

# Building a Nurturing Math Talk Community for Sense Making

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My website is [karenfusonmath.com](http://karenfusonmath.com)  
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*Author of Math Expressions*  
a PK to Grade 6 math program

# A nurturing meaning-making visual Math Talk Community

is an inquiry-based teaching/learning environment, and has a continual focus on sense-making by all participants.

Students are expected:

- to understand what they are doing,
- come to be able to explain their thinking,
- understand the thinking of other students,
- learn to seek help when they need it, and
- help others who need it.

## Create a Nurturing Sense-Making Math Talk Community

The teacher orchestrates collaborative instructional conversations focused on the mathematical thinking of students, using these responsive means of assistance that facilitate learning and teaching by all:

- Engaging and involving
- Managing
- Coaching which is modeling, clarifying, instructing/explaining, questioning, feedback

## Support Sense-Making in the Math Talk Community

The teacher supports the sense-making of all classroom members **by using and assisting students to use and relate:**

Mathematical situations

Pedagogical supports especially drawings

Cultural mathematical symbols and words

Student's own explanations of their thinking

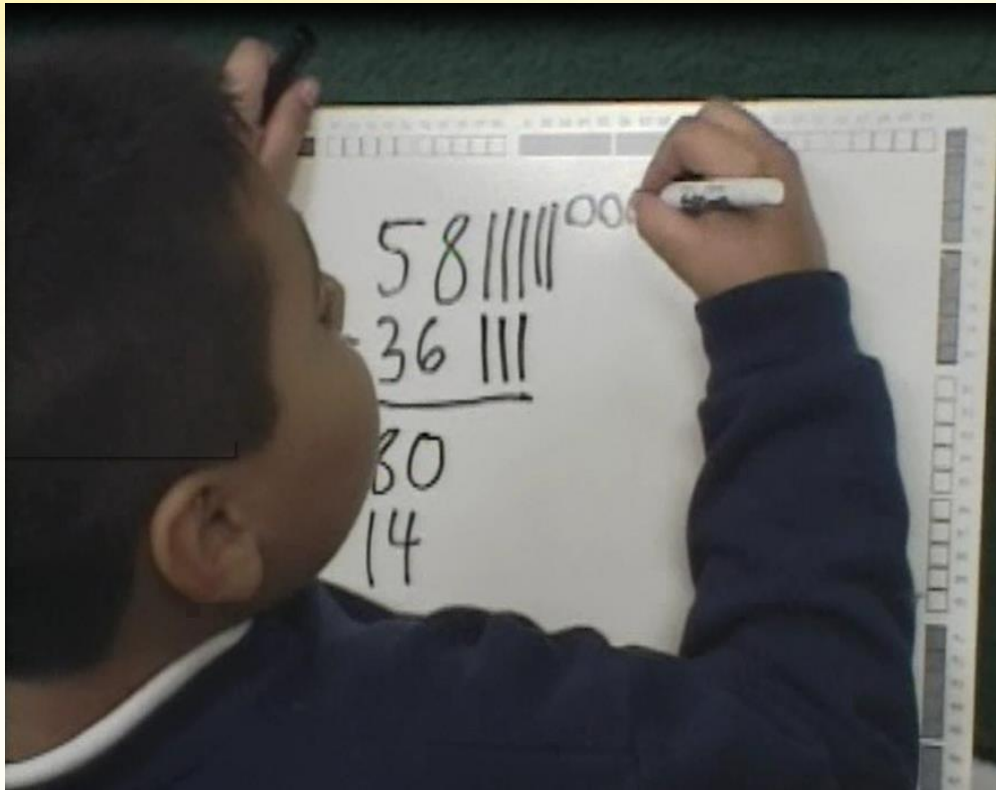
## Solve and Discuss Classroom Structure

<b>Solve</b>	<b>Explain</b>	<b>Question</b>	<b>Justify</b>
<p><b>All students solve.</b> Some solve at the board, and the rest at their seats.</p>	<p><b>One student at the board</b> explains and then asks, “Are there any questions?”</p>	<p><b>Other students</b> ask questions to clarify or extend.</p>	<p>The original explainer <b>responds to the questions</b> by explaining more (justifying the original explanation).</p>

Any student at any time can ask for help from anyone.

For more practice, Solve and Discuss can take place in pairs or small groups.

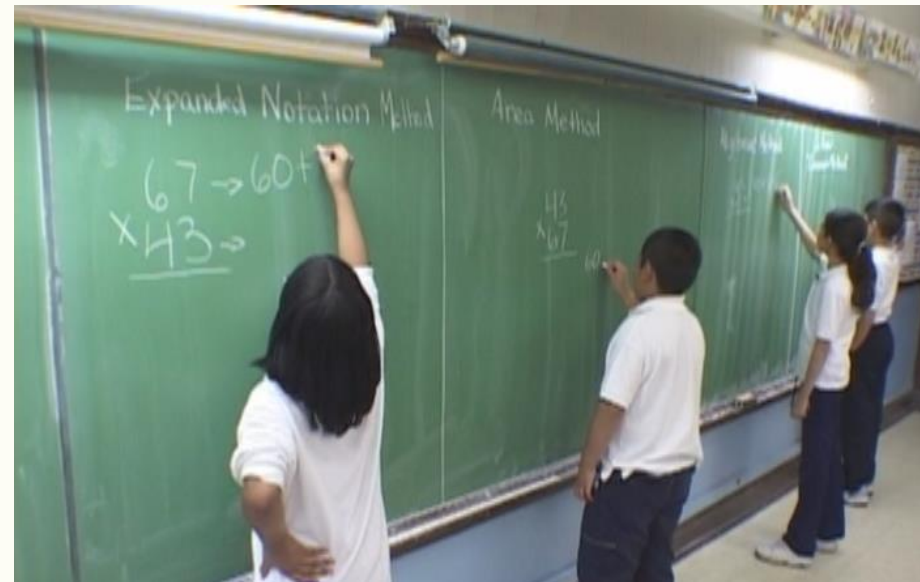
# Make the math thinking visible



- Students must make some kind of math drawing related to the math symbols to show their thinking.
- This supports understanding by the listeners and promotes meaning.

# Make the math thinking visible

- This is important for **equity**: less advanced students and English Learners are helped by the math drawing linked to the explanation by pointing.
- Be sure that **important methods remain** on the board or can be made visible again (e.g., on a Math Board) so they can be compared with other methods.





2. “Bite your tongue” to provide wait time. Students will explain, ask questions, or add a comment if you wait.

## Students must speak and not just listen

1. Structure opportunities to explain to a partner and repeat what the partner says, if needed. Students eventually find their own words, but may need the security of saying an explanation they know is correct.
3. Help students speak to classmates by moving to the side or back of the room. Later remind students with a silent gesture to address each other.



A nurturing meaning-making visual  
Math Talk Community  
is what students and teachers need  
to recover from the deprivations  
of the covid years.

It is therapy for the soul and the self.  
Everyone needs to belong to a community in  
which your thinking and your self is valued and  
in which you can help others.

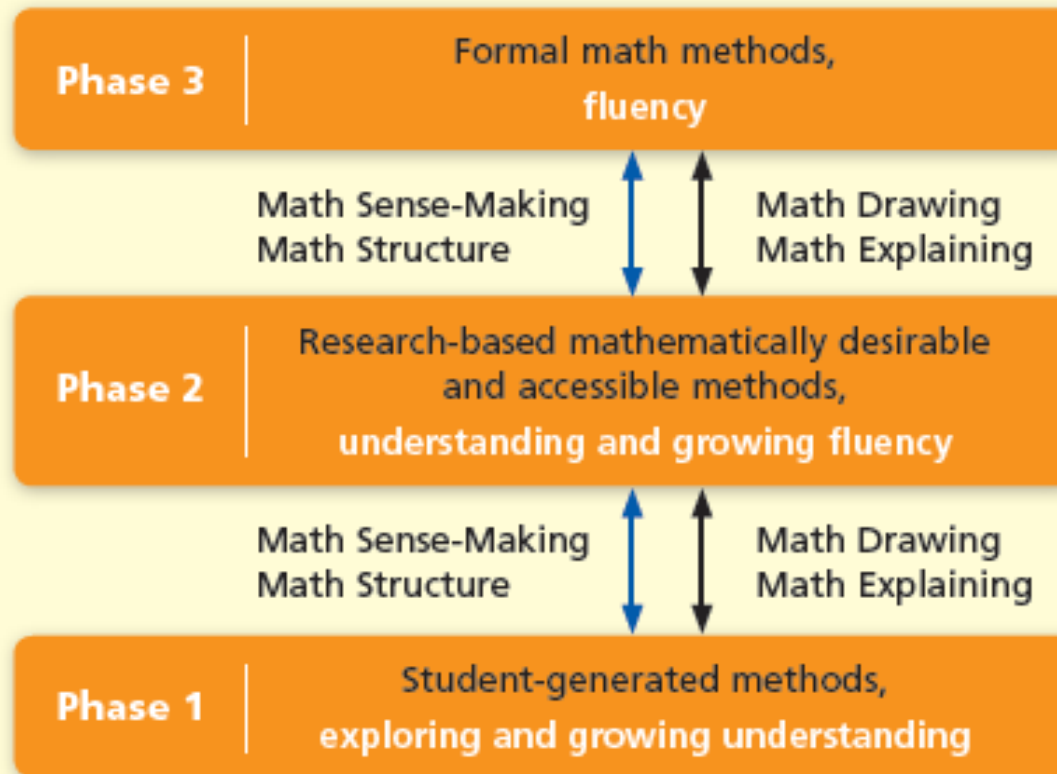
A nurturing meaning-making visual  
Math Talk Community  
allows you to individualize within  
whole-class discussions:

Several methods arise and are discussed, and  
you can introduce research-based methods that  
are accessible and mathematically desirable.

With the classroom math talk support, all  
students move to a strong method.

# Inquiry Learning Path in the Math Talk Community

Bridging for teachers  
and students by coherent  
learning supports



Learning  
Path



Students develop math drawings to show their thinking.

## Mathematical Practices

Math Sense-Making	Math Structure	Math Drawings	Math Explaining
Make sense and use of appropriate precision.	See structure and generalize.	Model and use tools.	Reason, explain, and question.
MP1 Make sense of problems and persevere in solving them. MP6 Attend to precision.	MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	MP4 Model with mathematics. MP5 Use appropriate tools strategically.	MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others.

Teachers continually assist students to do math sense-making about math structure using math drawings to support math explaining.

Teachers continually assist students to do **math sense-making** about **math structure** using **math drawings** to support **math explaining**.

**Balanced Inquiry Learning Path teaching** requires situational diagrams, drawings of numbers, and mathematically-desirable and accessible computational methods.

My many years of classroom research focused on finding, developing, and testing these in varied classrooms of students.

These all involve **learning paths** to bring students from where they start to fluency with advanced enough methods.

You can find details of all of these on my website in the Teaching Progressions and in the Classroom Videos.

**[karenfusonmath.com](http://karenfusonmath.com)**

# Teaching Progressions on karenfusionmath.com

## A Functioning Math Talk Community

### Source of Math Ideas

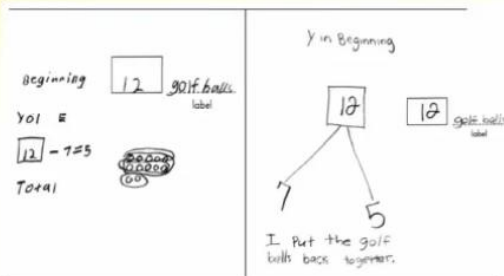
- The students contribute their ideas as the teacher or other students are teaching, confident that **their ideas are valued**.
- The students spontaneously compare and contrast and **build on ideas**.
- The teacher is **still engaged and deciding what is important** to continue exploring.

### Responsibility for Learning

- The students listen to understand, then **initiate clarifying other students' work** and ideas for themselves **and for others**.
- The students **assist each other** in understanding and correcting errors.
- The teacher **supports students** as they help one another.



Yolanda has a box of golf balls. Eddie took 7 of them. Now Yolanda has 5 left. How many golf balls did Yolanda have in the beginning?

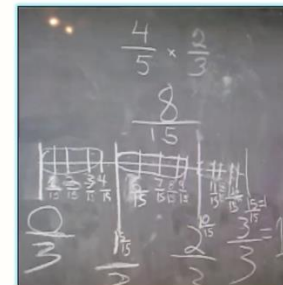
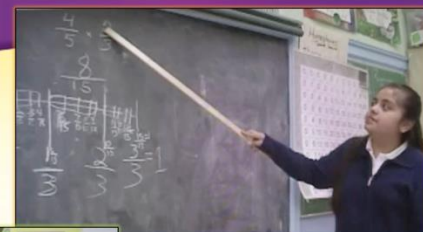


$$\begin{array}{r} 7 \text{ E} \\ + 5 \text{ Yolanda} \\ \hline 12 \text{ mall} \end{array}$$

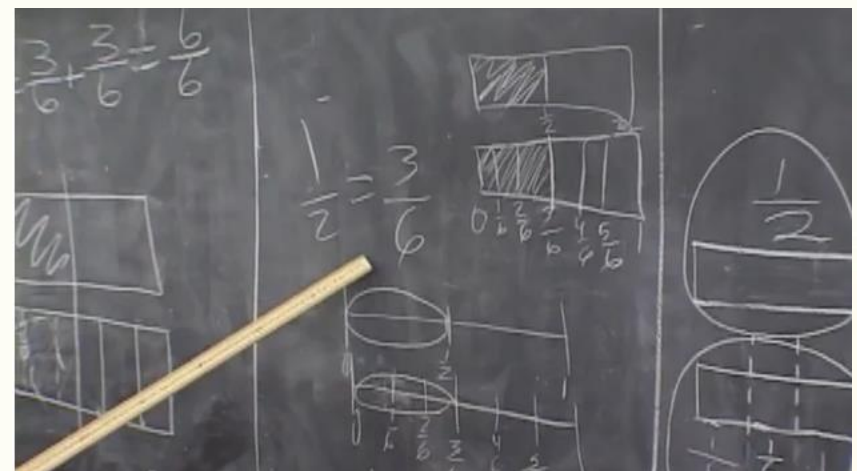
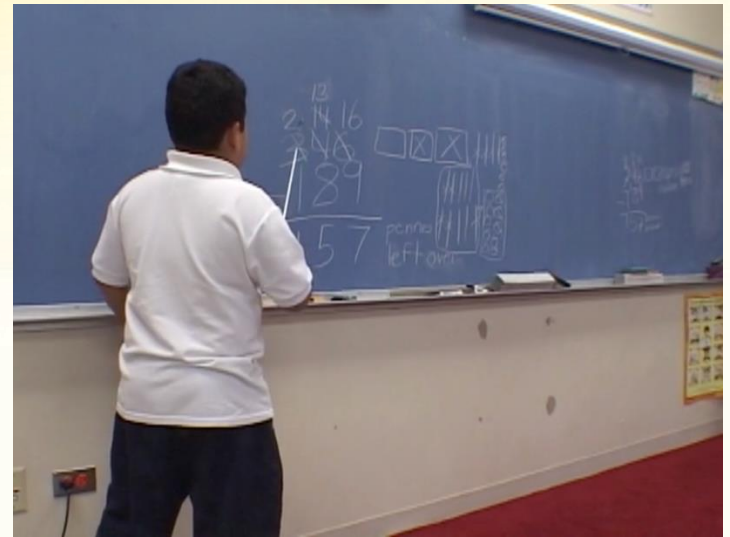
The key to solving story problems is **understanding the situation**. Students' equations often show the situation rather than the solution. Students drawings should be labeled to show which numbers or objects show which parts of the story situation.

### Math Talk Structure

- Solve
- Explain
- Question
- Justify



# Classroom Videos on [karenfusionmath.com](http://karenfusionmath.com)



The About menu on [karenfusonmath.com](http://karenfusonmath.com) gives my background and describes each of the menu choices. These include my publications and visual presentations about my research.

The Teaching Progressions and Classroom Videos show the visual representations that help students show their thinking. They have been used in my math program *Math Expressions* to show that they work, but they can be used by anyone.

**My email is [karenfuson@mac.com](mailto:karenfuson@mac.com)**



# **Expanding Number Talks to Build Equitable Math Talk Classrooms**

by Karen Fuson and Steve Leinwand

The paper above appeared in the March NCTM journal  
**Mathematics Teacher: Learning and Teaching PK to G12.**

It describes how limiting Number Talks are and how they can be extended to Math Talk in all lessons by having students use and discuss drawings and written methods.

**It discusses some kinds of drawings students can make for different math domains, so it is a good resource for you in starting your Math Talk Community.**

There is no one “standard algorithm.” There are variations in ways to record efficient, accurate, and generalizable methods that form the collection of standard algorithms.

**There are better methods;** my research is about these. These are in Classroom Videos, papers, and Teaching Progressions on my website.

These are the mathematically desirable and accessible methods that are standard algorithms.

Most taken to be “standard algorithms” are difficult or misleading.

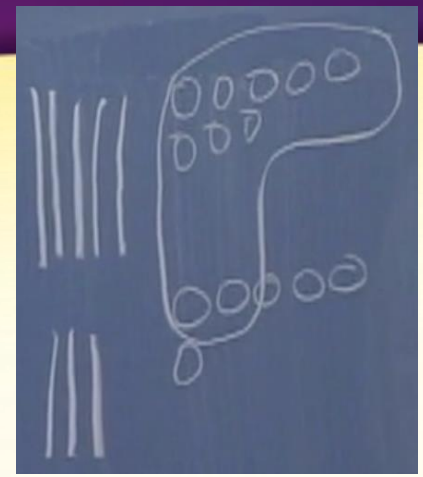
The CCSS say in the critical area for the **first year** of a given computation: “Students develop, discuss, and use **efficient, accurate, and generalizable methods.**”

They **do not say** to wait until Grade 4 to do “standard algorithms.”

More details are in the paper below (it is on my website under publications).

Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership*, 14 (2), 14-30.

# G1 New Groups Below 1.NBT.4

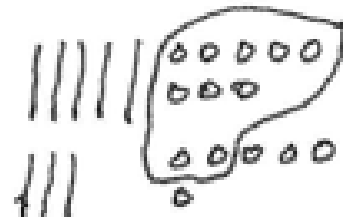


$$\begin{array}{r} 58 \\ + 36 \\ \hline 94 \end{array}$$



### New Groups Below

$$\begin{array}{r} 58 \\ + 36 \\ \hline 4 \end{array} \quad \begin{array}{r} 58 \\ + 36 \\ \hline 94 \end{array}$$



### New Groups Above

$$\begin{array}{r} 1 \\ 58 \\ + 36 \\ \hline 4 \end{array} \quad \begin{array}{r} 1 \\ 58 \\ + 36 \\ \hline 94 \end{array}$$

## Common Subtraction Error

$$\begin{array}{r} 346 \\ - 159 \\ \hline 213 \end{array}$$

## Error Created by Alternating in the Current Common Method

**Ungroup  
Tens**

$$\begin{array}{r} 316 \\ 346 \\ - 159 \\ \hline \end{array}$$

**Subtract  
Ones**

$$\begin{array}{r} 316 \\ 346 \\ - 159 \\ \hline 7 \end{array}$$

# Nonalternating Better Method

Write problem and make drawing

□ □ □    |||    ●●●●●●

$$\begin{array}{r} 346 \\ - 159 \\ \hline \end{array}$$

Ungroup 1 hundred (solving left to right)

□ □ ~~□~~    |||    ●●●●●●

$$\begin{array}{r} 214 \\ 346 \\ - 159 \\ \hline \end{array}$$

Ungroup 1 ten

□ □ ~~□~~    |||    ●●●●●●

$$\begin{array}{r} 13 \\ 214 \\ 346 \\ - 159 \\ \hline \end{array}$$

## Misleading Current Common Method

$$\begin{array}{r} 1 \leftarrow \\ 2 \\ 94 \\ \times 36 \\ \hline 564 \\ \overset{1}{2}82 \\ \hline 3384 \end{array}$$

From  $30 \times 4 = 120$ .  
The 1 is 1 hundred,  
not 1 ten.



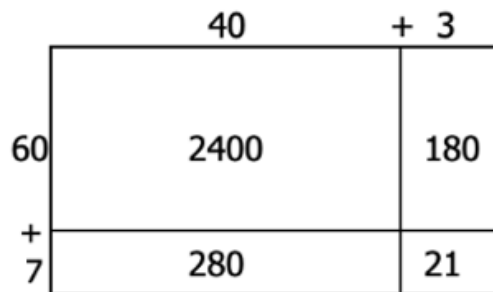
Drawn Quantity Model



Accessible Standard Algorithms

Current Common

Area Model



Place Value Sections

$$\begin{array}{r} 2400 \\ 180 \\ 280 \\ + 21 \\ \hline 2881 \end{array}$$

Expanded Notation

$$\begin{array}{l} 43 = 40 + 3 \\ \underline{\times 67} = \underline{60 + 7} \\ 60 \times 40 = 2400 \\ 60 \times 3 = 180 \\ 7 \times 40 = 280 \\ 7 \times 3 = 21 \\ \hline 2881 \end{array}$$

1-Row

$$\begin{array}{r} 1 \\ 2 \\ 43 \\ \underline{\times 67} \\ 301 \\ 258 \\ \hline 2881 \end{array}$$

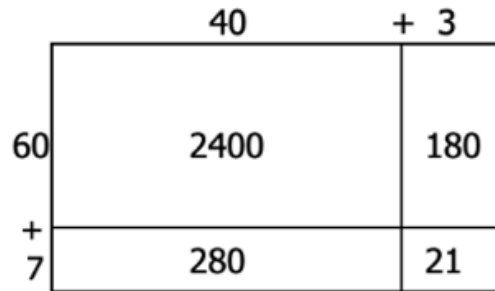
**Drawn Quantity Model**



**Accessible Standard Algorithms**

**Current Common**

**Area Model**



**Place Value Sections**

$$\begin{array}{r} 2400 \\ 180 \\ 280 \\ + 21 \\ \hline 2881 \end{array}$$

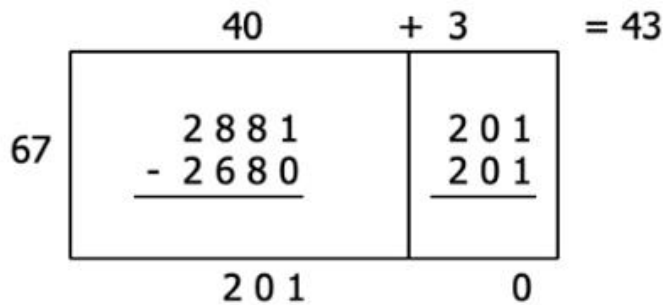
**Expanded Notation**

$$\begin{array}{l} 43 = 40 + 3 \\ \times 67 = 60 + 7 \\ \hline 60 \times 40 = 2400 \\ 60 \times 3 = 180 \\ 7 \times 40 = 280 \\ 7 \times 3 = 21 \\ \hline 2881 \end{array}$$

**1-Row**

$$\begin{array}{r} 1 \\ 2 \\ 43 \\ \times 67 \\ \hline 301 \\ 258 \\ \hline 2881 \end{array}$$

**Rectangle Sections**



**Expanded Notation**

$$\begin{array}{r} 3 \text{ ---} \\ 40 \text{ ---} \\ 67 \overline{) 2881} \\ - 2680 \\ \hline 201 \\ - 201 \\ \hline \end{array}$$

**Digit by Digit**

$$\begin{array}{r} 43 \\ 67 \overline{) 2881} \\ - 268 \\ \hline 201 \\ - 201 \\ \hline \end{array}$$

# Drawings and Written Variations of Standard Algorithms

**Quantity Model** ← → **Good Variations** **Current Common**

**New Groups Below** **Show All Totals** **New Groups Above**

$$\begin{array}{r} 100 \\ + 10 \\ \hline 110 \end{array}$$

**Left → Right** **Right → Left** **R → L Ungroup, Then Subtract**

$$\begin{array}{r} 132 \\ - 46 \\ \hline 86 \end{array}$$

**Area Model** **Place Value Sections** **Expanded Notation** **1-Row**

$$\begin{array}{r} 40 \quad + 3 \\ 60 \quad 2400 \quad 180 \\ + \\ 7 \quad 280 \quad 21 \\ \hline \end{array}$$

$$\begin{array}{r} 2400 \\ 180 \\ 280 \\ + 21 \\ \hline 2881 \end{array}$$

$$43 = 40 + 3$$

$$\begin{array}{r} 67 \\ \times 43 \\ \hline 201 \\ 2680 \\ \hline 2881 \end{array}$$

**Rectangle Sections** **Expanded Notation** **Digit by Digit**

$$\begin{array}{r} 40 \quad + 3 \\ 67 \quad 2881 \quad 201 \\ - 2680 \quad 201 \\ \hline 201 \quad 0 \end{array} = 43$$

$$67 \overline{) 2881}$$

$$\begin{array}{r} 43 \\ 67 \overline{) 2881} \\ - 2680 \\ \hline 201 \\ - 201 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ 67 \overline{) 2881} \\ - 268 \\ \hline 201 \\ - 201 \\ \hline \end{array}$$

# Remote Teaching Materials on [karenfusonmath.com](http://karenfusonmath.com)

Name: \_\_\_\_\_

Counting Mat 5 groups Unit 2 to end of unit

Thousands	Hundreds	Tens	ONES	Tenths	Hundredths	Thousandths
1,000	100	10	1	0.1	0.01	0.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

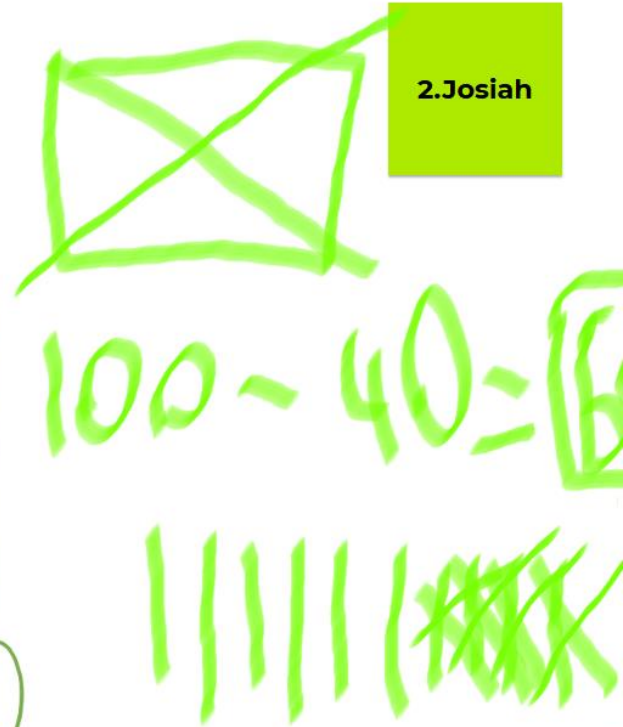
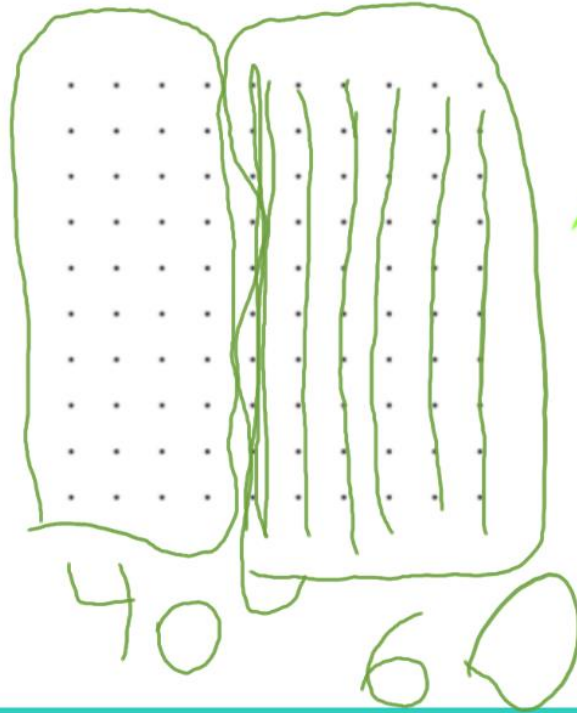
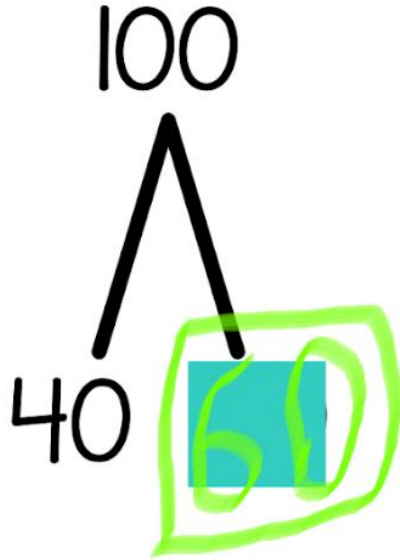
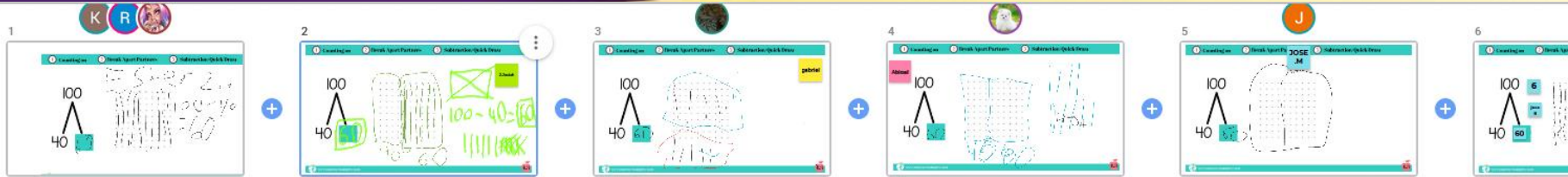
1,629.261      7,345.437      2,568.652

Handwritten addition:  $278 + 156 = 434$

Handwritten place value diagram for 100 and 10.

Number line: 31 to 70

Number line: 100 to 1000



The Digital Learning Environments are free and can be accessed on my website [karenfusonmath.com](http://karenfusonmath.com) by clicking in the top right menu choice Remote Teaching Materials.

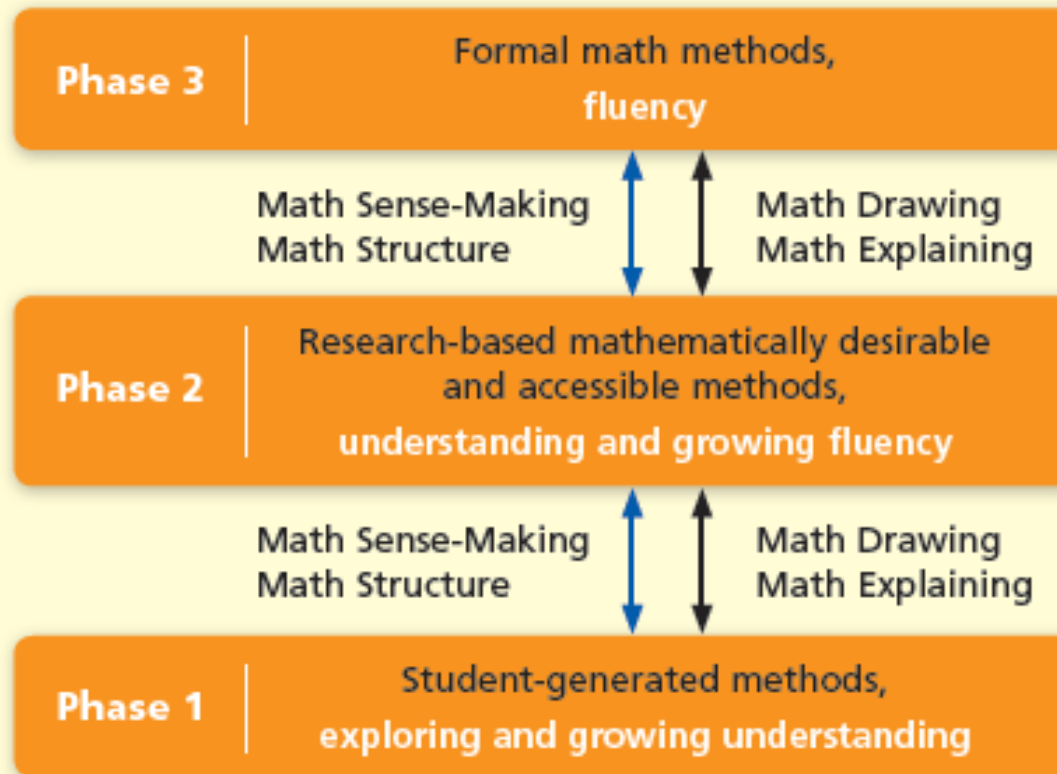
There are **Quick Practices** for grades K to 6 and **Daily Routines** for grades K to 2 that help to build and practice vital knowledge.

The Math Expressions **strategy/fluency cards** are available to use with the whole class or by individual students at home (as of December 2022).

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