

Karen C. Fuson

Research Shorter Publications for Teachers

- Fuson, K. C. (1991). Children's early counting: Saying the number-word sequence, counting objects, and understanding cardinality. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 27-39). Milton Keynes, GB: Open University Press.
- Fuson, K. C. & Kwon, Y. (1991). Chinese-based regular and European irregular systems of number words: The disadvantages for English-speaking children. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 211-226). Milton Keynes, GB: Open University Press.
- Fuson, K. C. (1992). Relationships between counting and cardinality from age 2 to age 8. In J. Bideaud, J. P. Fischer, & C. Meljac (Eds.), *Pathways to number* (pp. 127-149). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 159-179). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
- Fuson, K. C. & Kwon, Y. (1992). Learning addition and subtraction: Effects of number words and other cultural tools. In J. Bideaud & C. Meljac (Eds.), *Pathways to number* (283-306). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 351-374). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
- Fuson, K. C., & Burghardt, B. H. (1993). Group case studies of second graders inventing multidigit addition procedures for base-ten blocks and written marks. In J. R. Becker & B. J. Pence (Eds., 1993), *Proceedings of the Fifteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 240-246). The Center for Mathematics and Computer Science Education, San Jose State University, San Jose, CA.
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- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instructions: The case of mathematics. *Educational Researcher*, Vol. 25, No. 4., pp. 12-21.
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- Fuson, K. C., Lo Cicero, A., Hudson, K., & Smith, S. T. (1997). Snapshots across two years in the life of an urban Latino classroom. In Hiebert, J., Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P., *Making sense: Teaching and learning mathematics with understanding* (pp. 129-159). Portsmouth, NH: Heinemann.
- Fuson, K. C., Sherin, B., & Smith, S. T. (1998). A Vygotskian action-research model for developing and assessing conceptual models and instructional materials inter-actively. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 541-548). Raleigh, NC: North Carolina State University.
- Ron, M. P. (1998). My parent taught me this way: The European/Latino multidigit subtraction algorithm and confusions with a U.S. algorithm. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of*

algorithms in school mathematics (pp. 115-119). Reston, VA: The National Council of Teachers of Mathematics.

- De La Cruz, Y. (1999). Reversing the trend: Latino families in real partnerships with schools. *Teaching Children Mathematics*, 5 (5), pp. 296-300.
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- Fuson, K. C., De La Cruz, Y., Smith, S., Lo Cicero, A., Hudson, K., Ron, P., & Steeby, R. (2000). Blending the best of the 20th century to achieve a Mathematics Equity Pedagogy in the 21st century. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 197-212). Reston, VA: NCTM.
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- Kalchman, M. & Fuson, K. (2001). Conceptual understanding of functions: A tale of two schemas. In R. Speiser, C. S. Maher, & C. Walter (Eds., 2001), *Proceedings of the Twenty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 1* (pp. 195-205). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
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- Abrahamson, D. (2003). Text talk, body talk, table talk: A design of ratio and proportion as classroom parallel events. *Proceedings of the 27th annual meeting of the International Group for the Psychology of Mathematics Education, Honolulu, Hawaii, Vol. 2* (pp. 1-8). Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education.
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- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2015). Describing levels and components of a Math-Talk Learning Community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices* (pp. 125-134). Reston, VA: NCTM.
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- Fuson, K. C. (2020). The best multidigit computation methods: A cross-cultural cognitive, mathematical, and empirical analysis. *Universal Journal of Educational Research*, 8(4): 1299-1314, 2020 DOI: 10.13189/ujer.2020.080421
- Fuson, K. C. (2022). *Learning Path Teaching in a Math Talk Community: An overview of my research papers describing effective and nurturing ways to teach and learn in math classrooms*. Published on karenfusonmath.com.
- Fuson, K. C. & Leinwand, S. (2023). Building equitable Math Talk classrooms. *Mathematics Teacher Learning and Teaching*, 116 (3), March, 164-173. 2023 DOI: <https://doi.org/10.5951/MTLT.2022.0285>
- Fuson, K. C., Kiebler, S., & Decker, R. (April, 2024a). Accessible standard algorithms for understanding and equity. *Mathematics Teacher: Learning and Teaching K-12*. Volume 117, Issue 04, April, 268–275. DOI: 10.5951/MTLT.2023.0212
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Longer Research Publications

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- Stigler, J., Fuson, K. C., Ham, M., & Kim, M. S. (1986). An analysis of addition and subtraction word problems in Soviet and American elementary textbooks. *Cognition and Instruction*, 3, 153-171.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. [Table of Contents and Chapter 1: Introduction and overview of different uses of number words]

- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. [Chapter 11: An overview of changes in children's number word concepts from age 2 through 8]
- Fuson, K. C., Stigler, J., Bartsch, K. (1988). Grade placement of addition and subtraction topics in China, Japan, the Soviet Union, Taiwan, and the United States. *Journal for Research in Mathematics Education*, 19, 449-458.
- Fuson, K. C., & Willis, G. B. (1988). Subtracting by counting up: More evidence. *Journal for Research in Mathematics Education*, 19, 402-420.
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