**Comparing multidigit numbers using math drawings and Secret Code Cards**

You will need these student materials:

* The dot column student page on page 8, one for each student; a pencil for each student.
* Secret Code Cards made as directed on pages 9 to 14; one set for each student or pair of students.
* 1 or 2 pieces of paper for each student

**Students learn to draw ten-sticks and hundred-boxes**

**Draw ten-sticks.** Each student needs a page of dot columns that is on page 10. Have everyone turn the page so that the columns are vertical and the extra space is on the right. Ask students to explore what is on the page and share what they find. Be sure that the discussion includes that there are 10 dots in each column, the dots in the columns are in groups of five, two 5-groups of dots make a group of 10 dots, the columns are in groups of five columns, and 5-groups of columns make a group of 10 columns.

Tell students that they are doing this activity to learn to make meaningful drawings for hundreds, tens, and ones. Have students draw a stick through the top left column of 10 dots. Tell students that they can call that stick a *ten-stick* because it has 10 dots on it. Have students draw a stick through the next nine columns of that top left group of dots.

A group of blue sticks

Description automatically generated with medium confidence

**Draw hundred-boxes.**

* Ask students how many dots they have in their ten ten-sticks and how they know how many.
* With the whole class count the tens-sticks by tens to verify that there are 100 dots in the ten ten-sticks.
* Ask students to draw a rectangle around the outside of all ten of the ten-sticks to make a hundred-box.

A blue line drawing on a white background

Description automatically generated

* Ask students how they could make ten-sticks in the next ten columns of dots [by drawing sticks through the ten columns of ten dots].
* After they have all done that, ask them how to make a hundred-box from those ten ten-sticks [draw a rectangle around all of those ten-sticks] and have them do so.
* Repeat for the next ten columns of ten dots (see below).
* Count all of the ten-sticks in the top row by tens to verify that there are 300 dots in the three hundred-boxes.
* Then ask students if they really need to draw all of the 10 sticks inside 100 box or if they could just draw the box around all of the dots. [Yes, it would be faster to just draw around the 100 dots and they could imagine the ten ten-sticks inside.]

**Make numbers**

* Have students make 279 in the second row of dot columns and explain how they did it [by drawing hundred-boxes around the first two groups of ten ten-columns, then drawing sticks through 7 columns to make 7 ten-sticks, and then looping 9 dots or circles: see below].
* Ask students to discuss how the separation of the columns of dots into a group of five and another group of five lets them find 70 very quickly as 5 tens plus 2 tens.
* Ask students to discuss how the separation of the dots into a group of five and another group of five lets them find 9 very quickly as 5 plus 4.
* In the next row have students make 92, 66, and 87 and discuss how they used the groups of five tens to quickly make 90 as five tens plus four tens, 60 as five tens plus one ten, and 80 as five tens plus three tens. Also discuss various ways to use the 5-groups to make 6 and 7. The drawings shown make individual dots or circles instead of a loop to transition to the free drawings that students will make next (see examples on the right).
* Use the bottom two rows to make numbers that students suggest. Specify that at least three examples should use numbers of 6, 7, 8, or 9 in the tens and the ones.

A sheet of a paper with a number and a number

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**Begin free drawing**

* To the right of the top row in the empty space ask students to draw three small 100-boxes (these are just small rectangles with nothing inside them) and write 300 under the boxes (see above).
* Ask students to pretend to see ten tiny ten-sticks inside each box because each box represents 100 just as the big boxes on the left do with their actual ten ten-sticks each with ten dots. Tell students that it is so much faster to make these small quick hundred-boxes that they will use these small drawings to show numbers.
* In the second row, out on the right, ask students to make 279 (they draw two small hundred-boxes and then draw seven small ten-sticks and then draw nine dots or a circle for each dot--circles are faster and easier to see). Remind students to draw the seven sticks using five groups: five small ten-sticks and then a space and then two more ten-sticks.
* Then have students discuss how they showed nine dots including how they could show nine dots using five groups as 5 and 4. They can draw the dots horizontally or vertically. They then write 279 under the drawing to remind them of what it shows.
* To the right of the third column have students draw ten-sticks and circles for 92 and then 66 and then 87. Have them write the numbers under or above each drawing.
* Then have them discuss how they can see the five groups in the ten-sticks and in the ones circles. [Students need to leave a space after the first five ten-sticks to show that 5-group of tens. For the ones students often make a row of five small circles with one, two, three, or four circles below that to show six through nine, but they can make a column of five with one, two, three, or four in a new column to the right.] The 5-groups are crucial for seeing the numbers drawn so that everyone can see and discuss such numbers when comparing, adding, or subtracting 3-digit numbers.

**Draw more numbers.**

Use the bottom two rows of columns of dots to make more numbers, especially those that use 5-groups for ones and tens (that use the numbers 6, 7, 8, and 9 in the ones and the tens). Have students draw each number to the right, write the number nearby, and explain their drawings.

**Students connect the math drawings to the Secret Code Cards**

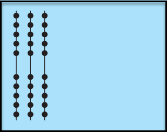
**Make 2-digit numbers**

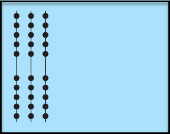
* Have students make 38 with their Secret Code Cards and then make a math drawing (they can use the back of the page) to show 38.
* Have students discuss how they made 8 using 5-groups (two ways are shown below).
* Then have students discuss the 0 on the tens card 30 that is hiding under the ones card 8 and what that 0 shows and why it is there (it shows that the 3 is three tens and so the 0 needs to be in the tens place).
* Have students turn over the Secret Code Cards to discuss any other relationships they see between their math drawing and the Secret Code Card tens and ones drawings (shown below). During this discussion have students use *thirty* and *three tens* for 30 to help *thirty* mean *three tens*.

A math equations and symbols

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Description automatically generatedA blue sign with black numbers

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A blue rectangle with black dots

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* Have students make more 2-digit numbers with the Secret Code Cards and with math drawings and discuss all of the relationships they see.
* For the tens cards and for the ten-stick drawings, have student use English decade words like *twenty, thirty, forty,* etc. and also tens words like *two tens, three tens, four tens*, etc. to describe the numbers and help the English words take on tens meanings. Using both kinds of words helps build place-value understanding about operations on multidigit numbers. Students need to think of the value of the place a digit is in (the hundreds, or tens, or ones place) because that changes the value of the digit. Having students (and you) say these values frequently as they work with multidigit numbers helps students remember the different quantities that are represented.
* Continue helping your students describe all of these connections as they make more 2-digit numbers.

**Make 3-digit numbers**

Move on to making and discussing 3-digit numbers when your students can say and explain 2-digit numbers as being composed of tens and of ones. Begin by having students make 38 again and then use the 100 card to make 138. Have students discuss what each zero on the 100 card means and how the zeros are shown on the back of the 100 card (there are no drawings of tens and ones).

A blue and black number

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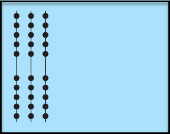
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A blue sign with black numbers

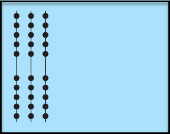
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A blue rectangle with black dots

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Have students make math drawings of other 3-digit numbers and show them with Secret Code Cards. It is important for students to discuss all of the relationships they see between the Secret Code Cards and their math drawings and also the drawings on the back of the Secret Code Cards.

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A blue and black number

Description automatically generatedA blue sign with black numbers

Description automatically generatedA blue line drawing of a square and a number

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Students should see and discuss relationships on the front and the backs of the cards with the cards expanded separately and also layered on top of each other to show the 3-digit number. Students can notice and discuss how the drawings on the back of the cards show all of the ten

tens on each 10-stick and all of the ten ten-sticks inside the hundred-box. This can help them remember the meanings of the quick hundred-boxes and quick ten-sticks they are drawing. You can start with numbers using only the 100 card to reduce the number of cards and permit students to focus on the relationships.

A blue line drawing of a square and bar code

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A blue rectangular object with a black circle

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Description automatically generatedA blue sign with black numbers

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A blue rectangular object with a black circle

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Then make and discuss numbers between 200 and 1000 by using all of the hundreds cards. For later grades students can make Secret Code Cards for thousands and even ten thousands if they want to make larger numbers. They can discuss what drawing they will use for each thousand and each ten thousand.

A blue sign with a black number three

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A blue and black number

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A blue line drawn on a white surface

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**Use student math drawings and Secret Code Cards**

**to support comparing of 2-digit and 3-digit numbers**

Student math drawings and Secret Code Cards are helpful in understanding how to compare 2-digit and 3-digit numbers to find which is more or less. It is useful to compare numbers in which the larger number has a small number of ones and the smaller number has a large number of ones. This is because students may look at the ones digit and make their comparing judgment based on which of the numbers in the ones place is greater. The task for understanding comparing of multidigit numbers is for students to understand that the number in the greater place value position (hundreds, tens, or ones) is what is important in the comparison. So we will start by asking students to compare 81 and 39.

* Ask students to make a math drawing for 81 and 39 and decide which number is greater and why. Have students discuss their drawings and responses.
* Then ask students to make Secret Code Cards to show 81 and 39 and again decide which number is greater and why.
* Ask students to give reasons that use the fronts and the backs of the Secret Code Cards and that use them apart and layered. Remind students to use English words for tens such as *eighty* and *thirty* but also to use tens words like *eight tens* and *three tens* to specify clearly their value.

A blue sign with black numbers

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Description automatically generatedBlue lines on a white background

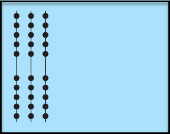
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A blue lines and dots

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A blue rectangular object with black dots

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A blue sign with a black number on it

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The cards above on the left show single digits. But students need to understand the size and location of the place values tens and ones to understand the quantities involved in the single digits *eight, one, three, nine*. The expanded unlayered Secret Code Cards in the middle show the values of the tens and ones as *eighty* and *thirty* or as *eight tens* and *three tens*. The math drawings on the backs of the Secret Code Cards in the third column and student math drawings in the fourth column show clearly that there are many more in 81 than in 39 and that the digit in the ones place really does not matter at all because even having one more 10 in the tens place would be bigger than nine ones. Students may need to remind each other that each quick ten-stick could have ten little circles or dots on it.

Have students make other 2-digit numbers with math drawings and Secret Code Cards, choosing the larger number to have a small number of ones and the smaller number to have a larger number of ones because this is the case that often creates errors as students ignore the value of the digit that comes from the place it is in.

* When students are secure with 2-digit numbers, move on to examples with 3-digit numbers. Begin with a misleading example 712 and 487 that has the larger number of hundreds having small tens and ones and the smaller number of hundreds having large tens and ones numbers.
* Ask students to make a math drawing for 712 and 487 and to make Secret Code Cards for 712 and 487 and decide which number is greater and why. Have students discuss their drawings and responses.
* Ask students to give reasons that use the fronts and the backs of the Secret Code Cards and that use them apart and layered. Remind students to use English words for tens such as *ten* and *eighty* but also to use tens words like *one ten* and *eight tens* to specify their value clearly. Students can make and describe many different connections among the quantities in the math drawings, the expanded notation in the Secret Code Cards, and the regular place-value single-digit notation. All of these connections deepen student understanding.
* Have as many students as possible articulate at least one relationship. Repetition is good in this situation. Students need to see and hear relationships repeatedly because it is challenging to them to make all of the possible connections.

Ask students to say the comparison in both directions to get practice in using the word *less*: *712 is more than 487* and *487 is less than 712*.

A blue rectangular numbers with black numbers

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Compare more pairs of 3-digit numbers like 396 and 821, 703 and 495, 279 and 904, and 561 and 529.

* Have students move toward general statements of relationships that they can use to compare numbers without using math drawings or the Secret Code Cards. An example would be: Look at the numbers in the biggest place and compare them. When students are working to make such general conclusions, it is helpful for them to think of math drawings or the Secret Code Cards to keep the conclusions grounded in why they are true.
* Ask students to summarize things they have learned today.

Many students will need more practice with the cards and math drawings. You can mix such practices with related topics. Over time give all students a chance to describe relationships they see in features of the cards and the math drawings.

A grid of dots on a white background

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**Directions to make the Secret Code Cards**

We call these Secret Code Cards because they show the secret code of numbers. These cards can be layered on top of each other to show the number as it is written in single digits (on the right below) or taken apart to see the place values in the expanded notation of the number (on the left below). The numeral side has tiny numbers on the top left so that the place-value expanded notation version is shown even when the cards are layered to just show single digits. The Secret Code Cards help students use both names for groups of ten, the English word *eighty* for 80 and the tens word *8 tens*. The backs of the cards show hundred-boxes, ten-sticks, and ones-circles so that when these backs are layered they show math drawings such as those students draw.

A blue and black number

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A blue sign with black numbers

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A blue rectangular object with a black circle

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On pages 11 to 14 are Secret Code Cards for ones, tens, and hundreds. The ones and tens are on pages 11 and 12, and the hundreds are on pages 13 and 14. You will need to make two-sided copies from pages 11 and 12 and also two-sided copies from pages 13 and 14. Select **Print Actual Size** (not **Fit to Page**) to print out pages 11 to 14 and select **2-sided Printing** and **Long Edge Binding** if you have such a choice. These cards last longer if you can make them from card stock. You can print on paper or cardstock of any color.

On the cards on pages 11 to 14, we have taken out the cut lines around each card (like those shown on page 12) because it is so difficult to align the front and the back exactly and not have the cut lines show after cutting. So instead the pages 13 to 16 have marks that show the cut lines but are outside the cards. There are small horizontal cut lines on each side of the page, and small vertical cut lines on the top and the bottom of the page. When printing 2-sided, you will align the corners of the small reverse corner marks that are at each corner of the rectangle making the cards and outside the cards. Cutting the cards apart works best if you make all of the horizontal cuts and then make the vertical cuts because the bottom row has the vertical cuts in a different place than the rows above. **Be sure that the small numbers at the top left of each card show clearly on the cut cards.**

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These cards below are smaller than the actual Secret Code Cards on pages 11 to 14. These small cards are just to show where the cards will be cut apart. The cut lines were removed from pages 13 to 16 because the cut lines are messy when the cards are cut apart. You will use the small cut marks outside the space of the cards on pages 11 to 14 to cut the cards apart as on the lines on these small pages.

A blue square with black numbers

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A blue rectangular box with numbers

Description automatically generated A blue rectangular object with black lines

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