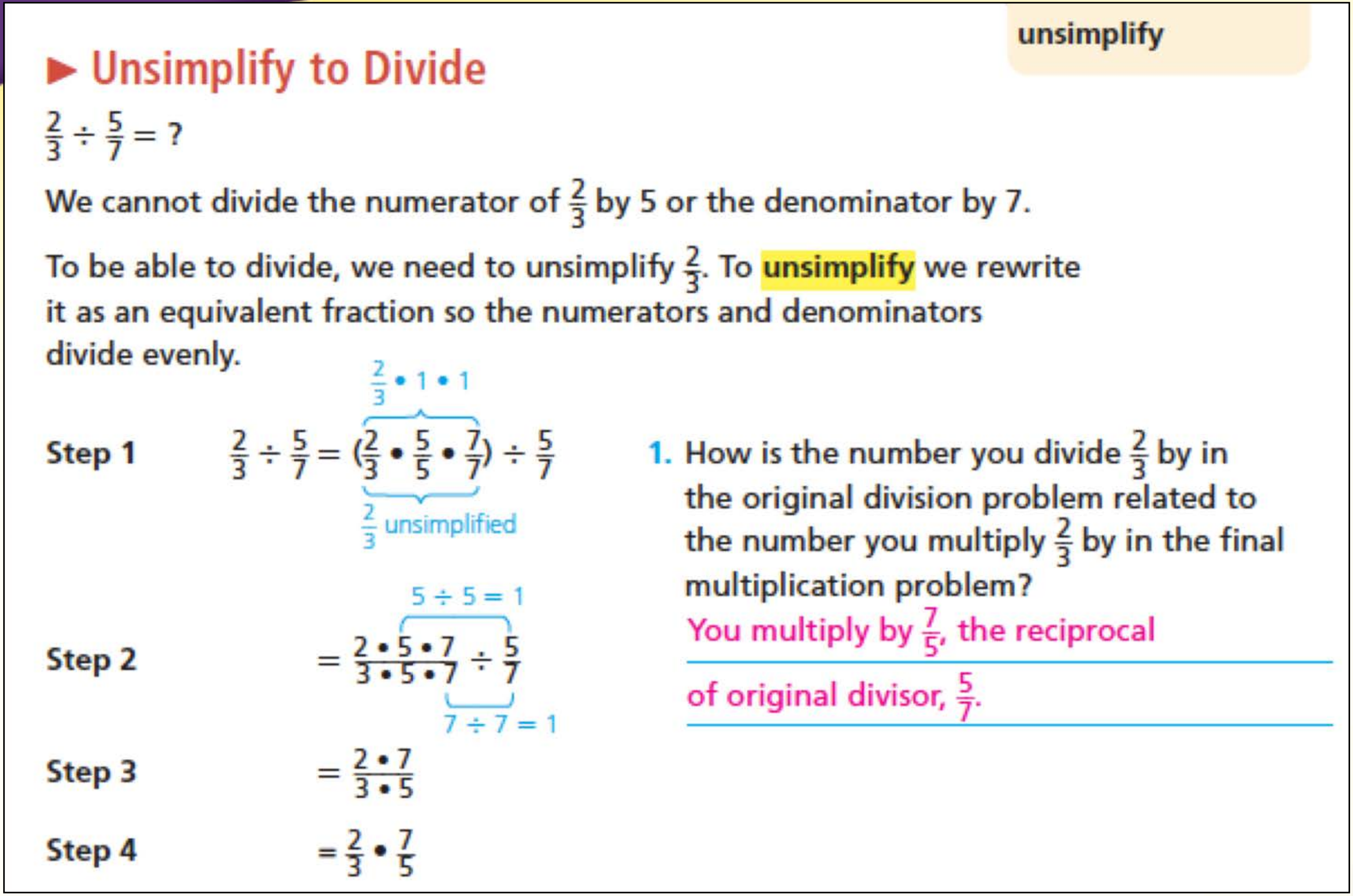
$\frac{?}{5} \bullet \frac{2}{3} = \frac{8}{15}$

Step 3 Look at the numerators.

Take 4 fifteenths from each of the 2 thirds to make $\frac{8}{15}$.

$\frac{4}{5} \bullet \frac{2}{3} = \frac{8}{15}$

G6 Unsimplify to Divide and See That You Have Multiplied by the Reciprocal



unsimplify

▶ Unsimplify to Divide

$\frac{2}{3} \div \frac{5}{7} = ?$

We cannot divide the numerator of $\frac{2}{3}$ by 5 or the denominator by 7.

To be able to divide, we need to unsimplify $\frac{2}{3}$. To **unsimplify** we rewrite it as an equivalent fraction so the numerators and denominators divide evenly.

Step 1 $\frac{2}{3} \div \frac{5}{7} = (\underbrace{\overbrace{\frac{2}{3} \cdot \frac{5}{5} \cdot \frac{7}{7}}^{\frac{2}{3} \cdot 1 \cdot 1}}_{\frac{2}{3}\text{ unsimplified}}) \div \frac{5}{7}$

Step 2 $\overset{5 \div 5 = 1}{\underset{7 \div 7 = 1}{= \frac{2 \cdot 5 \cdot 7}{3 \cdot 5 \cdot 7} \div \frac{5}{7}}}$

Step 3 $= \frac{2 \cdot 7}{3 \cdot 5}$

Step 4 $= \frac{2}{3} \cdot \frac{7}{5}$

1. How is the number you divide $\frac{2}{3}$ by in the original division problem related to the number you multiply $\frac{2}{3}$ by in the final multiplication problem?

You multiply by $\frac{7}{5}$, the reciprocal of original divisor, $\frac{5}{7}$.

G6 Two Ways to Divide Fractions

1. If you can, divide the top and bottom numbers (the numerator and denominator) of the product by the top and bottom numbers (the numerator and denominator) of the factor.

1. If you can't divide easily, flip the factor and multiply the product by it (multiply the product by the reciprocal of the factor).

Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings
that students can make and use in their own ways
in problem solving and explaining of thinking.

We want classrooms to be **using the mathematical practices**:
Students focus on math sense-making about math structure
using math drawings (visual models) to support math explaining.

Reason about unit fractions.

Student drawings of length models can support understanding of fraction computation

Professor Karen C. Fuson
Northwestern University

Please see the Teaching Progressions, Classroom Videos, and Publications on my website karenfusonmath.com for fractions and for other CCSS-M math topics. There are 18 hours of Teaching Progressions for the various math domains in the CCSS-M.