Base-Ten Knowledge:
Language, Meanings, and Units

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Please see my website karenfusonmath.com or karenfusonmath.net for 22 hours of audio-visual Teaching Progressions for all CCSS domains and for my papers, classroom videos, presentations, and supports for teaching remotely.

I am sorry that I am not able to be at the conference in person.

Please email any questions you have to karenfuson@mac.com

Some slides have some detail that I do not discuss.

Please request the PPT if you would like to see some slides again.

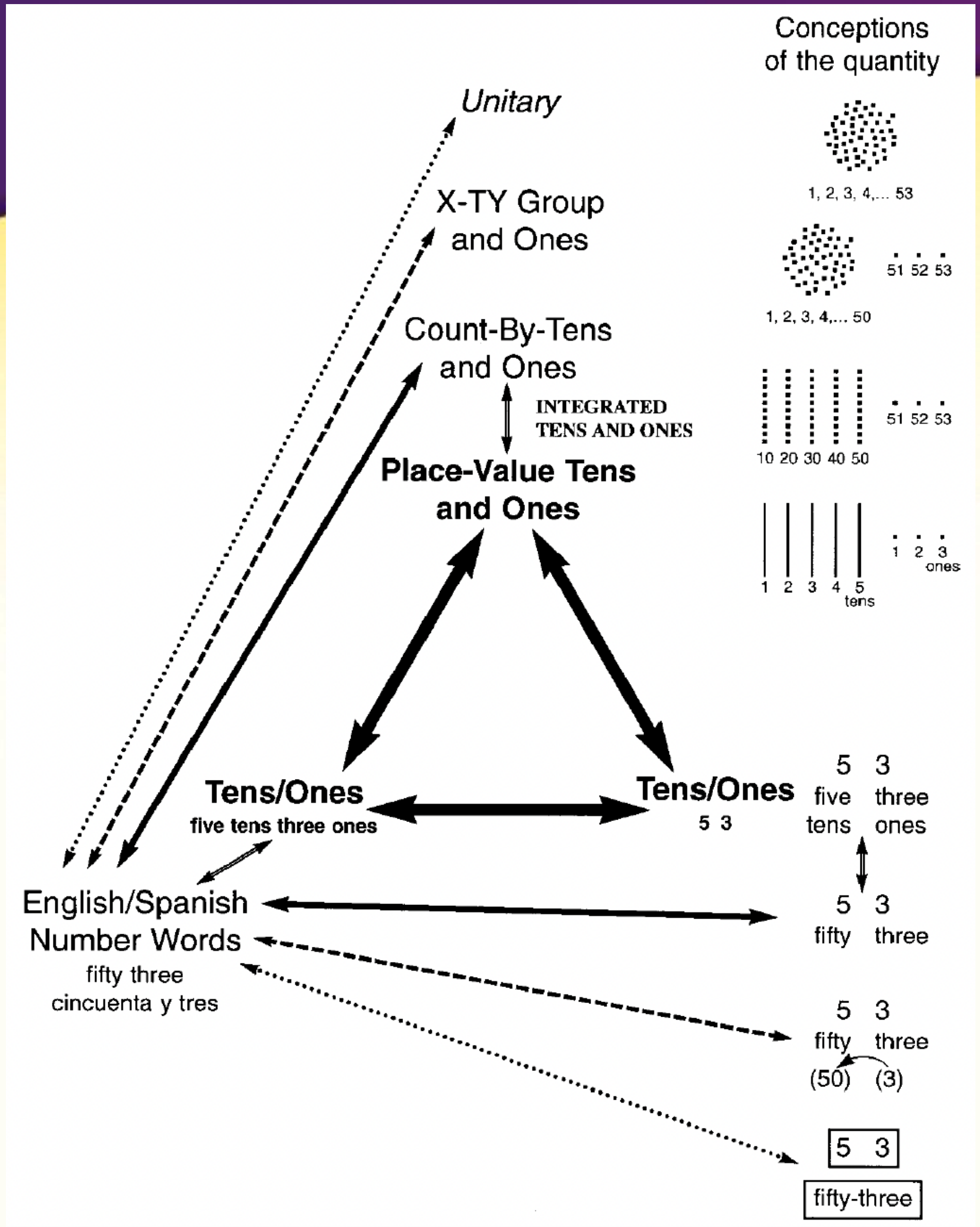
The symposium papers use creative and innovative ways to focus on the advanced place-value competence

composing and decomposing multiunits.

Visual salience or disguising of multiunits?

What knowledge is prerequisite to other knowledge?

What does such knowledge predict?

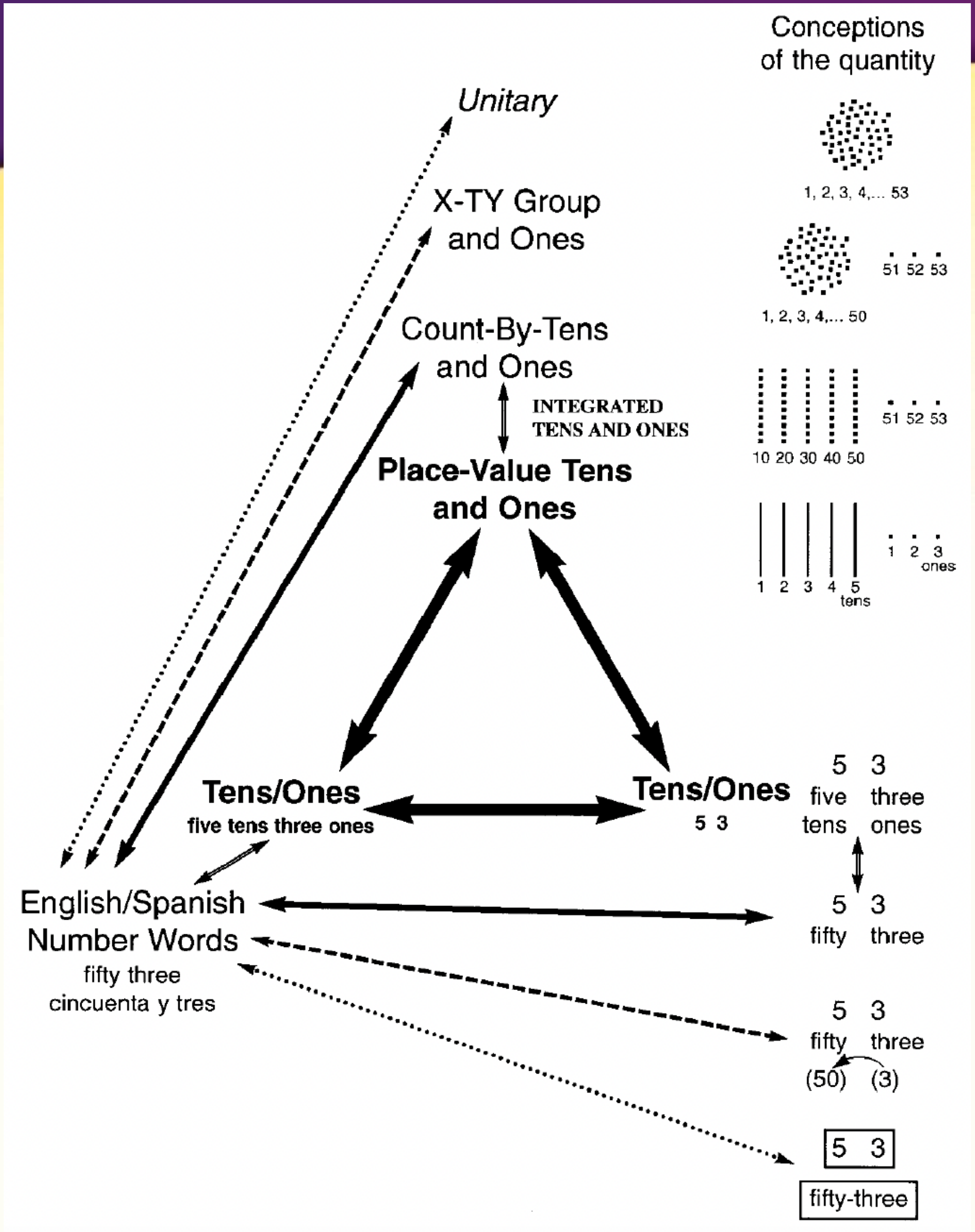


Children build successively more embedded concepts and counting skills for 2-digit numbers.

Only children using European languages or others that have irregular names for the tens use the Count-by-Tens and ones concept.

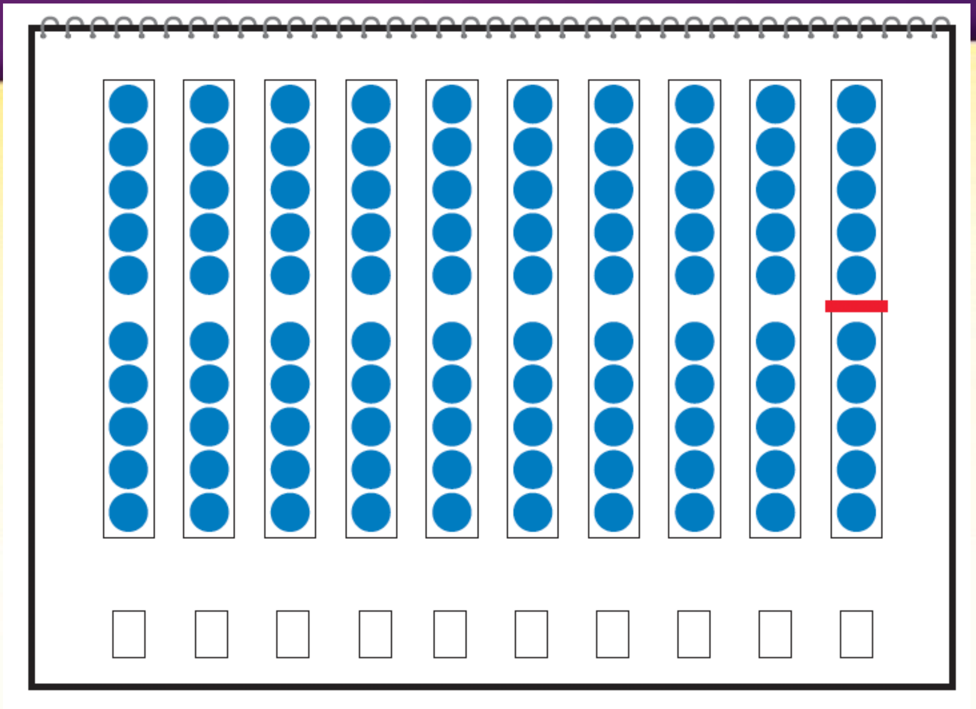
Children using East Asian and other languages that name the tens such as two ten, three ten, etc. use the Place-Value Tens and Ones concept.

Chan et al., 2014, established that Chinese children use the Unitary Multidigit and then the Decade and Ones (Counting on) and then the Separate Tens and Ones counting strategies.



Shifting from counting by tens to counting by ones is difficult for many students. It needs practice as in slide 6.

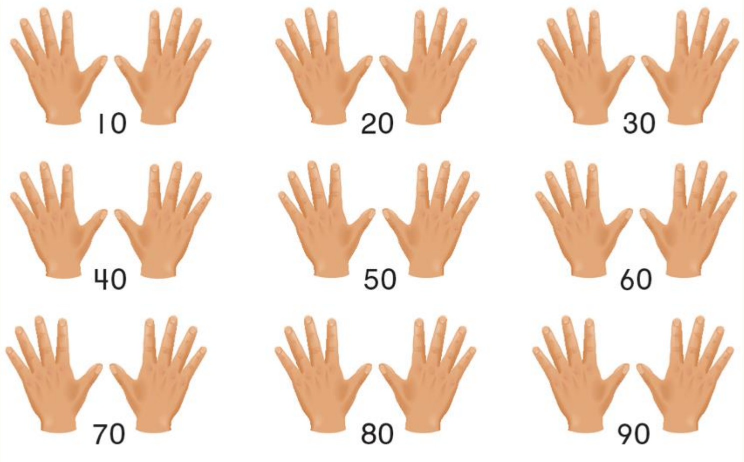
1	11	21	31	41	51	61	71	81	91	101	111
2	12	22	32	42	52	62	72	82	92	102	112
3	13	23	33	43	53	63	73	83	93	103	113
4	14	24	34	44	54	64	74	84	94	104	114
5	15	25	35	45	55	65	75	85	95	105	115
6	16	26	36	46	56	66	76	86	96	106	116
7	17	27	37	47	57	67	77	87	97	107	117
8	18	28	38	48	58	68	78	88	98	108	118
9	19	29	39	49	59	69	79	89	99	109	119
10	20	30	40	50	60	70	80	90	100	110	120



K and Grade 1

Student leader points down each column as students flash ten fingers then shift to ones.

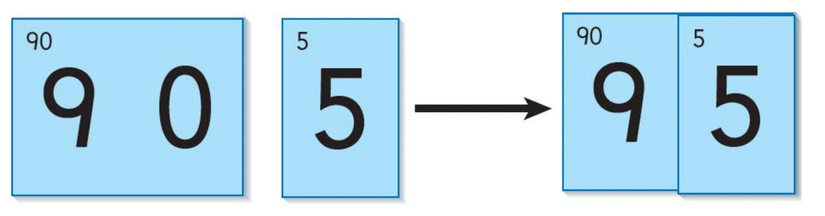
Sequence tens and ones structure



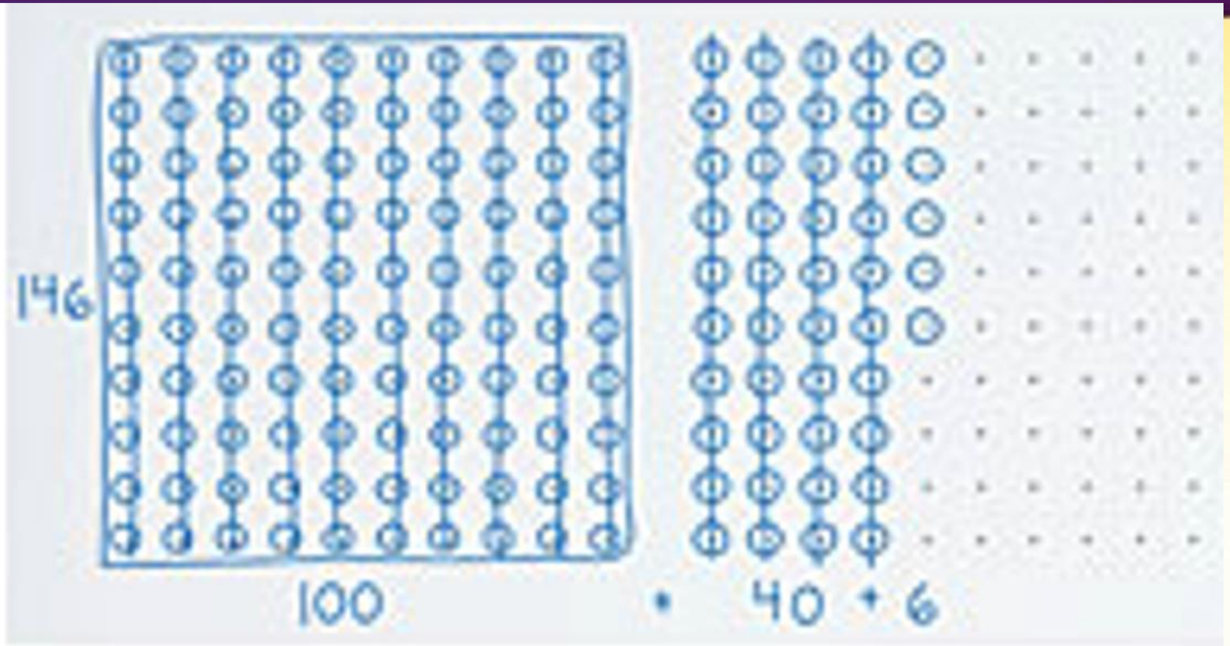
Stop. Freeze. Now count by ones.

Student leader: What number do you see? 95
How may tens and how many ones? 9 tens and 5 ones
Let's count the tens together (pointing down each column):
1, 2, 3, 4, 5, 6, 7, 8, 9 tens
Let's count the ones together (pointing to each circle):
1, 2, 3, 4, 5 ones

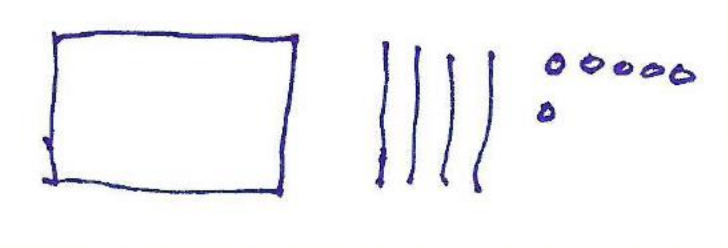
Separate tens and ones structure



Ninety-five is made from ninety and five.
Decade and ones structure

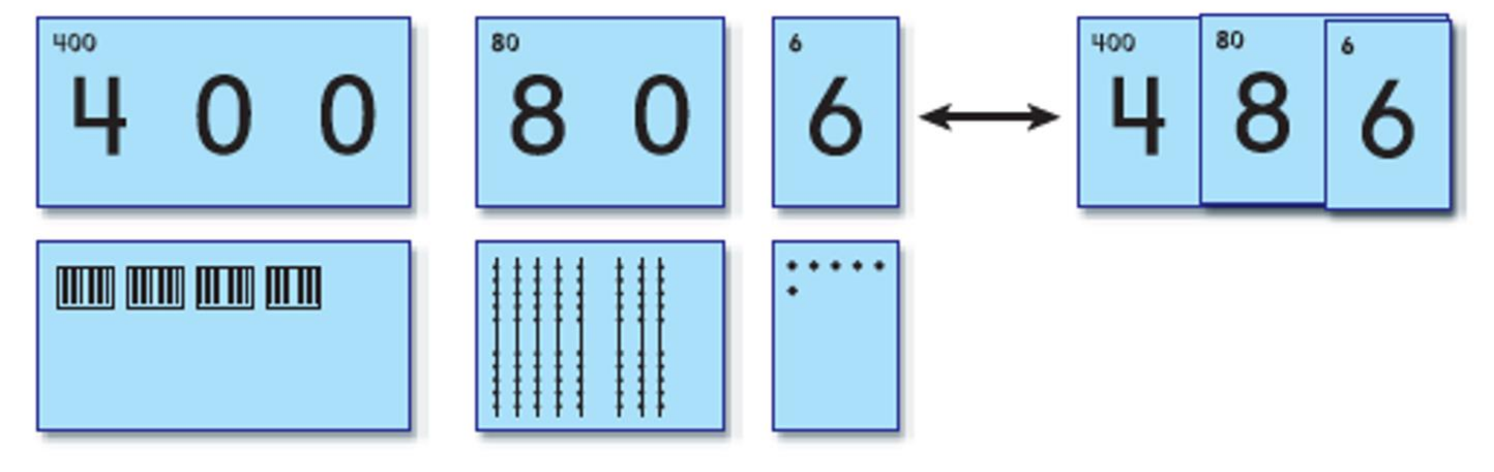


Student math drawing for 146

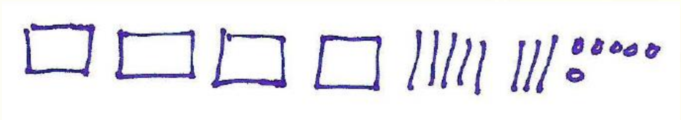


Grade 2

Secret code cards to show quantity meanings of the concatenated single digits

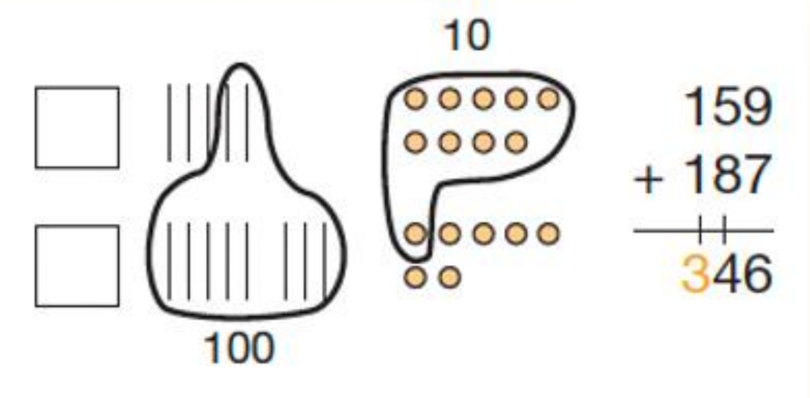


Student math drawing for 486



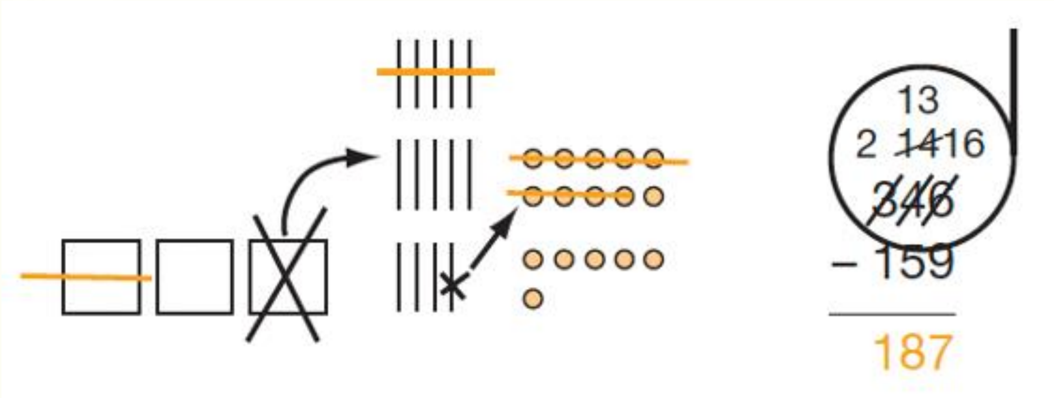
Grade 2

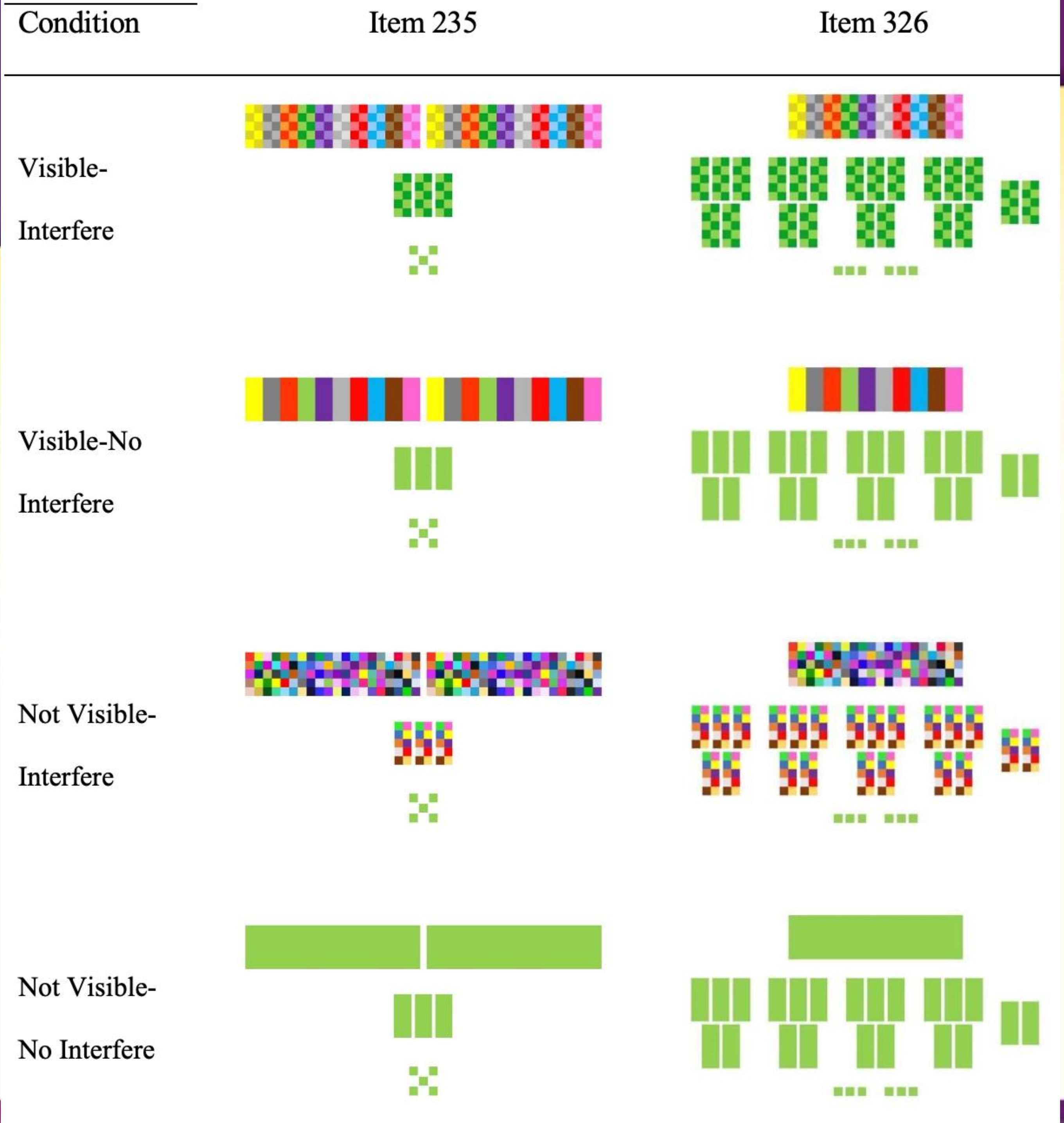
Compose ones to make ten and compose ten tens to make one hundred



Grade 2

Decompose ten to make ten ones and descompose one hundred to make ten tens





Which unit to count? Composing and decomposing multiunits

Partial knowledge: When composing, 84% of the children across all conditions were assigned to profiles that involved counting composite units. Of these children, 48% made errors when doing so.

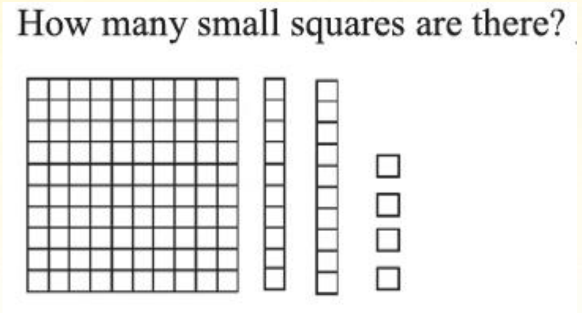
Need tens as prompts: When no tens were visible in the representation, children tended to use ones as the unit in their responses.

Can use tens as prompts: When the tens were made visually salient, children were more likely to count the target unit, resulting in a larger proportion of students producing optimal responses.

Tens as prompts can mislead: All eight students in the visible conditions who were assigned the Wrong Unit profile counted the tens instead of the hundreds, so making tens visible in this case led to incorrect responses on composing items.

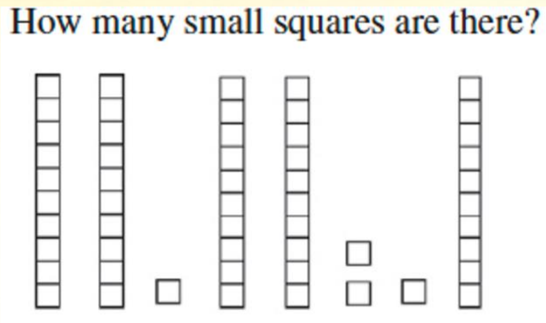
Barilaro, Osana, and Lafay

Initial place value knowledge
Hundreds tens and ones in order

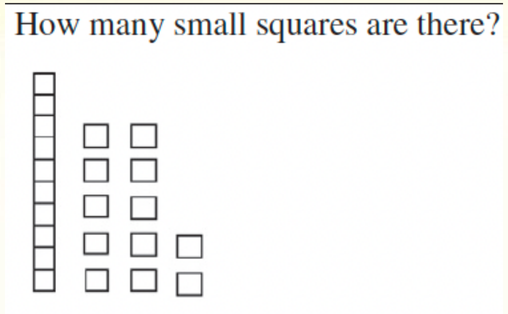


Advanced place value knowledge
Multiunits out of order or composing of units

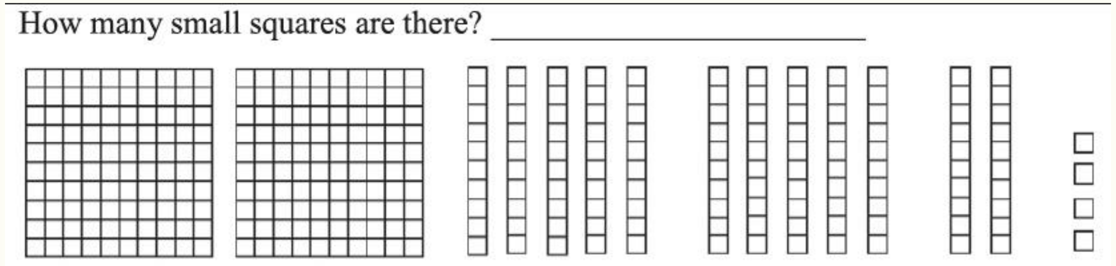
Out of order



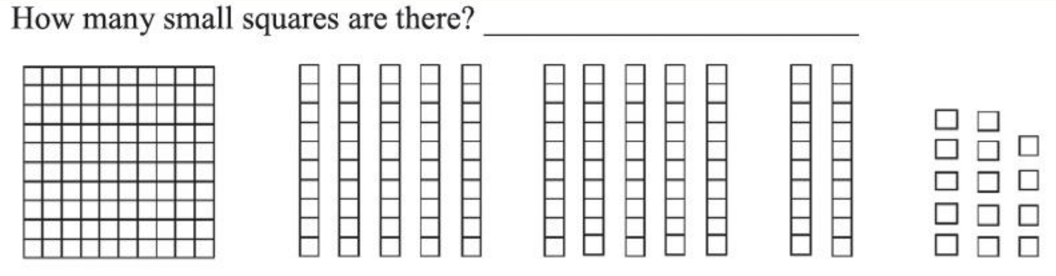
Composing ones to make a ten



Composing tens to make a hundred

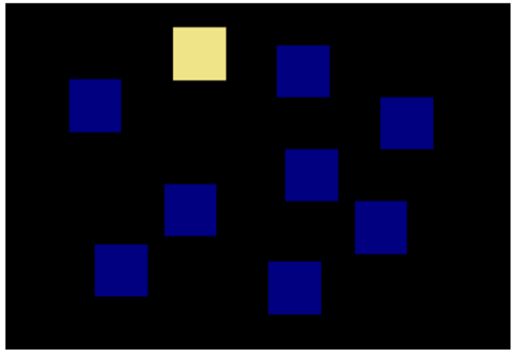


Composing tens to make a hundred AND composing ones to make a ten



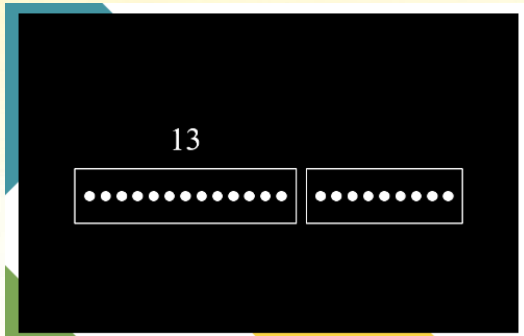
Chan et al., 2014

See and remember **a sequence** of spatial locations



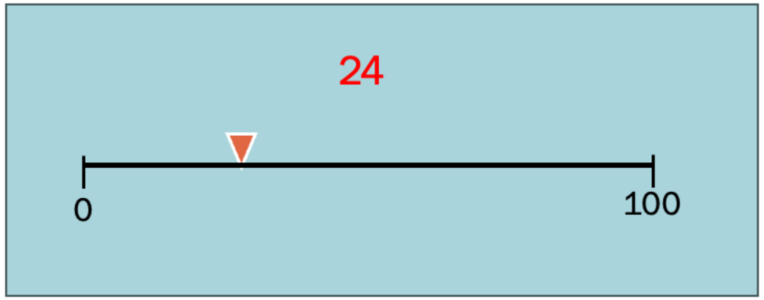
Corsi Block Task

Use embedded cardinality in **left to right sequence**



Count-on Task

Embed a cardinality in the **left to right sequence** of a number list from 1 to 100



Number Line Estimation Task

Count embedded tens in **left to right sequence** and compose ones to make a ten

Strategic Counting Task

Chan and Wong

Transcoding expanded errors

Ex: writing the numeral “two thousand seven hundred forty-three” as “200743” or “2000700403”or “200070043” or “2700403”, etc.

Predominantly in K

Base ten invented counting errors

Count sequence showed a transistion to a different unit but did not shift to the new count sequence by that unit [not shift or not know?]

Prompts to count by tens or by hundreds were given to focus children on counting by higher units

Predominantly in Grade 1

The invented counting errors-only group had higher multidigit calculation performance compared to those who made neither error, p =. 007

Bower, Mix, Yuan, Hancock, Smith

In domains like place value where there are multiplie conceptions that build and relate, use prompts to uncover less accessible knowledge.

Kamii digit meaning task

Show a card with 16 on it. Count out that many cubes.

16

The interviewer then pointed to the 6 and said, *What does this part mean? Show me with the cubes what this part means.*

The interviewer then pointed to the 1 and repeated the question.

Prompt 1: That is one thing it means. Can you think of something else that this might mean? Can you show me with the cubes?

Prompt 2: This is a teen number. What does this (point to the 1) mean in the teen number? Can you show me with the cubes?

Kamii (1989) G2 and G3 children only 16% and 30% showed ten chips for the 1

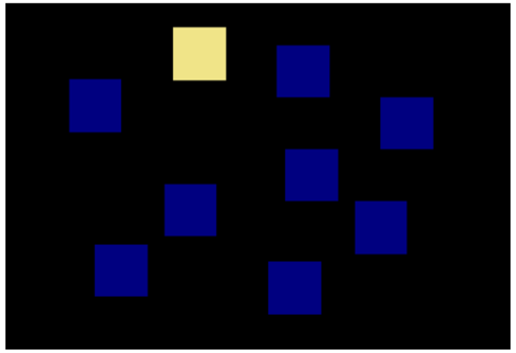
Math Expressions G1 children 64% immediately and 96% after Prompt 2.

Math Expressions full-day K children 51% immediately and 77% after Prompt 2.

Prompts revealed underlying knowledge not accessed first.

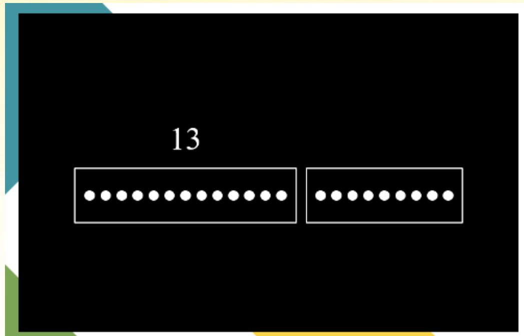
Opportunity to learn is crucial.

See and remember a sequence of spatial locations



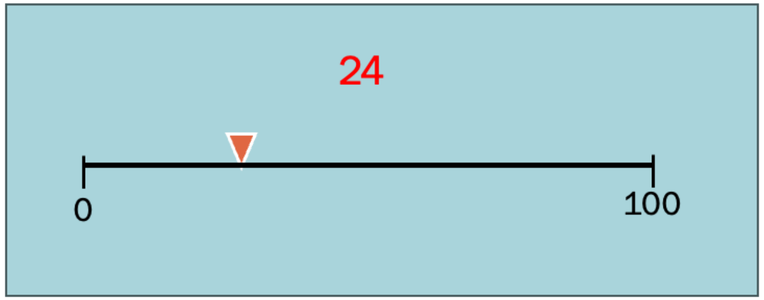
Corsi Block Task

Use embedded cardinality in left to right sequence



Count-on Task

Embed a cardinality in the left to right sequence of a number list from 1 to 100



Number Line Estimation Task

There is no such thing as a mental number line in children's heads. There is a mental number LIST: an internalized sequence of counting words.

Count embedded tens in left to right sequence and compose ones to make a ten

Strategic Counting Task

Please use clear specific language.

Describe partial understanding as accurately and as detailed as possible and do not introduce or use general terms like relative, approximate, positional principle because these are not well defined and muddy the waters.

Place Value

× 10 (larger) ←→ ÷ 10 (smaller)

Thousands	Hundreds	Tens	ONES	Tenths	Hundredths	Thousandths
1,000.	100.	10.	1.	0.1	0.01	0.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
\$1,000.00 	\$100.00 	\$10.00 	\$1.00 	\$0.10 	\$0.01 	\$0.001 

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Papers drawn from for this PPT

These all are under publications on my websites karenfusonmath.net and karenfusonmath.com

Fuson, K. C. (1990). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction, and place value. *Cognition and Instruction, 7,* 343–403

Fuson, K. C. (1990). Issues in place-value and multidigit addition and subtraction learning. Journal for Research in Mathematics Education, 21, 273-280.

Fuson, K. C., & Briars, D. J. (1990). Base-ten blocks as a first- and second-grade learning/teaching approach for multidigit addition and subtraction and place-value concepts. Journal for Research in Mathematics Education, 21, 180-206.

Fuson, K. C., Wearne, D., Hiebert, J., Human, P., Murray, H., Olivier, A., Carpenter, T., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers at work in addition and subtraction. Journal for Research in Mathematics Education, 28, 130-162.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. Journal for Research in Mathematics Education, 28, 738-766.

Ho, C. S., & Fuson, K. C. (1998). Effects of language characteristics on children's knowledge of teens quantities as tens and ones: Comparisons of Chinese, British, and American kindergartners. Journal of Educational Psychology, 90, 536-544.

Fuson, K. C. (1998). Pedagogical, mathematical, and real-world conceptual-support nets: A model for building children’s mathematical domain knowledge. Mathematical Cognition, 4(2), 147-186.

Sequence Tens and Ones are conceptions and skills in European languages NOT in East Asian regular ten counting systems

Chan et al., 2014, established that East Asian children use the Unitary Multidigit and then the Decade and Ones (Counting on) and then the Separate Tens and Ones counting strategies.

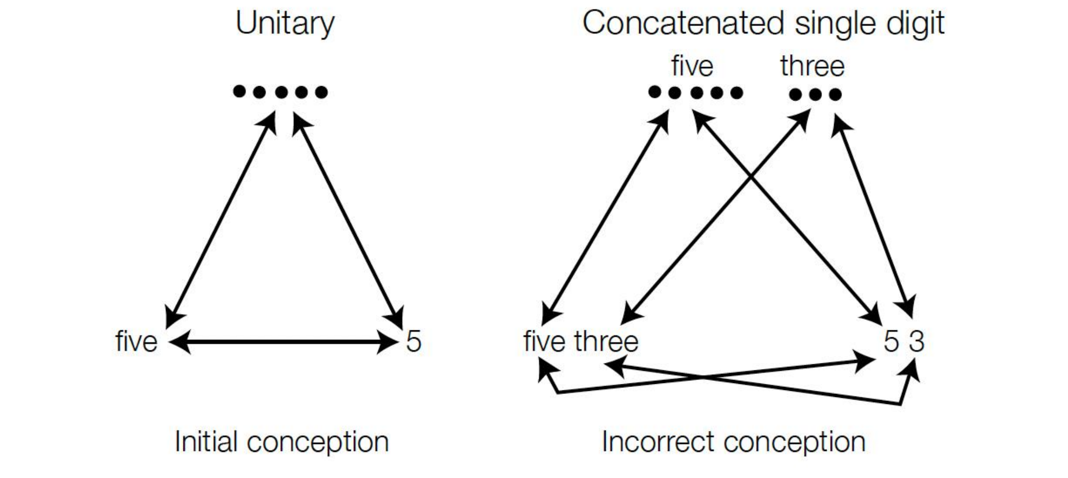


Figure 1. Unitary triad (quantity, number word, written numeral) and common incorrect multidigit conception derived from the appearance of the multidigit numbers

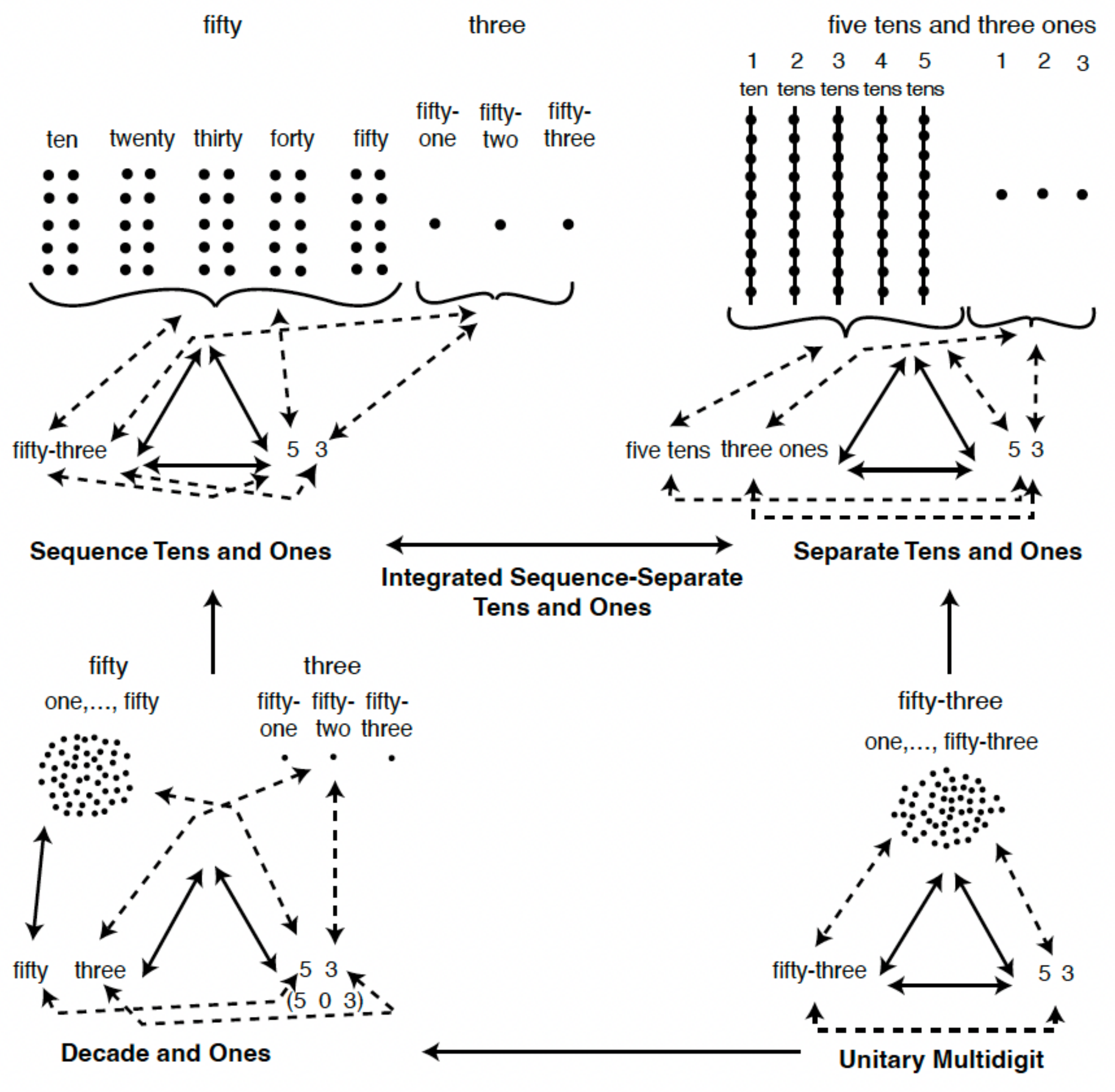
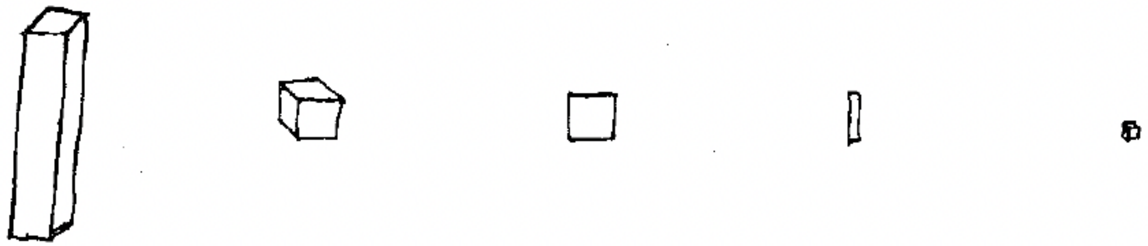


Figure 2. A developmental sequence of conceptual structures for two-digit numbers: the UDSSI triad model

TABLE 2
Conceptual Structures for Multiunit Numbers

Name of the Conceptual Structure	*Nature of the Conceptual Structure*				
Features of the marks					
Visual layout					
Positions ordered in increasing value from the right	___ Fifth	___ Fourth	___ Third	___ Second	___ First
Features of the words					
Multiunit names	*Wan*	*Qian*	*Bai*	*Shi*	*Yi*
Words ordered in decreasing value as they are said	Ten-thousand	Thousand	Hundred	Ten	Ones
Multiunit structures					
Multiunit quantities					
Regular ten-for-one and one-for-ten trades	Ten thousands one	Ten hundreds ten one	Ten tens ten one	Ten ones ten one	ten
Positions/values as cumulative trades	Four trades	Three trades	Two trades	One trade	No trades
Positions/values as cumulative multiples of ten	Four multiples of ten (t × t × t × t)	Three multiples of ten (t × t × t)	Two multiples of ten (t × t)	One multiple of ten (t)	No multiples of ten
Positions/values as exponential words for multiples of ten	Ten to the fourth power	Ten to the third power	Ten to the second power	Ten to the first power	Ten to the zero power
Positions/values as exponential marks for multiples of ten	10^4	10^3	10^2	10^1	10^0

Knowledge of counting one to nine,
written digits 1 to 9,
cardinal quantities one to nine

Move left

Relative spatial locations

Match multiunit names to location and to multiunit quantities

Move right

See quantities as so many ones

Composing/decomposing by tens

Repeated trading to give positions and values

Exponential symbols and meanings as repeated grouping by ten

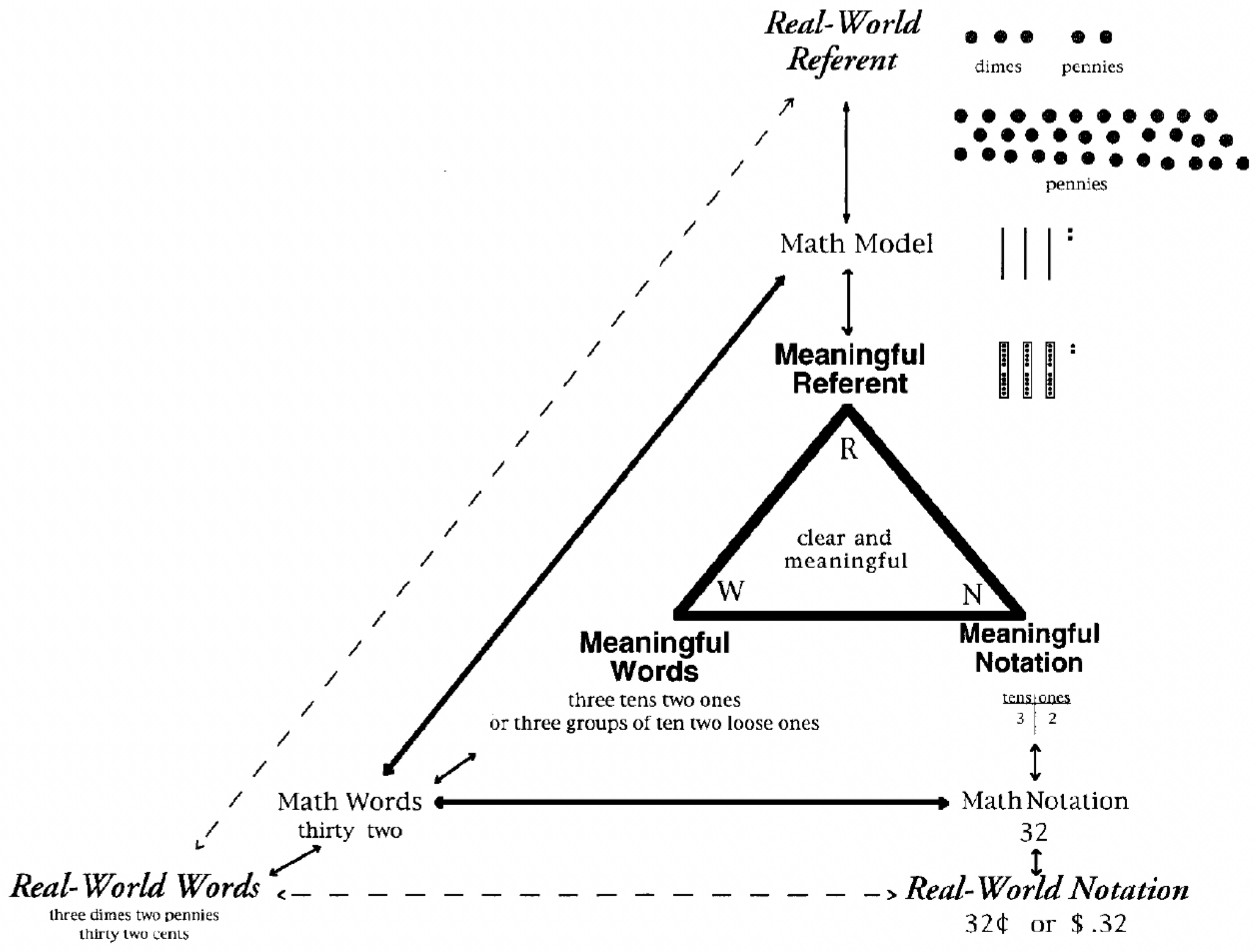


FIG. 3. Conceptual-support net: Real-world, mathematical, and meaningful referent-word-notation triads

Traditional teaching

Conceptual-support net

Including all referents including student drawn quantity math model

Use manipulatives

TABLE 1
Named-Value and Unnamed Position-Value Words and Written Marks

Numbers Expressed As	*Named-Value System*	*Unnamed Position-Value System*
Spoken words	*Two ten-thousand nine thousand five hundred eight ten three* or *two wan nine qian five bai eight shi three*	Two nine five eight three
Written marks	*2 TTh 9 Th 5 H 8 T 3*	2 9 5 8 3

Concatenated single digit conception

Place value/positional system

Value of a position n places to the left or right of 1 is Base x Base x Base n-1 times

Symmetry is around the ones/single units place

This regular repetition creates multiple meanings of each place as so many ones and as so many of each of the units smaller than it:

700 is seven hundred ones and 70 tens and 7 hundreds

Base-ten system

needs nine unique symbols [now usually 1 2 3 4 5 6 7 8 9]

to put in places to say how many of the units in that place

and it needs 0 to make the next larger unit as

10 (one ten made from ten ones) of the place to the right

Values are created by powers of ten/repeated multiples of ten or of one-tenth

that are composed n-1 times to make adjacent positions