**Directions for drawing ten-sticks and hundred-boxes**

The page to give to students is on page 5. Turn the page so that the dot columns are vertical and the extra space is on the right. Page 4 shows the page after the directions below have been followed.

**Drawing ten-sticks**

Each student needs a page of dot columns. Have everyone turn the page so that the columns are vertical and the extra space is on the right.

Tell students that they are doing this activity to learn to make meaningful drawings for hundreds, tens, and ones. Ask students to explore what is on the page and share what they find. Be sure that they discuss that

* each column has two groups of five dots,
* there are 10 dots in each column because two fives make ten, and
* the columns are in groups of five with two of these groups of five columns making a big group of 100 dots.

Have students draw a stick through the top left column of 10 dots. Tell students that they can call that stick a *ten-stick* because it has 10 dots on it. Have students draw a stick through the next nine columns of that top left group of dots.

**Drawing hundred-boxes**

* Ask students how many dots they have in their ten ten-sticks and how they know how many. Then with the whole class count the tens-sticks by tens to verify that there are 100 dots in the ten ten-sticks.
* Ask students to draw a rectangle around the outside of all ten of the ten-sticks to make a hundred-box (see the example on page 4).
* Ask students how they could make ten-sticks in the next ten columns of dots (by drawing sticks through the ten columns of ten dots).
* After they have all done that, ask them how to make a hundred-box from those ten ten-sticks (draw a rectangle around all of those ten-sticks) and have them do so.
* Repeat for the next ten columns of ten dots.
* Count all of the ten-sticks in the top row by tens to verify that there are 300 dots in the three hundred-boxes.
* Then ask students if they really need to draw all of the 10 sticks inside 100 box or if they could just draw the box around all of the dots. (It would be faster to just draw around the 100 dots, and they could imagine the ten ten-sticks inside.)

**Making numbers**

Have students make 279 in the second row of dot columns and explain how they did it (by drawing hundred-boxes around the first two groups of ten ten-columns, then drawing sticks through 7 columns to make 7 ten-sticks, and then drawing 9 dots or circles: see below).

* Have students discuss how the separation of the columns of dots into a group of five and another group of five lets them find 70 very quickly.
* In the next row have students make 92, 66, and 87 and discuss how they used the groups of five tens to quickly make 90, 60, and 80.
* Use the bottom two rows to make numbers that students suggest. Specify that at least three examples should use the groups of five so digits need to be between 6 and 9.

**Begin free drawing**

To the right of the top row in the empty space ask students to draw three small 100-boxes (these are just small rectangles with nothing inside them) and write 300 under the boxes (see example on page 4).

* Ask students to pretend to see ten tiny ten-sticks inside each box because each box represents 100 just as the big boxes on the left do with their actual ten ten-sticks each with ten dots. Tell students that it is so much faster to make these small quick hundred-boxes that they will use these small drawings to show numbers.
* In the second row ask students to make 279 by drawing two small hundred-boxes and then drawing seven small ten-sticks and then drawing nine dots or a circle for each dot (circles are faster and easier to see). Remind students to draw the seven sticks using five groups: five small sticks and then a space and then two more sticks.
* Then have students discuss how they showed nine dots and ask how they could show nine dots using five groups. Students can draw the dots horizontally or vertically. They can write 279 under the drawing to remind them of what it shows.
* To the right of the third column have students draw ten-sticks and circles for 92. Have them write the numbers under or above each drawing. Then have them discuss how they can see the five groups in the ten-sticks and in the ones. (Students need to leave a space after the first five ten-sticks to show that 5-group. For the ones students often make a row of five small circles with one, two, three, or four circles below that to show six through nine, but they can make a column of five with one, two, three, or four in a new column to the right.)
* Have students repeat the steps above for 66 and then for 87.

**Draw more numbers**

Use the bottom two rows of columns of dots for students to make more numbers, especially those that use 5-groups for ones and tens (use the numbers 6, 7, 8, and 9 in the ones and the tens). Then have students free draw each number to the right, write the number below or above, and explain their drawings.

**Building more understandings**

**Connect the math drawings to the Secret Code Cards**

Find in the second row of the table of Teaching Materials on the Math Expressions Users page the file How to make Secret Code Cards to show place-value numerals. Follow the directions and make enough secret code cards for all of your students. A detailed description of how to introduce the Secret Code Cards and relate them to math drawings is in the same file. Here is a brief overview of those detailed steps with drawings.

* Give a set of the Secret Code Cards to each student.
* Have students discuss the features they see on the Secret Code Cards.
* Have students make a 2-digit or 3-digit number with the Secret Code Cards and then make a math drawing to show that number.
* Have students discuss the 0’s hiding under the ones and tens digits and what those 0s show and why they are there.
* Have students discuss how the math drawings they made relate to the Secret Code Cards.
* For the tens cards and for the ten-stick drawings, have student use English decade words like *twenty, thirty, forty,* etc. and tens words like *two tens, three tens, four tens*, etc. to describe the numbers. Using both kinds of words helps build place-value understanding especially understanding about operations on multidigit numbers.
* Continue helping your students describe all of these connections.

**Use the math drawings to support multidigit adding and subtracting**

For discussion and an example of using math drawings of hundreds, tens, and ones to support student explanation and discussion of steps in an accessible standard algorithm for multidigit addition see the paper

* Accessible Standard Algorithms for Understanding and Equity (Fuson, Kiebler, Decker, 2024) on the NCTM website nctm.org

For discussion and an example of using math drawings of hundreds, tens, and ones to support student explanation and discussion of an accessible standard algorithm for multidigit subtraction see the paper

* Accessible Standard Algorithms for Understanding and Equity Part 2: Multidigit and Decimal Subtraction, Multiplication, and Division (Fuson, Kiebler, Decker, 2024) on karenfusonmath.net

For more detailed explanations and videos of students using math drawings to solve and explain multidigit addition and subtraction see these sources:

|  |  |
| --- | --- |
| What to see | Where to go |
| To watch first graders explaining math drawings and three multidigit addition standard algorithms | Please go to <https://karenfusonmath.net/classroom-videos/#C-Longer-Classroom-Teaching-Examples> and play Grade 1 |
| To watch the first author explaining math drawings and three multidigit addition standard algorithms along with students from several grades solving problems | Please go to <https://karenfusonmath.net/classroom-videos/#B-Math-Explanations> and play Multidigit Addition |
| To watch third graders explain 3-digit subtraction accessible standard algorithms | Please go to <https://karenfusonmath.net/classroom-videos/#C-Longer-Classroom-Teaching-Examples> and play G3 Multidigit Subtraction.  |
| To watch fifth graders explain 7-digit subtraction accessible standard algorithms | Please go to <https://karenfusonmath.net/classroom-videos/#G-Place-Value-and-Multidigit-Addition-and-Subtraction> and play the last video G4 Explaining 7-digit subtraction. |



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